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Modular Structure of the Local Algebras Associated with the Free Massless Scalar Field Theory

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Abstract. The modular structure of the von Neumann algebra of local observables associated with a double cone in the vacuum representation of the free massless scalar field theory of any number of dimensions is described. The modular automorphism group is induced by the unitary implementation of a family of generalized fractional linear transformations on Minkowski space and is a subgroup of the conformal group. The modular conjugation operator is the anti-unitary implementation of a product of time reversal and relativistic ray inversion. The group generated by the modular conjugation operators for the local algebras associated with the family of double cone regions is the group of proper conformal transformations. A theorem is presented asserting the unitary equivalence of local algebras associated with lightcones, double cones, and wedge regions. For the double cone algebras, this provides an explicit realization of spacelike duality and establishes the known type III_1 factor property. It is shown that the timelike duality property of the lightcone algebras does not hold for the double cone algebras. A different definition of the von Neumann algebras associated with a region is introduced which agrees with the standard one for a lightcone or a double cone region but which allows the timelike duality property for the double cone algebras. In the case of one spatial dimension, the standard local algebras associated with the double cone regions satisfy both spacelike and timelike duality.

1. Introduction

In algebraic quantum field theory, one considers a family of von Neumann algebras $R(O)$ indexed by regions O of Minkowski space M. The modular structure of a von Neumann algebra $R(O)$ consists of a one-parameter strongly continuous unitary group $\Delta^{i\lambda}$, $\lambda \in \mathbb{R}$, which acts as a KMS automorphism group on $R(O)$ and its commutant $R(O)'$, and an anti-unitary involution J_O such that $J_O R(O) J_O = R(O)'$.

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