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THE YAMABE PROBLEM

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1. Introduction. Riemannian differential geometry originated in attempts to generalize the highly successful theory of compact surfaces. From the earliest days, *conformal* changes of metric (multiplication of the metric by a positive function) have played an important role in surface theory. For example, one consequence of the famous uniformization theorem of complex analysis is the fact that every surface has a conformal metric of constant (Gaussian) curvature. This provides a "standard model" for each homeomorphism class of surfaces, and reduces topological questions to differential geometric ones.

Life would be simple if the naive generalization of this theorem held in higher dimensions: every n-manifold would have a conformal metric of constant curvature, and questions in differential topology would be reduced to geometric questions about the constant-curvature models. However, it is easy to see that this cannot be true. In general the problem is highly overdetermined: the curvature tensor has on the order of n^4 independent components, while a conformal change of metric allows us to choose only one unknown function. For example, if $n \geqslant 4$, the Weyl tensor, formed from the components of the Riemannian curvature tensor, is conformally invariant and vanishes if and only if the metric is locally conformally equivalent to the Euclidean metric. From this point of view it seems natural instead to seek a

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