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Black Holes in the Brans-Dicke

Theory of Gravitation

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Abstract. It is shown that a stationary space containing a black hole is a solution of the Brans-Dicke field equations if and only if it is a solution of the Einstein field equations. This implies that when the star collapses to form a black hole, it loses that fraction (about 7%) of its measured gravitational mass that arises from the scalar interaction. This mass loss is in addition to that caused by emission of scalar or tensor gravitational radiation. Another consequence is that there will not be any scalar gravitational radiation emitted when two black holes collide.

1. Introduction

In this paper I shall extend the arguments of the previous paper [1] to the Brans-Dicke theory of gravitation. Most of the results of the previous paper did not depend on the field equations in detail, but only on certain inequalities on the Ricci tensor such as

$$R_{ab} l^a l^b \geqq 0$$

for any null vector l^a. These inequalities are also satisfied in the Brans-Dicke theory if one expresses it in the conformal frame in which the gravitational constant is constant and the masses of particles vary with position. I shall call this the Einstein frame. In particular, the results that stationary black holes must be axi-symmetric and have spherical topology will hold in the Brans-Dicke theory also. It then follows that the scalar field which occurs in the Brans-Dicke theory must be constant everywhere in a stationary black hole solution. From this it follows that stationary black holes in Brans-Dicke theory are precisely the same as in general relativity and so presumably are represented by the Kerr family of solutions. What this seems to indicate is that if a massive body collapses behind an event horizon, its effect as a source of the scalar field decreases to zero. This has two important consequences. Firstly, the scalar monopole moment represents a fraction $1/(2\omega+4)$ of the measured active gravitational mass of a normal body (ω is the coupling constants