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REPRESENTATION TYPE OF ONE POINT EXTENSIONS OF TILTED EUCLIDEAN ALGEBRAS

By

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Abstract. We know, after [P1], that, given a tame algebra Λ, the Tits form q_Λ is weakly non negative. Moreover, the converse has been shown for some families of algebras, but it is not true in general. In the same article [P1], De la Peña proved that if Λ is a tame concealed algebra, not of type $\tilde{A}_n$ and M is an indecomposable Λ-module then $\Lambda[M]$ is tame if and only if $q_{\Lambda[M]}$ is weakly non negative. The purpose of this work is to show the same result for Λ a strongly simply connected tilted algebra of euclidean type.

1. Preliminaries

Throughout this paper, k denotes an algebraically closed field. By an algebra Λ we mean a finite-dimensional, basic and connected k-algebra of the form $\Lambda \cong kQ/I$ where Q is a finite quiver and I an admissible ideal. We assume that Q has no oriented cycles. Let Λ-mod denote the category of finite-dimensional left Λ-modules, and Λ-ind a full subcategory of Λ-mod consisting of a complete set of non-isomorphic indecomposable objects of Λ-mod.

We shall use freely the known properties of the Auslander-Reiten translations, τ and τ^{-1}, and the Auslander-Reiten quiver of Λ-mod, Γ_Λ. For basic notions we refer to [R2] and [ARS]. See also [A] and [CB].

Tame algebras have the Tits form weakly non negative and for some classes of algebras, as for instance tilted or quasi-tilted algebras, this fact is determinant, that is, if Λ is tilted or quasi-tilted, then Λ is tame if and only if the Tits quadratic form is weakly non negative. Also, we have

THEOREM 1.1 (De la Peña) [P1]. *Let $\Lambda = B[M]$ be a one point extension, where B is a tame concealed algebra, not of type $\tilde{A}_n$, and M an indecomposable B-module. Then Λ is tame if and only if q_Λ is weakly non negative.*

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It is natural to ask when a similar result extends to tilted algebras. In this work we will give a partial answer, that is, we prove the following:

Let B be a strongly simply connected tilted algebra of euclidean type and M an indecomposable B-module, then the one point extension $B[M]$ is tame if and only if $q_{B[M]}$ is weakly non negative.

Modules over a one point extension $B[M]$ can be identified with triples (X, U, φ) where $X \in B$-mod, U is a k-vectorspace and $\varphi : U \to Hom(M, X)$ is k-linear.

See [R1] for other notions and notations related to vectorspace categories.

We assume that B is such that $gldim\ B \leq 2$. Then for any B-module M we have $gldim\ B[M] \leq 3$. Hence we would be able to relate the Euler and the Tits form for $A = B[M]$.

DEFINITION 1.2 [R2]. *Let C_B be the Cartan matrix of B and let x and y vectors in $\boldsymbol{K}_0(B)$. Then we have a bilinear form $\langle , \rangle = xC_B^{-T}y^T$, where the corresponding quadratic form $\chi_B(x) = \langle x, x \rangle$ is called the Euler form of B.*

DEFINITION 1.3 [Bo]. *The Tits quadratic form is given by*:

$$q_B(x_1, x_2, \ldots, x_l) = \sum_{i \in Q_0} x_i^2 - \sum_{i,j \in Q_0} x_i.x_j.dim_k\ Ext_B^1(S_i, S_j) + \sum_{i,j \in Q_0} x_i.x_j.dim_k\ Ext_B^2(S_i, S_j).$$

By [R2] the Euler form of $A = B[M]$ can be calculated in terms of χ_B: Let X be a A-module and let:

$$\underline{dim}_A(X) = \underline{dim}_B(Y) + n.\underline{dim}_A(S_e),$$

where e is the new vertex. Then

$$\chi_A(\underline{dim}\ X) = \chi_B(\underline{dim}\ Y) + n^2 - n(dim_k\ Hom_B(M, Y) - dim_k\ Ext_B^1(M, Y) + dim_k\ Ext_B^2(M, Y))$$

On the other hand, as $gldim\ B \leq 2$ then $\chi_B = q_B$, its Tits form is computed in following:

$$q_A(x_1, x_2, \ldots, x_l, n) = q_B(x_1, x_2, \ldots, x_l) + n^2 - \sum_{j \in Q_0} n.x_j(dim_k\ Ext_A^1(S_e, S_j) + dim_k\ Ext_A^1(S_j, S_e)) + \sum_{j \in Q_0} n.x_j(dim_k\ Ext_A^2(S_e, S_j) + dim_k\ Ext_A^2(S_j, S_e))$$

Comparing, we have:

PROPOSITION 1.4. *With the above notation*:

$$\chi_A(\underline{dim}\ X) = q_A(\underline{dim}\ X) - n.dim_k\ Ext_B^2(M, Y)$$

THEOREM 1.5 (De la Peña) [P1].
If B is a tame algebra, then q_B is weakly non negative.

An algebra Λ is tilted of type Δ if there exists a *tilting* module T over a path algebra $k\Delta$ such that $\Lambda = End_{k\Delta}(T)$. Tilted algebras are characterized by the existence of *complete slices* in a component of their Auslander-Reiten quiver, called the *connecting component*. The structure of the Auslander-Reiten quiver of a tilted algebra is given in [R2] and in [K]. Other facts about this subject can be seen in the survey of Assem, [A].

THEOREM 1.6 [K]. *Let B be a tilted algebra of infinite representation type. The following conditions are equivalent*:

(1) *B is tame*

(2) *χ_B is weakly non negative*

2. Modules of the Separating Tubular Family

Let us assume that B is a tilted algebra of euclidean type, and that M is an indecomposable B-module. We begin studying the case that M is not directed. We observe that 2.1 is very similar to [T], but we do not assume that B is a good algebra, but that the preinjective component of B be of tree type.

Let B be a tilted tame algebra of euclidean type with

1) the complete slice in the preinjective component.

2) the preinjective component of tree type.

Let M be an indecomposable module, in the separating tubular family.

PROPOSITION 2.1. *In the above conditions, if $B[M]$ is wild then $q_{B[M]}$ is strongly indefinite.*

To prove this proposition, we need some preliminar results, concerning derived categories. We refer to Happel ([H]) and Keller ([Ke]) for definitions and basic results.

LEMMA 2.2 [T]. *Let $B = End_A(T)$ with T an A-tilting module and $M = Hom(T, R)$ with $R \in \mathscr{G}(T)$. Then there exists a $A[R]$-tilting module T' such that $B[M] = End_{A[R]}(T')$.*

PROOF OF THE PROPOSITION. Let $B[M]$ be of wild type. Suppose that $H[R]$ is tame, in this case we have the possibilities: $H[R]$ is domestic tubular, tubular algebra or $H[R]$ is a 2-tubular algebra. But, in any case, $H[R]$ is derived tame (by [P5]) and $H[R]$ and $B[M]$ are derived equivalent (by [H], pag. 110), and so, $B[M]$ is also derived tame, and therefore tame, a contradiction. So, we have $H[R]$ wild.

Since B is tilted of euclidean type and the preinjective component of B is of tree type, H is tame, euclidean and $\tilde{A}_n$-free so, by [P1], there exist $V_1, V_2, \dots V_n$, preinjective H-modules with $q_{H[R]}(dim(\oplus V_i \oplus nS'e)) < 0$ and each $V_i \in \mathscr{G}(T)$, in this case let $W_i = Hom(T, V_i)$, W_i is a preinjective B-module that belongs to $\mathscr{Y}(T)$. So, we have: $\chi_{B[M]}(\underline{dim} \oplus W_i \oplus nSe) = \chi_B(\underline{dim} \oplus W_i) + n^2 - n\langle \underline{dim}\ M, \underline{dim} \oplus W_i \rangle_B$.

By [R2], pag. 175, there is an isometry $\sigma_T = \boldsymbol{K}_0(H) \to \boldsymbol{K}_0(B)$ such that: $\sigma_T(\underline{dim}\ V_i) = \underline{dim}\ W_i$ and $\sigma_T(\underline{dim}\ R) = \underline{dim}\ M$ so: $\chi_H(\underline{dim} \oplus V_i) = \chi_B(\underline{dim} \oplus W_i)$ and $\langle \underline{dim}\ M, \underline{dim} \oplus W_i \rangle_B = \langle \underline{dim}\ R, \underline{dim} \oplus V_i \rangle_H$ then: $\chi_{H[R]}(\underline{dim}(\oplus V_i \oplus nS'e)) = \chi_{B[M]}(\underline{dim}(\oplus W_i \oplus nSe)) < 0$ by [P1]. But $q_{B[M]}(\underline{dim}(\oplus W_i + nSe)) = \chi_{B[M]}(\underline{dim}(\oplus W_i \oplus nSe) + n\ dim_k\ Ext^2_B(M, \oplus W_i)$ and again, since $Hom(M, W_i) \neq 0\ \forall i$ and W_i is a directed module, we have: $Ext^2(M, \oplus W_i) = 0$ so $q_{B[M]}(\underline{dim}(\oplus W_i \oplus nSe)) < 0$. Clearly, $\underline{dim}(\oplus W_i \oplus nSe)$ is a vector of positive coordenates. □

We will see now that the same result see in 2.1 is true for algebras of euclidean type, with a complete slice in the postprojective component.

THEOREM 2.3. *Let B be a tilted algebra of euclidean type whose preinjective component is of tree type and let M be a indecomposable B-module in the separating tubular family such that the one-point extension $B[M]$ is wild.*

Then $q_{B[M]}$ is strongly indefinite.

PROOF. Since B is of euclidean type, either B has a complete slice in the preinjective component, and the result follows from 2.1, or B has a complete slice in the postprojective component. Let us see the case when

1) there is a complete slice of B in the postprojective component, and
2) the preinjective component of B is of tree type.

By [R2], B is a branch coextension of a tame concealed algebra B_0 and the preinjective component of B is the same preinjective component of B_0, and so B_0 is $\tilde{A}_n$-free. Assume that $B = {}_{i=1}^{t}[E_i, R_i]B_0$ where E_i is a B_0-ray module and R_i is a branch, for all i. Let us consider separately the following situations: A) $M_0 = M|_{B_0}$ is such that $M_0 = 0$;

B) $M_0 = M|_{B_0}$ is such that $M_0 \neq 0$.

In case A, *supp* M is contained in a branch R and the vectorspace category $Hom(M, B-mod)$ is the same as $Hom(M, R-mod)$. By [MP], if $Hom(M, R-mod)$ is wild then $q_{R[M]}$ is strongly indefinite. As $R[M]$ is a convex subcategory of $B[M]$, if $q_{R[M]}$ is strongly indefinite then $q_{B[M]}$ is strongly indefinite.

In case B, we can distinguish two situations:

B1: $B_0[M_0]$ is wild;

B2: $B_0[M_0]$ is tame.

We begin by B1. If $B_0[M_0]$ is wild, since the preinjective component of B is the same preinjective component of B_0, B_0 is tame concealed and $\tilde{A}_n$-free. So, by [P1], $q_{B_0[M_0]}$ is strongly indefinite. But $B_0[M_0]$ is a convex subcategory of $B[M]$ and so $q_{B[M]}$ is strongly indefinite.

Let us see B2, that is $B_0[M_0]$ is tame, but $B[M]$ wild.

Again, since $B_0[M_0]$ is tame, we have two possibilities:

B2.1 M_0 is a ray module.

B2.2 M_0 is a module of regular length two in the tube of rank $n-2$ and B_0 is tame concealed of type $\tilde{D}_n$. In the case B.2.1, we have that if M is a ray module over B, by [R2] 4.5 and 4.6, the component $\mathscr{T}[M]$ is a standard inserted-co-inserted tube. Moreover, all indecomposable projectives of $B[M]$ lie in $\mathscr{P}$, the postprojective component, or on $\mathscr{T}[M]$ (where is the unique projective that is outside of $\mathscr{P}$) therefore, $B[M]$ is an algebra with acceptable projectives (see [PT]) and in this case, $B[M]$, it is wild if and only if $q_{B[M]}$ is strongly indefinite. On the other hand, if $M = M_0$ and therefore, M is a ray module over B_0, then $B[M] = B[M_0]$ is an iterated tubular algebra and in this case, $B[M]$ is tame, a contradiction. So, we can assume that M is not a ray module over B and moreover that $M \neq M_0$ and, therefore, that there exists an indecomposable injective I in $\mathscr{T}$, the tube where M lies, such that $Hom(M, I) \neq 0$ and that there are two arrows starting in M. Also, we can assume that i, the coextension vertex belongs to *supp* M, so that there exists a morphism $M \to I_i$.

Let E be the ray module which is the root of the branch.

Let $B_i = [E]B_0$ and $M_i = M|_{B_i}$. Then we have: $Hom_{B_i}(M_i, M_0) \neq 0$, but $Hom_{B_i}(M_0, M_i) = 0$, and again we have two cases:

B.2.1.1 The branch is co-inserted in E, $E \neq M_0$;

B.2.1.2 The branch is co-inserted in $E = M_0$.

In the first case, since M is not a ray module over B, we can assume that there exists an arrow that start in M and points to the mouth of the tube, say $M \to Y$. Moreover, by [[R2], 4.5] there exists a sectional path $M \to M_t \to M_{t-1} \to \cdots M_0$ that does not contain injectives. So, we can consider that all of these modules $\tau^{-1}M_i$, and in particular $\tau^{-1}M_1$, are non zero.

Since M_0 is a B_0-ray module, then $\tau^{-1}M_1$ cannot be a B_0-module. But in this case, it is a co-ray module and therefore M_0 is a co-ray module, contradiction. So, the situation B.2.1.1 does not occur.

If the branch is co-inserted in $E = M_0$, $M_0 = M|_{B_0}$, M is not a ray module. Again, we can assume that there exists an arrow starting in M and pointing to the mouth of the tube. Moreover, since the branch is co-inserted in M_0, there is a sectional path $M \to I$ the injective of the co-insertion. Let us look at the category $Hom(M, B - mod)$. This category has three pieces. Since B is tilted, $Hom(M, X) \neq 0$ only for modules X that are preinjective or in the same tube $\mathcal{T}$ where M lies. Let X be a B_0-module. Since M is a co-inserted module, $Hom_B(M, X) \neq 0$ and, hence, $Hom_{B_0}(M_0, X) \neq 0$. Since B_0 is a tame concealed algebra and M_0 is a ray module over B_0, $Hom(M, B - mod)$ contains the following subcategories: the ray of $\mathcal{T}$ that starts in M_0, $Hom(M_0, \mathcal{I}(B_0)$ where $\mathcal{I}(B_0)$ is the preinjective component of B_0 and the subcategory given by the sucessors of M in the tube, that are not B_0-modules. Since $B_0[M_0]$ is tame, $Hom(M_0, \mathcal{I}(B_0))$ is given by some of the patterns given in [[R1], pag. 254]. Let us assume that one of the following two situations occur:

Either M is injective and so the vectorspace category restricted to the tube is given by two sectional paths: one, finite, pointing to the mouth of the tube and one, infinite, (the ray) or M is not injective but the vectorspace category restricted to the tube is given by two parallel paths. We will see that in this situation, since $B_0[M_0]$ is tame, $B[M]$ is tame, in contradiction to the hypothesis, because $A = B[M]$ is a coil enlargement of B_0, by [AS] because $A^+ = B_0[M_0]$, $A^- = B$, are both tame. As that $A = B[M]$ is tame.

Let us assume then that M is not injective and that there exists a sectional path $M \to Y_t$ with $t \geq 1$. In first place, we observe that $Hom_B(Y_i, X) = 0$ for all preinjective X. But Y_i being on the coray, and to the right of M_0, there does not exist an infinite path coming out of it, and similarly $Hom(\tau^{-1}M, X) = 0$ for all preinjective X.

In particular, $Hom(Y_i, X) = Hom(\tau^{-1}M, X) = 0$ for all X such that $Hom(M_0, X) \neq 0$ with X in the preinjective component. Moreover $Hom(Y_i, \tau^{-1}M) = 0 = Hom(\tau^{-1}M, Y_j)$ for $\forall j \geq 1$. Hence, by [[R1] (3.1)] we can find one of the following path-incomparable (see [Ch]) subcategories in $\mathcal{I}(B_0)$, with the only exception of the case $(\tilde{D}_n, n-2) : \boldsymbol{K}_1 = \{A, B, C\}$, (in cases: $(\tilde{D}_4, 1)$, $(\tilde{D}_6, 2)$, $(\tilde{D}_7, 2)$, $(\tilde{D}_8, 2)$, $(\tilde{E}_6, 2)$, $(\tilde{E}_7, 3)$, $(\tilde{E}_7, 4)$, $(\tilde{E}_8, 5)$ and $\boldsymbol{K}_2 = \{A, B \to C\}$ in cases $(\tilde{D}_5, 2)$ and $(\tilde{E}_6, 3)$. So, in each case, adding the objects Y_1, $\tau^{-1}M$ to the categories $\boldsymbol{K}_1$ or $\boldsymbol{K}_2$ we have that $Hom(M, B - mod)$ is wild and that $q_{B[M]}$ is strongly indefinite.

Let us calculate the quadratic form for the case $(\tilde{D}_5, 2)$, the other cases are similar. Let $\tilde{L}$ be the B-module $\tilde{L} = 2Y_1 \oplus 2\tau^{-1}M \oplus 2A \oplus B \oplus C$ and $L = \tilde{L} \oplus 4S_e$, then $q_{B[M]}(\underline{dim}\ L) = \chi_{B[M]}(\underline{dim}\ L) + 4\ dim_k\ Ext^2(M, \tilde{L}) = \chi_{B[M]}(\underline{dim}\ L) = \chi_{B[M]}(\underline{dim}\ \tilde{L}) + 4^2 - 4(8) = 15 + 16 - 32 = -1$. Let us see the case $(\tilde{D}_n, n-2)$. In this case, the pattern is given by:

$$\begin{tikzcd}
 & & & \overset{A}{\circ} \arrow[dr] & & \\
\circ \arrow[r] & \cdots \circ \arrow[r] & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[r] & \cdots \\
 & & & \underset{B}{\circ} \arrow[ur] & &
\end{tikzcd}$$

If $t > 1$, considering that $K = \{A, B, \tau^{-1}M,\ Y_1 \to Y_2\}$ is wild, again the quadratic form is strongly indefinite. On the other hand, if $t = 1$ we have two possibilities:

Case 1

$$\begin{tikzcd}
 & & Y_1 & & \\
 & Y_0 \arrow[ur] \arrow[dr] & & & \\
M \arrow[ur] \arrow[dr] & & \tau^{-1}M \arrow[dr] & & \\
 & Z_1 \arrow[ur] \arrow[dr] & & \tau^{-1}Z_1 & \\
 & & Z_2 \arrow[ur] \arrow[dr] & & \\
 & & & \cdots & \\
 & & & I \arrow[dr] & \\
 & & & & M_0
\end{tikzcd}$$

and case 2

$$\begin{tikzcd}
 & & Y_1 \\
 & Y_0 \arrow[ur] \arrow[dr] & \\
M \arrow[ur] \arrow[dr] & & \tau^{-1}M \\
 & I \arrow[ur] \arrow[dr] & \\
 & & M_0
\end{tikzcd}$$

In case 1, we can again consider the wild subcategory $\{Y_1, \tau^{-1}M \to \tau^{-1}Z_1, A, B\}$ and the quadratic form is strongly indefinite. On the other hand, in case 2, we have a vectorspace category which is in fact tame, by Nazarova Theorem, so that $B[M]$ is tame.

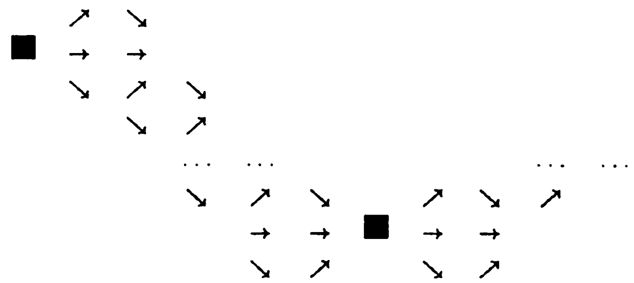
Let us examine now B.2.2, M_0 is a module of regular length 2 in a tube of rank $n-2$ and B_0 is tame concealed of type $\tilde{D}_n$. If $M = M_0$ lies in a stable tube, then $Hom(M, B - mod) = Hom(M_0, B_0 - mod)$ and therefore both are tame or wild simultaneosly. So, we can assume that M belongs to a co-inserted tube. Since M_0 has regular length 2, there exist E_1 and E_0 ray-modules over B_0 such that $\tau E_0 = E_1 \to M_0 \to E_0$ is the ARS for E_0. Let $E_0, E_1, \ldots E_{n-3}$ be the ray-modules over B_0 of the tube where M lies. Again, we divide in possibilities.

B.2.2.1 The branch is co-inserted in E_0.

B.2.2.2 The branch is co-inserted in E_1.

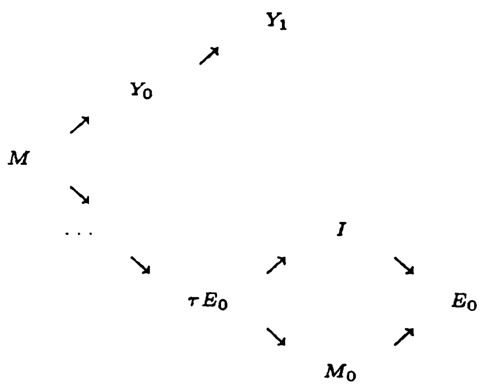
B.2.2.3 The branch is co-inserted in E_j for $j \neq 0$ or 1.

Let us observe that if $M = M_0$, then $Hom(M, B - mod)$ has the same pattern as $Hom(M_0, B_0 - mod)$. If M is a B_0-module, then $Hom_B(M, N) \neq 0$ for modules N in the same tube as M or for modules N in the preinjective component. Hence, being $Hom(M, N) = Hom(M_0, N_0)$ it has the following pattern



which is tame, by [R1]. (In this picture we indicate the non zero modules in the category with ■ indicating the objects of dimension 2.) We can assume that M belongs to the co-ray and that there exists an injective I in the tube $\mathscr{T}$ such that $Hom(M, I) \neq 0$.

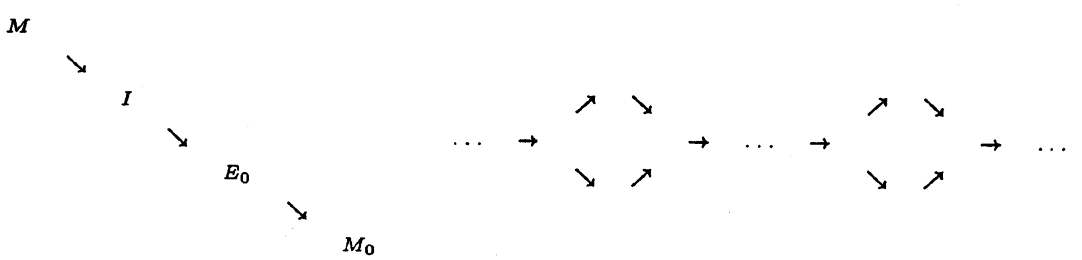
Let us consider B.2.2.1. We have a co-inserted branch in E_0, and



If there exists a sectional path $M \to Y_0 \to Y_1$, then, $Hom(M, Y_1) \neq 0$. Let us observe that $Y_1|_{B_0} = 0$ and $Hom(Y_1, X) = 0$ for all preinjective module X and in particular, $Hom(Y_1, X_i) = 0$ for each of the preinjective $X_i's$ such that

$Hom(M_0, X_i)$ has dimension 2. Hence $q_{B[M]}$ is strongly indefinite. Let us assume that the longest sectional path starting at M in the direction of the mouth of the tube has length 1. In this case, again, $Hom(M, B-mod)$ has the same pattern than $Hom(M_0, B_0-mod)$ and so it is tame.

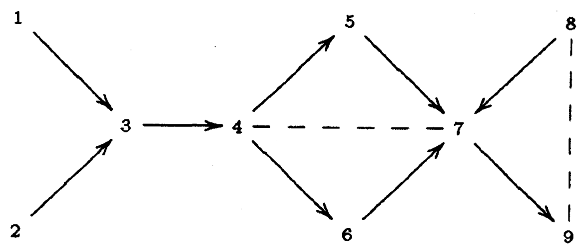
Let us consider B.2.2.2. Since $Hom(E_1, E_0) = 0$, the morphisms from M to X, for X preinjective, are just the ones that factor through the successor of M_0, M_1, and those that factor through E_0 are equal to zero and the vectorspace category $Hom(M, B-mod)$ is of the form:



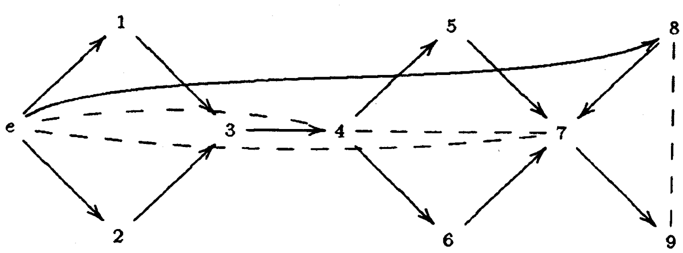
and we can repeat the arguments of the case B.2.1.2.

Finally, let us look at B.2.2.3. The branch is inserted in E_j with $j \neq 0$ or 1. But, in this case, $M = M_0$, $Hom(M_0, I) = 0$ for any I injective in $\mathscr{T}$ and we fall again in a already examined case. □

Example 2.4. Let us see an example.
Let B be given by:



B is tilted of type $\tilde{D}_8$, with a complete slice in the postprojective component. Let us consider M_1 a module of the separating tubular family, such that the ordinary quiver of $\Lambda_1 = B[M_1]$, is given below. Then Λ_1 is wild and $q_{\Lambda_1}(I_3 \oplus I_3 \oplus I_8 \oplus 2S_e) = -1$.



3. Directed Modules

PROPOSITION 3.1. *Let B be a tilted algebra of euclidean type, with the postprojective component of tree type and M an indecomposable B-module in this component. Then, if $B[M]$ is wild, the Tits form $q_{B[M]}$ is strongly indefinite.*

PROOF. Since B is of euclidean type we have two possibilities

1) B has a complete slice in the preinjective component, or
2) B has a complete slice in the postprojective component.

In the first case, all injectives are in the preinjective component, so for any I such that $Hom(M, I) \neq 0$, M and I are separated by a separating tubular family and the result follows from [PT].

In case 2 all projectives are in the postprojective component.

Let us consider $\mathscr{C}'$ the component in the Auslander-Reiten quiver of $B[M]$ that contains the new projective module P_e, we will see that $\mathscr{C}'$ is a π-component (as in [Co]). For this, it is enough to prove that $l(Hom(\_, B[M]) < \infty$, but as $B[M] = B \oplus P_e$ and the number of indecomposable modules that are predecessors of $B[M]$ is finite, so, $\mathscr{C}'$ is a π-component. Again two situations can occur:

1) The new simple injective I_e belongs to $\mathscr{C}'$, or
2) The new simple injective I_e does not belong to $\mathscr{C}'$.

Recall that the $B[M]$-indecomposable injectives are of the form $\bar{I}_i = (I_i, Hom(M, I_i), id.)$ when $Hom(M, I_i) \neq 0$, $(I_i, 0, 0)$ when $Hom(M, I_i) = 0$, where I_i are the indecomposable injectives of B and the new injective I_e is equal to $(0, k, 0)$.

Let us consider 1), so $I_e \in \mathscr{C}'$, again by [Co], since $\mathscr{C}'$ contains a projective module then $l(Hom(\_, I_e)) < \infty$. But in this case the number of $B[M]$-modules that are not B-modules is finite and so $B[M]$ is tame.

Let us consider 2). The new injective I_e does not belong to $\mathscr{C}'$. If no other injective belongs to $\mathscr{C}'$, by [Co] $\mathscr{C}'$ is a postprojective component that contains all projectives and no injectives. In this case $B[M]$ is a tilted algebra and the representation type is given by the corresponding quadratic form. Let us see that no injective belongs to $\mathscr{C}'$. Let I be a B-indecomposable injective, if $Hom(M, I) \neq 0$, there exists a non zero morphism $(I, 0, 0) \to (I, Hom(M, I), id.)$ Consider P the B-indecomposable projective associated to I, then $(P, 0, 0)$ is the $B[M]$-projective associated to $(I, Hom(M, I), id.)$ and $Hom((P, 0, 0), (I, 0, 0)) \neq 0$. As in B-mod, P and I are in different components, there exists infinite B-modules X_i such that $Hom(X_i, I) \neq 0$ but in this case, $Hom_{B[M]}((X_i, 0, 0), (I, 0, 0)) \neq 0$ for infinite modules, a contradiction to the fact that $(l(Hom(\_,(I,0,0)) < \infty$. So $\mathscr{C}$ does not contain any injective. $\square$

We have been assuming that some of the directed components of B are of tree type. In general these hypothesis does not imply that the algebra is a good algebra or is strongly simply connected (see [S3] for definitions). But for tilted tame algebras, this is the case.

THEOREM 3.2 [ALP]. *Let B be a tame tilted algebra. Then B is strongly simply connected if and only if the orbit quiver of each directed component of $\Gamma(mod\ B)$ is a tree.*

COROLLARY 3.3. *Let B be a strongly simply connected tilted algebra of euclidean type and M an indecomposable B-module. If $B[M]$ is wild then $q_{B[M]}$ is strongly indefinite.*

PROOF. If M is a postprojective module, we have the result by 3.1. If M is a module of the tubular family, the result follows by 2.3. Let us assume that M is preinjective. If B has a complete slice in the postprojective component the result follows from [P1]. Let us assume that B has a complete slice in the preinjective component, we are going to use the same argument used by De la Peña in [P4]. Let $\mathscr{S}(M \to) = \{Y \in B - mod$ such that there exist a sectional path $M \to Y\}$ and let P_e denote the new projective in $B[M]$. Let us call $\mathscr{S} = \mathscr{S}(M \to) \cup \{P_e\}$. Then $\mathscr{S}$ is a slice (in general not complete) in $B[M]$, and we can consider C the full subcategory of $B[M]$ determined by the vertices i such that $Y(i) \neq 0$ for $Y \in \mathscr{S}$. In this case, C is a convex subcategory of $B[M]$, and $\mathscr{S}$ is a complete slice in C, so C is tilted. Moreover all $B[M]$-modules are B-modules or are C-modules. If $B[M]$ is wild, then C is wild, and as C is convex in $B[M]$ $q_{B[M]}$ is strongly indefinite. $\square$

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