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# Volume Dependence of the Energy Spectrum in Massive Quantum Field Theories

## I. Stable Particle States

M. Lüscher

Theory Division, Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52
Federal Republic of Germany

**Abstract.** Due to polarization effects, the mass $M$ of a stable particle in a quantum field theory enclosed in a large (space-like) box of size $L$ and periodic boundary conditions in general differs from its infinite volume value $m$. As $L$ increases, the finite size mass shift $\Delta m = M - m$ goes to zero exponentially with a rate, which depends on the particle considered and on the spectrum of light particles in the theory. This behaviour follows from an apparently universal asymptotic formula, already presented earlier, which relates $\Delta m$ to certain forward elastic scattering amplitudes. A detailed proof of this basic relation is given here to all orders of perturbation theory in arbitrary massive quantum field theories.

## 1. Introduction

From experience with large scale numerical simulations of lattice gauge theories over the past few years, it has become plausible that with this method a reliable calculation of the hadron masses and other low energy parameters in QCD will ultimately be possible. Because of the limited capacity of today's computer systems, the lattices one can simulate are however rather small so that, for example, a lattice of size $L = 5$ fermi and spacing $a = (2\,\text{GeV})^{-1}$ would already be considered huge by present standards. Hadrons contained in such small volumes occupy a significant fraction of the available space and one therefore expects that the calculated masses show some dependence on $L$. Thus, for the correct interpretation of the data obtained from Monte Carlo simulations, a theoretical understanding of these finite size effects is needed and studies with variable $L$ must be made to check the theoretical expectations.

Finite volume effects are also interesting in their own right and their investigation may prove useful for purposes other than merely controlling a systematic error source. The reason for this is that they probe the system at distances large compared to the lattice spacing. In general, they are therefore universal (i.e. independent of the form and magnitude of the ultra-violet cutoff) and often contain useful information on the infinite volume system. In statistical