

## COMPLEX MANIFOLDS AND MATHEMATICAL PHYSICS

BY R. O. WELLS, JR.<sup>1</sup>

### TABLE OF CONTENTS

1. Introduction
2. The interaction of complex manifold theory with mathematical physics: A summary
3. Minkowski space
4. Twistors and the Penrose correspondence
5. Homogeneous spaces and group actions
6. Tensors and spinors
7. Maxwell's equations and the zero-rest-mass field equations
8. Cohomology and holomorphic vector bundles
9. Holomorphic representation of solutions of the zero-rest-mass field equations
10. Deriving the zero-rest-mass field equations from integral geometry.

**1. Introduction.** In the past several years there have been some remarkable links forged between two rather distinct areas of research, namely complex manifold theory on the one hand, and mathematical physics on the other. Complex manifold theory has its roots in the theory of Riemann surfaces and in algebraic geometry, and has seen significant progress in this century based on the introduction of ideas from algebraic topology, differential geometry, partial differential equations, etc. Mathematical physics has been involved in this century in the developments of relativity theory, quantum mechanics, quantum electrodynamics, and quantum field theory, to mention some major developments. Most of these disciplines are formulated in forms of field equations, i.e. partial differential equations whose solutions (under some boundary conditions) represent physical or measurable quantities. The link mentioned above between complex manifold theory and mathematical physics is that in many cases, the solutions of a given field equation can be represented entirely in terms of complex manifolds, holomorphic vector bundles, or cohomology classes on open complex manifolds with coefficients in certain holomorphic vector bundles. In simplistic terms the field equations can be reduced to the Cauchy-Riemann equations by making suitable changes in the geometric background space.

The purpose of this paper is to survey some of these interactions which

---

Received by the editors July 1, 1978.

*AMS (MOS) subject classifications* (1970). Primary 32-02, 32C35, 32L05, 32L10, 32G05, 53C65, 83C50.

<sup>1</sup>The preparation of this paper was supported by NSF MCS 75-05270 at Rice University, by the University of Paris VI, and by Oxford University.