

On Realizations of Families of Strongly Pseudo-Convex CR Structures

Kimio MIYAJIMA

Kagoshima University

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Introduction.

Let (V, o) be a normal isolated singularity and M its link (i.e. a cut locus with a small sphere in an ambient space). It was proven in [M4] that, if $\dim_{\mathbb{C}}(V, o) \geq 4$ and $\text{depth}(V, o) \geq 3$ then the Kuranishi family of CR structures on M (constructed in [AK2]) induces the versal family of (V, o) (cf. [G]).

However, in spite of some interesting non-versal families of isolated singularities (e.g. modular deformations [K-S], [P]) or non-Kuranishi families of CR structures (e.g. [A-M2]), the argument in [M4] hardly enables us to compare non-Kuranishi families of CR structures with families of isolated singularities.

In this paper, we will consider realizations of families of strongly pseudo-convex CR structures as families of real hypersurfaces of complex manifolds (i.e. a relative version of [O]). A family of (1,1)-convex-concave ambient manifolds of the family of CR manifolds can be completed to a flat family of normal isolated singularities, if $\text{depth}(V, o) \geq 3$, by A. Fujiki's unpublished work [F].

Let M be a strongly pseudo-convex real hypersurface of a complex manifold X , the complex structure of X induces a CR structure $CTM \cap T''X|_M$ on M , we will denote it by ${}^{\circ}T''$ and fix a (non-canonical) splitting $CTM = {}^{\circ}T'' + {}^{\circ}T' + CF$, where ${}^{\circ}T' = {}^{\circ}\bar{T}''$ and F is a real line subbundle of TM . Denote $A_b^{0,q}({}^{\circ}T' + CF) := \Gamma(M, ({}^{\circ}T' + CF) \otimes \Lambda^q({}^{\circ}T'')^*)$ and denote by $\mathcal{A}_k^{0,q}({}^{\circ}T' + CF)$ its completion with respect to the k -th order Sobolev norm. The following notion of families of CR structures is an analogy of ones of complex structures: Fix an integer $k \geq n+2$ and let $(S, o) \subset (\mathbb{C}^m, 0)$ be a germ of complex space defined by an ideal $\mathcal{I}_{S,o} \subset \mathbb{C}\{s_1, s_2, \dots, s_m\}$. A family of deformations of CR structures on M parametrized by (S, o) is a

$$\omega(s) \in \mathcal{A}_k^{0,1}({}^{\circ}T' + CF)\{s_1, s_2, \dots, s_m\} \cap A_b^{0,1}({}^{\circ}T' + CF)[[s_1, s_2, \dots, s_m]]$$

satisfying $\omega(0)=0$ and

$$P_b(\omega(s)) \equiv 0 \pmod{\mathcal{I}_{S,o} \mathcal{A}_{k-1}^{0,2}({}^\circ T' + CF)\{s_1, s_2, \dots, s_m\}},$$

where $P_b(\omega)$ is the integrability condition (cf. §2 below).

Then our main theorem is

MAIN THEOREM. *Let M be a strongly pseudo-convex real hypersurface of a complex manifold X with $\dim_{\mathbb{C}} X \geq 4$. Fix an interger $k \geq n+2$. If a family $\omega(s)$ of deformations of CR structures on M is in $A_b^{0,1}({}^\circ T')[[s_1, s_2, \dots, s_m]]$, then there exists a family $\mathcal{W} \rightarrow (S, o)$ of complex manifolds with a family of CR-embeddings of Sobolev $(k+1/2)$ -class $G: M \times (S, o) \rightarrow \mathcal{W}$ with respect to $\omega(s)$ over (S, o) .*

The assumption $\omega(s) \in A_b^{0,1}({}^\circ T')[[s_1, s_2, \dots, s_m]]$ means that $\omega(s)$ is a family of CR structures fixing the original contact structure $\mathcal{R}_e({}^\circ T' + {}^\circ T'')$. And, in view of deformations of isolated singularities, this assumption is not restrictive, because any family of isolated singularities corresponds to some family of CR structures fixing the contact structure (cf. [Ak2], [A-M1]).

Let $\pi: \mathcal{U} \rightarrow (T, o)$ be the (formally) versal family of deformations of complex manifolds near M , constructed in [M4]. Then the main theorem will be proven by constructing a holomorphic map $\tau: (S, o) \rightarrow (T, o)$ and a family of CR-embeddings $G: M \times (S, o) \rightarrow \mathcal{U} \times_{(T,o)} (S, o)$ with respect to $\omega(s)$. We will construct the maps τ and G by formal extensions to infinitesimal neighbourhoods of (S, o) . Vanishings of obstructions to formal extensions follow from the fact that the (non-canonical) restriction map $\check{H}^q(\Omega, \Theta) \rightarrow H_{\check{D}_b}^q(M, {}^\circ T' + CF)$ is isomorphism at $q=1, 2$, where Ω is a $(1, 1)$ -convex-concave neighbourhood of M in X and $\check{H}^q(\Omega, \Theta)$ denotes the q -th Čech cohomology group with coefficients in the sheaf of germs of holomorphic vector field Θ . In this argument, we need not the assumption $\omega(s) \in A_b^{0,1}({}^\circ T')[[s_1, s_2, \dots, s_m]]$. However we need this restriction for the convergence procedure, because the Sobolev estimate $\|\mathcal{D}_b N_b X f\|_k \leq c \|f\|_k$ does not hold unless $X \in {}^\circ T'' + {}^\circ T'$.

As a corollary of (the proof of) the main theorem, we have that the versal family of strongly pseudo-convex CR structures on M is determined uniquely in the following sense.

COROLLARY 5.1. *Let M be as in the main theorem. Then any versal family of CR structures on M (in the sense of Kuranishi) is realized as a real hypersurface of the (formally) versal family of deformations of X near M . In particular, their parameter spaces coincide with each other.*

The arrangement of this paper is as follows. We will define the concept of families of CR structures in §§1 and 2. In §3, we will recall the construction of the (formally) versal family of tubular neighbourhoods of M in [M4]. The proof of the main theorem will depend deeply on this construction, and it will be given in §4. In §5, we will prove Corollary 5.1.

Throughout this paper, as parameter spaces of families, we will consider germs of not necessarily reduced complex analytic spaces. About basic concepts, refer to [T] for holomorphic vector bundles on a CR manifold, to [Ak1] for deformations of CR structures, and to [Ak2] (resp. [Ak3]) for the new harmonic analysis associated to the construction of the Kuranishi family of strongly pseudo-convex CR structures (resp. of complex structures on a (1,1)-convex-concave domain). In more general situation dealt with in [M3], the method of this paper works well and the main theorem will be valid.

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§1. CR structures.

Let M be a real C^∞ manifold with $\dim_{\mathbb{R}} M = 2n - 1$. An almost CR structure on M is a subbundle E of CTM satisfying

$$(1.1) \quad E \cap \bar{E} = \{0\} \quad \text{and} \quad \text{rank}_{\mathbb{C}}(CTM/(E + \bar{E})) = 1.$$

We call E a CR structure if, moreover, it satisfies

$$(1.2) \quad [u, v] \in \Gamma(M, E) \quad \text{for any} \quad u, v \in \Gamma(M, E).$$

A real manifold M with a CR structure E is called a CR manifold (M, E) . A differentiable embedding g of M into a complex manifold X is called a CR embedding of (M, E) into X if it satisfies $dg(E) = dg(CTM) \cap T''X$.

Any differentiable embedding into a complex manifold as a real hypersurface naturally induces a CR structure with respect to which the embedding is a CR-one. Conversely, [O] proved that any C^∞ compact strongly pseudo-convex CR manifold (M, E) with $\dim_{\mathbb{R}} M \geq 5$ has a CR embedding into a complex manifold as a real hypersurface. Here let us define “strongly pseudo-convex”: If we choose a non-vanishing real one-form θ (called a contact form) annihilating $E + \bar{E}$, the Levi form is defined as the Hermitian form on E by $\langle X, Y \rangle := -\sqrt{-1} \theta([X, \bar{Y}])$. (The Levi form is defined up to a non-vanishing real factor. So the rank and the signature depend only on the CR structure E .) The CR structure is called strongly pseudo-convex if the Levi form has a definite sign at each point of M .

Throughout this paper, we will assume that M is a strongly pseudo-convex compact real hypersurface of a complex manifold X and ${}^\circ T'' = CTM \cap (T''X|_M)$ is a naturally induced CR structure on M .

On a CR manifold $(M, {}^\circ T'')$, We have the tangential Cauchy-Riemann operator $\bar{\partial}_b : \mathcal{C}^\infty(M) \rightarrow A_b^{0,1} := \Gamma(M, ({}^\circ T'')^*)$ given by $\bar{\partial}_b f(\bar{X}) = \bar{X}f$ for $\bar{X} \in \Gamma(M, {}^\circ T'')$.

A holomorphic vector bundle on a CR manifold M is a complex vector bundle V with a differential operator $\bar{\partial}_b : \Gamma(M, V) \rightarrow A_b^{0,1}(V) := \Gamma(M, V \otimes ({}^\circ T'')^*)$ satisfying

$$(1.3) \quad \bar{\partial}_b(fs) = (\bar{\partial}_b f) \otimes s + f(\bar{\partial}_b s) \quad \text{for } f \in \mathcal{C}^\infty(M) \text{ and } s \in \Gamma(M, V),$$

and

$$(1.4) \quad \bar{\partial}_b \bar{\partial}_b = 0,$$

where $\bar{\partial}_b : A_b^{0,1}(V) \rightarrow A_b^{0,2}(V)$ is the differential operator naturally induced from the above $\bar{\partial}_b$.

REMARK 1.1. If \tilde{V} is a complex analytic vector bundle on X and $V = \tilde{V}|_M$ then V is a holomorphic vector bundle on a CR manifold M by

$$(1.5) \quad \bar{\partial}_b s = (\bar{\partial} \tilde{s})|_{T''} \quad \text{where } \tilde{s} \in \Gamma(X, \tilde{V}) \text{ is an extension of } s.$$

Let ${}^\circ T' = {}^\circ \bar{T}''$ and F be a real line subbundle of TM such that $CF \simeq CTM / ({}^\circ T'' + {}^\circ T')$. Fix a splitting $CTM = {}^\circ T'' + {}^\circ T' + CF$ as differentiable vector bundles and denote ${}^\circ T' + CF$ by T' . Throughout this paper, we will denote by subscripts ${}^\circ T''$, ${}^\circ T'$, CF and T' the projection onto them respectively.

REMARK 1.2 (cf. [Ak1] and [T]). T' is a holomorphic vector bundle on M by $(\bar{\partial}_b s)(u) = [s, u]_{T'}$ for $s \in \Gamma(M, T')$ and $u \in \Gamma(M, {}^\circ T'')$. The projection $\rho'|_{T'} : T' \rightarrow T'X|_M$ induces an isomorphism of holomorphic vector bundles $(T', \bar{\partial}_b) \simeq (T'X|_M, \bar{\partial}_b)$ where $(T'X|_M, \bar{\partial}_b)$ denotes the holomorphic vector bundle induced from the complex analytic vector bundle $T'X$ on X as in Remark 1.1.

For a holomorphic vector bundle V we denote $A_b^{0,q}(V) := \Gamma(M, V \otimes \wedge^q ({}^\circ T'')^*)$, then we have a differential complex

$$(1.6) \quad 0 \longrightarrow A_b^{0,0}(V) \xrightarrow{\bar{\partial}_b} A_b^{0,1}(V) \xrightarrow{\bar{\partial}_b} A_b^{0,2}(V) \xrightarrow{\bar{\partial}_b} \cdots \xrightarrow{\bar{\partial}_b} A_b^{0,n-1}(V) \longrightarrow 0$$

defined from the above $\bar{\partial}_b$ satisfying (1.3) and (1.4) by the usual way (cf. [T] where he denote $\bar{\partial}_V$ instead of $\bar{\partial}_b$).

If we fix an Hermitian metric on M (i.e. a Riemannian metric which is hermitian in the fibres of ${}^\circ T'' + {}^\circ T'$) and an Hermitian inner product along fibres of V , then we have the formal adjoint operator $\mathfrak{D}_b : A_b^{0,q+1}(V) \rightarrow A_b^{0,q}(V)$ of $\bar{\partial}_b : A_b^{0,q}(V) \rightarrow A_b^{0,q+1}(V)$.

THEOREM 1.1 (cf. [F-K]). For $1 \leq q \leq n-2$,

- (1) $H_b^q = \{\psi \in A_b^{0,q}(V) \mid \square_b \psi = 0\}$ is finite dimensional where $\square_b = \bar{\partial}_b \mathfrak{D}_b + \mathfrak{D}_b \bar{\partial}_b$,
- (2) there exists a linear map (so called the tangential Neumann operator) $N_b : A_b^{0,q}(V) \rightarrow A_b^{0,q}(V)$ such that $\square_b N_b \psi = \psi - H_b \psi$, $\square_b N_b = N_b \square_b$, $N_b H_b = H_b N_b = 0$, where H_b denotes the L^2 -orthogonal projection $A_b^{0,q}(V) \rightarrow H_b^q$,
- (3) $\|\mathfrak{D}_b N_b \psi\|_{s+1/2} \leq C_s \|\mathfrak{D}_b N_b \psi\|'_s \leq C'_s \|\psi\|_s$ for $\psi \in A_b^{0,q}(V)$, where $\|\cdot\|_{s+1/2}$ (resp. $\|\cdot\|'_s$) denotes the Sobolev norm of order $s+1/2$ (resp. the Folland-Stein norm of order $(s+1)$) and C_s and C'_s are positive constants independent of ψ .

§2. Families of CR structures.

Let M be a C^∞ compact real hypersurface of a complex manifold X . We call an almost CR structure E on M is at a finite distance from ${}^\circ T''$ if $p_1|_E: E \rightarrow {}^\circ T''$ is an isomorphism where p_1 denotes the projection $CTM \rightarrow {}^\circ T''$ with respect to the fixed splitting $CTM = {}^\circ T'' + {}^\circ T' + CF$.

PROPOSITION 2.1 ([Ak1] Proposition 1.1). *If E is an almost CR structure on M at a finite distance from ${}^\circ T''$, then there exists a unique $\omega \in A_b^{0,1}(M, T')$ such that $E = \{u - \omega(u) \mid u \in {}^\circ T''\}$.*

We denote $\{u - \omega(u) \mid u \in {}^\circ T''\}$ by ${}^\omega T''$.

PROPOSITION 2.2 ([Ak1] Theorem 2.1). *An almost CR structure ${}^\omega T''$ is a CR structure if and only if ω satisfies a non-linear partial differential equation $P_b(\omega) = \bar{\partial}_b \omega + R_2(\omega) + R_3(\omega) = 0$, where $\bar{\partial}_b \omega$, $R_2(\omega)$ and $R_3(\omega)$ are all in $A_b^{0,2}(T')$ given by, for $u, v \in {}^\circ T''$,*

$$(2.1) \quad \bar{\partial}_b \omega(u, v) = [u, \omega(v)]_{T'} - [v, \omega(u)]_{T'} - \omega([u, v]_{\circ T''}),$$

$$(2.2) \quad R_2(\omega)(u, v) = -[\omega(u), \omega(v)]_{T'} + \omega([u, \omega(v)]_{\circ T''} + [\omega(u), v]_{\circ T''})$$

and

$$(2.3) \quad R_3(\omega)(u, v) = -\omega([\omega(u), \omega(v)]_{\circ T''}).$$

Let (S, o) be a germ of an analytic set in (\mathbb{C}^m, o) defined by an ideal $\mathcal{I}_{S,o} \subset \mathbb{C}\{s_1, \dots, s_m\}$. We use the abbreviation (s) instead of (s_1, \dots, s_m) .

Fix an integer $k \geq n+2$. And denote by $\mathcal{A}_k^{0,q}(T')$ the completion of $A_b^{0,q}(T')$ with respect to the k -th order Sobolev norm. Then the following definition of families of CR structures on M is an analogy of the one of families of complex structures.

DEFINITION 2.1. By a family of deformations of CR structures on M parametrized by (S, o) we mean a

$$\omega(s) \in \mathcal{A}_k^{0,1}(T')\{s_1, \dots, s_m\} \cap A_b^{0,1}(T')[[s_1, \dots, s_m]]$$

satisfying

- (1) $\omega(0) = 0$,
- (2) $P_b(\omega(s)) \in \mathcal{I}_{S,o} \mathcal{A}_{k-1}^{0,2}\{s_1, \dots, s_m\}$.

Next we will consider a realization of a family of CR structures. Let (S, o) and $\omega(s)$ be as above. And let $\pi: \mathcal{X} \rightarrow (S, o)$ be a family of complex manifolds. We may assume that the complex space \mathcal{X} is defined by a collection $\{F_{ij}(\zeta_j, s)\}_{i,j \in A}$ satisfying (3.10) and (3.11) below.

Let $f_{ij}(z_j) = F_{ij}(z_j, 0)$ and $\Lambda_0 = \{i \mid U_i \cap M \neq \emptyset\}$.

DEFINITION 2.2. By a CR embedding $g: M \times (S, o) \rightarrow \mathcal{X}$ with respect to a family

$\omega(s)$, we mean a collection $\{g_i(s)\}_{i \in A_0}$ such that, if we denote $g_i(s) = \sum_{\alpha=1}^n g_i^\alpha(s)(\partial/\partial z_i^\alpha)$,

- (1) $g_i(s) \in \mathcal{A}_k^{0,0}(U_i, T'X|_M)\{s_1, \dots, s_m\} \cap A_b^{0,0}(U_i, T'X|_M)[[s_1, \dots, s_m]]$,
- (2) $g_i^\alpha(0) = z^\alpha \quad (\alpha = 1, \dots, n)$,
- (3) $g_i^\alpha(s) - F_{ij}^\alpha(g_j(s), s) \in \mathcal{F}_{S,0} \mathcal{A}_k^{0,0}(U_i, 1)\{s_1, \dots, s_m\} \quad (\alpha = 1, \dots, n)$,
- (4) $(\bar{\partial}_b - \omega(s))g_i^\alpha(s) \in \mathcal{F}_{S,0} \mathcal{A}_k^{0,1}(U_i, 1)\{s_1, \dots, s_m\} \quad (\alpha = 1, \dots, n)$,

where we denote by $\mathcal{A}_k^{0,0}(U_i, T'X|_M)$ the completion of $A^{0,0}(U_i, T'X|_M)$ with respect to the $(k+1)$ -th order Folland-Stein norm $\|\cdot\|'_k$.

§3. Deformations of (1,1)-convex-concave domains near a real hypersurface.

Let X be an ambient complex manifold of M with the following condition: There exists a smooth strictly plurisubharmonic function $\Phi: X \rightarrow (a_*, b^*)$ ($-\infty \leq a_* < 0 < b^* \leq +\infty$) such that $d\Phi \neq 0$ on X , $M = \{x \in X \mid \Phi(x) = 0\}$, and $\Omega_{a,b} = \{x \in X \mid a < \Phi(x) < b\}$ is relatively compact for any $a_* < a < b < b^*$.

The following analysis on the deformation complex $(A^{0,q}(\bar{\Omega}, T'X), \bar{\partial})$ is due to T. Akahori (cf. [Ak3] and [Ak4]), where Ω denotes $\Omega_{a,b}$ for some $a_* < a < b < b^*$. Let

$$\mathcal{E}^q = \left\{ \varphi \in A^{0,q}(\bar{\Omega}, T'X) \mid r\varphi \in A_b^{0,q}(\partial\Omega, T'X|_{\partial\Omega} \cap CT\partial\Omega) \text{ and } r\bar{\partial}\varphi \in A_b^{0,q+1}(\partial\Omega, T'X|_{\partial\Omega} \cap CT\partial\Omega) \right\},$$

where $r: A^{0,q}(\bar{\Omega}, T'X) \rightarrow A_b^{0,q}(\partial\Omega, T'X|_{\partial\Omega})$ is the natural restriction map given by $r\varphi(u_1, \dots, u_q) = \varphi(u_1, \dots, u_q)$ for $u_1, \dots, u_q \in \Gamma(\partial\Omega, T''X|_{\partial\Omega} \cap CT\partial\Omega)$. Then we have a subcomplex $(\mathcal{E}^q, \bar{\partial})$ of $(A^{0,q}(\bar{\Omega}, T'X), \bar{\partial})$.

PROPOSITION 3.1 (cf. [Ak3] Theorem 3.4).

- (1) $H_{\bar{\partial}}^1(\mathcal{E}^0) \rightarrow H_{\bar{\partial}}^1(\bar{\Omega}, T'X)$ is surjective,
- (2) $H_{\bar{\partial}}^q(\mathcal{E}^0) \rightarrow H_{\bar{\partial}}^q(\bar{\Omega}, T'X)$ is an isomorphism for $2 \leq q \leq n$.

Next, we fix an hermitian metric on X which equals to the Levi metric (an Hermitian metric which restricted to ${}^\circ T'' + {}^\circ T'$ coincides with the Levi form) near $\partial\Omega$ and let

$$\mathcal{D}^q(\mathcal{E}) = \left\{ \varphi \in \mathcal{E}^q \mid \langle \sigma(\vartheta, d\Phi)\varphi, \psi \rangle = 0 \text{ on } \partial\Omega \text{ for any } \psi \in \mathcal{E}^{q-1} \right\}.$$

PROPOSITION 3.2 (cf. [Ak3] Theorem 5.1, [Ak4] Theorem 4.1 and [M2] Main Theorem). For $2 \leq q \leq n-2$,

- (1) $H^q(\mathcal{E}) = \{\varphi \in \mathcal{D}^q(\mathcal{E}) \mid \bar{\partial}\varphi = \vartheta\varphi = 0\}$ is finite dimensional,
- (2) there exists a linear map $\mathcal{N}: A^{0,q}(\bar{\Omega}, T'X) \rightarrow A^{0,q}(\bar{\Omega}, T'X)$ such that $\varphi = \square \mathcal{N}\varphi + \mathcal{H}\varphi$, where \mathcal{H} denotes the orthogonal projection onto $H^q(\mathcal{E})$,
- (3) $\mathcal{N}\mathcal{H} = \mathcal{H}\mathcal{N} = 0$,
- (4) $\bar{\partial}\mathcal{N}\varphi \in \mathcal{D}^{q+1}(\mathcal{E})$ and $\vartheta\mathcal{N}\varphi \in \mathcal{E}^{q-1}$ if $\varphi \in \mathcal{E}^q$.

REMARK 3.1. From (2) and (4) of Proposition 3.2, we infer that the harmonic space $H^q(\mathcal{E})$ represents $H_{\bar{\partial}}^q(\mathcal{E}')$ for $2 \leq q \leq n-2$.

We will recall the outline of the construction of the family of tubular neighbourhoods of M in [M4] Theorem 1. We fix $a_* < a_1 < a < 0 < b < b_1 < b^*$ and an hermitian metric on X which is real analytic on a neighbourhood of $\bar{\Omega}_{a,b}$ and equals to the Levi metric near $\partial\Omega_{a_1,b_1}$. We denote $\Omega = \Omega_{a,b}$ and $\Omega_1 = \Omega_{a_1,b_1}$. Let $\| \cdot \|_{(0,k)}$ and $\| \cdot \|'_{(0,k)}$ be the norms on $A^{0,q}(\bar{\Omega}_1, T'X)$ introduced in [Ak3].

By the method in §7 of [Ak3], for a fixed $k > n+2$, we obtained a convergent powerseries $\varphi(t) \in \mathcal{A}_{(0,k)}^{0,1}\{t_1, \dots, t_r\} \cap \mathcal{E}^1[[t_1, \dots, t_r]]$ and an analytic space T having the following properties, where $\mathcal{A}_{(0,k)}^{0,1}$ denotes the completion of $A^{0,1}(\bar{\Omega}_1, T'X)$ with respect to $\| \cdot \|'_{(0,k)}$ -norm and $r = \dim_{\mathbb{C}} H^1(\Omega_1, T'X)$:

- (3.1) $\varphi(t)$ is real analytic on a neighbourhood of $\bar{\Omega} \times o$,
- (3.2) $\varphi(0) = 0$,
- (3.3) if $\varphi_1(t)$ denotes the linear term of $\varphi(t)$, then $\bar{\partial}\varphi_1(t) = 0$ and $[\varphi_1(t)]$ spans the first cohomology space $H_{\bar{\partial}}^1(\Omega_1, T'X)$,
- (3.4) $T = h^{-1}(0)$ where h is a complex analytic map from a neighbourhood W of $0 \in \mathbb{C}^r$ into $H^2(\mathcal{E})$ given by $h(t) = \mathcal{H}P(\varphi(t))$, where $P(\varphi(t)) = \bar{\partial}\varphi(t) - \frac{1}{2}[\varphi(t), \varphi(t)]$,
- (3.5) $P(\varphi(t)) \in \mathcal{I}_{T,o} \mathcal{A}_{(0,k)}^{0,2}\{t_1, \dots, t_r\}$, where $\mathcal{A}_{(0,k)}^{0,2}$ denotes the completion of $A^{0,2}(\bar{\Omega}, T'X)$ with respect to $\| \cdot \|_{(0,k)}$ -norm (cf. [M4] Proposition 2.1).

By (3.1)~(3.5), we have a smooth map $\pi : \mathcal{X} \rightarrow (T, o)$ such that

- (3.6) $\pi^{-1}(o)$ is a neighbourhood of $\bar{\Omega}$ in X ,
- (3.7) the Kodaira-Spencer map $\rho : T_o T \rightarrow H^1(\Omega, \Theta)$ of this family is an isomorphism.

Precisely speaking, we have a collection $\{U_i, \tilde{z}_i(t)\}_{i \in A}$ of real analytic complex charts of a neighbourhood of $\bar{\Omega}$ and a collection $\{F_{ij}(\zeta_j, t)\}_{i,j \in A}$ of transition functions such that

- (3.8) $\{U_i\}_{i \in A}$ is a finite open covering of a neighbourhood of $\bar{\Omega}$,
- (3.9) $\tilde{z}_i = (\tilde{z}_i^1, \dots, \tilde{z}_i^n) : U_i \times D \rightarrow \mathbb{C}^n$ is a real analytic map depending complex analytically on $t = (t_1, \dots, t_r)$, where D denotes a neighbourhood of 0 in \mathbb{C}^r .
- (3.10) $F_{ij} : \mathbb{C}^n \times D \supset W_{ij} \rightarrow W_{ji} \subset \mathbb{C}^n \times D$ is a complex analytic isomorphism between open sets W_{ij} and W_{ji} ,

if we denote $F_{ij}^\alpha(\zeta_j, t) = \zeta_i^\alpha \circ F_{ij}(\zeta_j, t)$ ($\alpha = 1, \dots, n$) then we have

$$(3.11) \quad F_{ij}^\alpha(F_{jk}(\zeta_k, t)) - F_{ik}^\alpha(\zeta_k, t) \in \Gamma(W_{ij} \cap F_{ij}^{-1}(W_{jk}) \cap W_{ik}, \mathcal{I}_T \otimes_{\mathcal{O}_D} \mathcal{O}_{\mathbb{C}^n \times D}),$$

and

$$(3.12) \quad t^\lambda \circ F_{ij}(\zeta_j, t) = t^\lambda \quad \text{for } \lambda = 1, \dots, r,$$

$$(3.13) \quad \tilde{z}_i^\alpha(t) - F_{ij}^\alpha(\tilde{z}_j(t), t) \in \Gamma(U_i \cap U_j, \mathcal{I}_{T,o} \otimes_{\mathcal{O}_D} \mathcal{C}_{X \times D}^\omega) \text{ where } \mathcal{C}_{X \times D}^\omega \text{ denotes the sheaf of germs of real analytic functions on } X \times D \text{ which are complex analytic in } t \in D.$$

$$(3.14) \quad (\bar{\partial} - \varphi(t))\tilde{z}_i^\alpha(t) \in \Gamma(U_i, \mathcal{I}_{T,o} \otimes_{\mathcal{O}_D} a_\omega^{0,1}) \text{ where } a_\omega^{0,1} \text{ denotes the sheaf of germs of real analytic } (0, 1)\text{-forms along fibres of } X \times D \rightarrow D \text{ depending complex analytically on } t.$$

§4. Proof of Main Theorem.

First of all, we discuss the relation between the Čech cohomologies and the Dolbeault cohomologies over a CR manifold.

Let V be a holomorphic vector bundle on a CR manifold M and denote by $\mathcal{O}_{CR}(V)$ the sheaf of germs of CR sections of V . Since $\bar{\partial}_b$ -Poincaré lemma holds for $1 \leq q \leq n-2$ (cf. [A-H1] Theorem 2 and [A-H2] Theorem 3), we have the CR version of Dolbeault isomorphism, for $q \leq n-2$,

$$H^q(M, \mathcal{O}_{CR}(V)) \ (\simeq \varinjlim \check{H}^q(\mathcal{U} \cap M, \mathcal{O}_{CR}(V))) \simeq H_{\bar{\partial}_b}^q(M, V) \ (= Z_b^q(V)/B_b^q(V))$$

where $Z_b^q(V) = \{\varphi \in A_b^{0,q}(M, V) \mid \bar{\partial}_b \varphi = 0\}$ and $B_b^q(V) = \bar{\partial}_b A_b^{0,q-1}(M, V)$.

Let V be a restriction on M of a complex analytic vector bundle \tilde{V} on X and $\mathcal{U} = \{U_i\}_{i \in \Lambda}$ a Leray covering of Ω . By restricting the correspondence under Dolbeault isomorphism onto M , we have

PROPOSITION 4.1. *We have a commutative diagram of cohomologies;*

$$\begin{array}{ccc} \check{H}^q(\mathcal{U}, \mathcal{O}(\tilde{V})) & \xrightarrow{\alpha^q} & H_{\bar{\partial}_b}^q(\Omega, \tilde{V}) \\ \downarrow \tilde{r}^q & & \downarrow r^q \\ \check{H}^q(\mathcal{U} \cap M, \mathcal{O}_{CR}(V)) & \xrightarrow{\beta^q} & H_{\bar{\partial}_b}^q(M, V) \quad (1 \leq q \leq n-2). \end{array}$$

It is well known that α^q is an isomorphism but β^q is not in general. β^1 is injective. By the same arguments in pp. 81 and 82 of [Y], we have

PROPOSITION 4.2. *r^q is an isomorphism for $1 \leq q \leq n-2$.*

PROOF OF MAIN THEOREM. Let X and Φ be as in §3 and $M = \{x \in X \mid \Phi(x) = 0\}$. Let $\pi: \mathcal{X} \rightarrow (T, o)$ be the family of deformations of complex manifolds near M obtained in [M4], Theorem 1 (cf. also §3) and $(S, o) \subset (\mathbb{C}^m, 0)$ be a germ of analytic space defined by an ideal $\mathcal{I}_{S,o} \subset \mathbb{C}\{s_1, \dots, s_m\}$. We denote the maximal ideal of $\mathbb{C}\{s_1, \dots, s_m\}$ by m . We may assume that the complex space \mathcal{X} is given by a collection of transition functions

$\{F_{ij}(\zeta_j, t)\}_{i,j \in A}$ with a collection of families of complex charts $\{U_i, \tilde{z}_i(t)\}_{i \in A}$ satisfying (3.8)–(3.14). Let $A_0 = \{i \mid U_i \cap M \neq \emptyset\}$ and set $f_{ij}(z_j) = F_{ij}(z_j, 0)$.

We will construct sequences $\{g_i^\mu(s)\}_{i \in A_0}$ and $\tau^\mu(s)$ for $\mu = 0, 1, \dots$ satisfying the following, where

$$g_i^\mu(s) = \sum_{\alpha=1}^n (g_i^\alpha)^{\mu}(z_i, s) \frac{\partial}{\partial z_i^\alpha} \in A_b^{0,0}(T'X|_M)[[s_1, s_2, \dots, s_m]] \quad \text{and}$$

$$\tau^\mu(s) \in \mathbb{C}^r [[s_1, s_2, \dots, s_m]] :$$

$$(4.1) \quad (g_i^0)^\alpha(z_i) = z_i^\alpha \quad (\alpha = 1, \dots, n) \quad \text{and} \quad \tau^0 = 0,$$

$$(4.2)_\mu \quad g_i^\mu(s) \equiv g_i^{\mu-1}(s) \pmod{m^\mu} \quad \text{and} \quad \tau^\mu(s) \equiv \tau^{\mu-1}(s) \pmod{m^\mu},$$

$$(4.3)_\mu \quad (g_i^\alpha)^\mu(f_{ij}(z_j), s) - F_{ij}^\alpha(g_j^\mu(z_j), s), \tau^\mu(s) \equiv 0 \pmod{(m^{\mu+1} + \mathcal{I}_{s,o}) \otimes A_b^{0,0}(U_i \cap U_j \cap M, 1)[[s_1, \dots, s_m]]} \quad (\alpha = 1, \dots, n),$$

$$(4.4)_\mu \quad (\bar{\partial}_b - \omega(s))(g_i^\alpha)^\mu(z_i, s) \equiv 0 \pmod{(m^{\mu+1} + \mathcal{I}_{s,o}) \otimes A_b^{0,1}(U_i \cap M, 1)[[s_1, \dots, s_m]]} \quad (\alpha = 1, \dots, n),$$

$$(4.5)_\mu \quad h_\lambda(\tau^\mu(s)) \equiv 0 \pmod{(m^{\mu+1} + \mathcal{I}_{s,o})} \quad (\lambda = 1, \dots, l).$$

For $\mu = 0$, because of (4.1), we set $(g_i^0)^\alpha(z_i) = z_i^\alpha$, $\tau^0 = 0$.

Suppose that $\{g_i^{\mu-1}(s)\}_{i \in A_0}$ and $\tau^{\mu-1}(s)$ are determined for some $\mu \geq 1$.

The proof will rely heavily on Grauert division theorem (cf. [G]). It provides a way specifying a representative of an element of (convergent) power series ring modulo an ideal. We call the specific representative the canonical modulus.

Let $\sigma_{ij|\mu}(s)$ be the canonical modulus of

$$g_i^{\mu-1}(s) - F_{ij}(g_j^{\mu-1}(s), \tau^{\mu-1}(s)) \pmod{(m^{\mu+1} + \mathcal{I}_{s,o}) \otimes A_b^{0,0}(U_i \cap U_j \cap M, T'X|_X)[[s_1, s_2, \dots, s_m]]},$$

and $\xi_{i|\mu}(s)$ the canonical modulus of $(\bar{\partial}_b - \omega(s))g_i^{\mu-1}(s)$

$$\pmod{(m^{\mu+1} + \mathcal{I}_{s,o}) \otimes A_b^{0,1}(U_i \cap M, T'X|_X)[[s_1, s_2, \dots, s_m]]}.$$

Note that $\sigma_{ij|\mu}(s)$ and $\xi_{i|\mu}(s)$ are homogeneous of order μ , by (4.3) $_{\mu-1}$ and (4.4) $_{\mu-1}$ respectively. Next let

$$g_i^{\alpha\mu}(s) = \sum_{j \in A_0} \sum_{\beta=1}^n \rho_j (\partial f_{ij} / \partial z_j^\beta) (\sigma_{ji|\mu}(s))^\beta. \quad (\alpha = 1, \dots, n),$$

where $\{\rho_j\}_{j \in A_0}$ is a partition of unity subordinate to $\mathcal{U} \cap M = \{U_j \cap M\}_{j \in A_0}$.

We note that $\{\xi_{i|\mu}(s)\}$ and $\{\bar{\partial}_b g_{i|\mu}(s)\}$ are in $A_b^{0,1}(M, T'X|_M)[[s_1, s_2, \dots, s_m]]$, by (4.3) $_{\mu-1}$ and (4.4) $_{\mu-1}$, and denote them by $\xi_\mu(s)$ and $\xi'_\mu(s)$ respectively. Let

$$\left\{ \Theta_{ij}^\sigma = \sum_{\alpha=1}^n \frac{\partial F_{ij}^\alpha}{\partial t^\sigma} \frac{\partial}{\partial z_i^\alpha} \right\} \in Z^1(\mathcal{U}, \Theta_X) \quad (\sigma = 1, \dots, r)$$

be a 1-cocycle and $\tau_\mu^\sigma(s)$ ($\sigma = 1, \dots, r$) homogeneous polynomials of s of order μ given by

$$\beta^1 \circ \tilde{r}^1 \left(\left[\sum_{\sigma=1}^r \Theta_{ij}^\sigma \tau_\mu^\sigma(s) \right] \right) = [H_b(\xi_\mu(s) + \xi'_\mu(s))].$$

Let

$$g''_{i|\mu}(s) = - \sum_{j \in \Lambda_0} \rho_j \left(\sum_{\sigma=1}^r \Theta_{ji}^\sigma \tau_\mu^\sigma(s) \right) \Big|_{U_i \cap M},$$

then $\{\bar{\partial}_b g''_{i|\mu}(s)\} \in A_b^{0,1}(M, T'X|_M)[[s_1, s_2, \dots, s_m]]$ and denote it by $\xi''_\mu(s)$. Let

$$g'''_\mu(s) = -\partial_b N_b(\xi_\mu(s) + \xi'_\mu(s) + \xi''_\mu(s)).$$

Finally we set

$$g_i^\mu(s) = g_i^{\mu-1}(s) + g'_{i|\mu}(s) + g''_{i|\mu}(s) + g'''_\mu(s) \Big|_{U_i \cap M}$$

and

$$\tau^\mu(s) = \tau^{\mu-1}(s) + \tau_\mu(s).$$

We will prove that $\{g_i^\mu(s)\}_{i \in \Lambda_0}$ and $\{\tau^\mu(s)\}$ satisfy (4.3) $_\mu \sim$ (4.5) $_\mu$.

PROOF OF (4.3) $_\mu$: By the same calculations of [A-M1] Lemma 3.2, it follows from (4.3) $_{\mu-1}$ and Proposition 4.4 (remark that Proposition 4.4 follows only from (4.3) $_{\mu-1} \sim$ (4.5) $_{\mu-1}$)

$$\sigma_{ij|\mu}^\alpha(s) - \sigma_{ik|\mu}^\alpha(s) + \sum_{\beta=1}^n \frac{\partial f_{ij}^\alpha}{\partial z_j^\beta} \sigma_{jk|\mu}^\beta(s) = 0 \quad (\alpha = 1, 2, \dots, n).$$

Since

$$\begin{aligned} & (g_i^\mu)^\alpha(s) - F_{ij}^\alpha(g_j^\mu(s), \tau^\mu(s)) \equiv \sigma_{ij|\mu}^\alpha(s) + (g'_{i|\mu})^\alpha(s) \\ & - \sum_{\beta=1}^n \frac{\partial f_{ij}^\alpha}{\partial z_j^\beta} (g'_{j|\mu})^\beta(s) + (g'_{i|\mu})^\alpha(s) - \sum_{\beta=1}^n \frac{\partial f_{ij}^\alpha}{\partial z_j^\beta} (g''_{j|\mu})^\beta(s) - \sum_{\sigma=1}^r \frac{\partial F_{ij}^\alpha}{\partial t^\sigma} \tau_\mu^\sigma(s) \\ & \text{modulo } (m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,0}(U_i \cap U_j \cap M, TX|_X)[[s_1, s_2, \dots, s_m]], \end{aligned}$$

(4.3) $_\mu$ follows from the definition of $\{g'_{i|\mu}(s)\}_{i \in \Lambda_0}$ and $\{g''_{i|\mu}(s)\}$.

PROOF OF (4.4) $_\mu$: Since $\bar{\partial}_b \xi_\mu(s) = \bar{\partial}_b \xi'_\mu(s) = 0$ (cf. Sublemma 4.6 below) and β^1 is injective, we infer from the definition of $\{g''_{i|\mu}(s)\}_{i \in \Lambda_0}$ that $H_b \xi''_\mu(s) = -H_b(\xi_\mu(s) + \xi'_\mu(s))$ holds. Therefore, since

$$\begin{aligned} & (\bar{\partial}_b - \omega(s))g_i^\mu(s) \equiv \xi_\mu(s) + \xi'_\mu(s) + \xi''_\mu(s) + \bar{\partial}_b g'''_\mu(s) \\ & \text{modulo } (m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,1}(M, T'X|_X)[[s_1, s_2, \dots, s_m]], \end{aligned}$$

(4.4) $_\mu$ follows from the definition of $g'''_\mu(s)$.

PROOF OF (4.5) _{μ} : Well will show

$$h_\lambda(\tau^{\mu-1}(s)) \equiv 0 \pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes \mathcal{C}[[s_1, \dots, s_m]]} \quad (\lambda = 1, \dots, l).$$

Let $p_\mu(s)$ be the canonical modulus of

$$P(\varphi(\tau^{\mu-1}(s))) \pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A^{0,2}(\bar{\Omega}_1, T'X)[[s_1, \dots, s_m]]}.$$

LEMMA 4.3. (1) $\bar{\partial}p_\mu(s) = 0$,

(2) $p_\mu(s) \in \mathcal{E}^2[[s_1, \dots, s_m]]$.

PROOF. (1) $\bar{\partial}P(\varphi(\tau^{\mu-1}(s))) = -[\bar{\partial}\varphi(\tau^{\mu-1}(s)), \varphi(\tau^{\mu-1}(s))]$
 $= -[P(\varphi(\tau^{\mu-1}(s))), \varphi(\tau^{\mu-1}(s))] \equiv 0$

$\pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A^{0,3}(\bar{\Omega}_1, T'X)[[s_1, \dots, s_m]]}$, by (3.5) and (4.5) _{$\mu-1$} .

(2) Since $\varphi(t) \in \mathcal{E}^1[[t_1, \dots, t_r]]$, by [M4] Sublemma 2 and [M1] Proposition 1.1, $rP(\varphi(t)) \in A_b^{0,2}(\partial\Omega_1, T'X|_{\partial\Omega_1} \cap CT\partial\Omega_1)[[t_1, \dots, t_r]]$. Hence we have $rp_\mu(s) \in A_b^{0,2}(\partial\Omega_1, T'X|_{\partial\Omega_1} \cap CT\partial\Omega_1)[[s_1, \dots, s_m]]$. Then $\bar{\partial}_b rp_\mu(s) \in A_b^{0,3}(\partial\Omega_1, T'X|_{\partial\Omega_1} \cap CT\partial\Omega_1)[[s_1, \dots, s_m]]$ follows from (1). Q.E.D.

It is enough to show

PROPOSITION 4.4. $r^2[p_\mu(s)|_{\bar{\Omega}}] = 0$.

In fact, by Proposition 4.2, $[p_\mu(s)|_{\bar{\Omega}}] = 0$ follows from Proposition 4.4. Since the restriction map $H_{\bar{\partial}}^2(\bar{\Omega}_1, T'X) \rightarrow H_{\bar{\partial}}^2(\bar{\Omega}, T'X)$ is an isomorphism by [H] Theorem 3.4.8, we have $[p_\mu(s)] = 0$ in $H_{\bar{\partial}}^2(\bar{\Omega}_1, T'X)$. This implies $[p_\mu(s)] = 0$ in $H_{\bar{\partial}}^2(\mathcal{E}')$ by Proposition 3.1. Therefore $\mathcal{H}p_\mu(s) = 0$ follows from Remark 3.1.

Now we will prove Proposition 4.4. Let $\Xi_{ijk|\mu}(\zeta_k, s)$ be the canonical modulus of

$$F_{ik}(\zeta_k, \tau^{\mu-1}(s)) - F_{ij}(F_{jk}(\zeta_k, \tau^{\mu-1}(s)), \tau^{\mu-1}(s)) \pmod{(m^{\mu-1} + \mathcal{I}_{S,o})}$$

in $C^0(U_i \cap U_j \cap U_k, \mathcal{O}_\Omega)[[s_1, \dots, s_m]]$. Then $\Xi_{ijk|\mu}(s)$ is a homogeneous polynomial of s of order μ and satisfies

$$\Xi_{ij|\mu}^\alpha(s) - \Xi_{ik|\mu}^\alpha(s) + \sum_{\beta=1}^n \frac{\partial f_{ij}^\alpha}{\partial z_j^\beta} \Xi_{jk|\mu}^\beta(s) = 0 \quad (\alpha = 1, 2, \dots, n).$$

By Proposition 4.1, Proposition 4.4 follows from

LEMMA 4.5. (1) $\beta^2 \circ \check{r}^2([\Xi_{ijk|\mu}(s)]) = 0$,

(2) $\alpha^2([\Xi_{ijk|\mu}(s)]) = -[p_\mu(s)|_{\bar{\Omega}}]$.

PROOF OF LEMMA 4.5 (1). The following Sublemma 4.6 implies Lemma 4.5 (1).

SUBLEMMA 4.6. (1) $\bar{\partial}_b \xi_{i|\mu}(s) = 0$,

(2) $\bar{\partial}_b \sigma_{ij|\mu}(s) = -\xi_{j|\mu}(s) + \xi_{i|\mu}(s)$,

(3) $(\Xi_{ijk|\mu}(s))|_M = \sigma_{jk|\mu}(s) - \sigma_{ik|\mu}(s) + \sigma_{ij|\mu}(s)$.

PROOF. (1) By [Ak1] Proposition 3.2 and (4.4) $_{\mu-1}$,

$$\begin{aligned}\bar{\partial}_b \xi_{i|\mu}(s) &\equiv (\bar{\partial}_b - \omega(s))^2 g_i^{\mu-1}(s) + \omega(s) \xi_{i|\mu}(s) \\ &\equiv P_b(\omega(s)) g_i^{\mu-1}(s) + \omega(s) \xi_{i|\mu}(s) \\ &\equiv 0 \pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,2}(U_i \cap M, T'X|_M)[[s_1, \dots, s_m]]}.\end{aligned}$$

(2) By (4.3) $_{\mu-1}$ and (4.4) $_{\mu-1}$, we have

$$\begin{aligned}\bar{\partial}_b \sigma_{ij|\mu}(s) &\equiv (\bar{\partial}_b - \omega(s)) g_i^{\mu-1}(s) - \frac{\partial f_{ij}}{\partial z_j} (\bar{\partial}_b - \omega(s)) g_j^{\mu-1}(s) \\ &\pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,1}(U_i \cap U_j \cap M, T'X|_M)[[s_1, \dots, s_m]]}.\end{aligned}$$

(3) By (4.3) $_{\mu-1}$,

$$\begin{aligned}\sigma_{ij|\mu}(s) &\equiv \sigma_{ik|\mu}(s) - \sigma_{jk|\mu}(s) + \Xi_{ijk|\mu}(g_k^{\mu-1}(s), \tau^{\mu-1}(s)) \\ &\pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,0}(U_i \cap U_j \cap U_k \cap M, T'X|_M)[[s_1, \dots, s_m]]}.\end{aligned}$$

Since $\Xi_{ijk|\mu}(\zeta_k, s)$ is homogeneous of order μ in s ,

$$\begin{aligned}\Xi_{ijk|\mu}(g_k^{\mu-1}(s), s) &\equiv \Xi_{ijk|\mu}(z_k, s) \\ &\pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A_b^{0,0}(U_i \cap U_j \cap U_k \cap M, T'X|_M)[[s_1, \dots, s_m]]}.\end{aligned}$$

Q.E.D.

PROOF OF LEMMA 4.5 (2). By (4.5) $_{\mu-1}$, we have

$$(4.6) \quad (\bar{\partial} - \varphi(\tau^{\mu-1}(s))) \tilde{z}_i^\alpha(z_i, \tau^{\mu-1}(s)) \equiv 0 \pmod{(m^\mu + \mathcal{I}_{S,o}) \otimes A^{0,1}(U_i, T'X)[[s_1, \dots, s_m]]},$$

$$(4.7) \quad \tilde{z}_i^\alpha(f_{ij}(z_j), \tau^{\mu-1}(s)) - F_{ij}^\alpha(\tilde{z}_j(z_j, \tau^{\mu-1}(s)), \tau^{\mu-1}(s)) \equiv 0 \pmod{(m^\mu + \mathcal{I}_{S,o}) \otimes A^{0,0}(U_i \cap U_j, T'X)[[s_1, \dots, s_m]]},$$

$$(4.8) \quad P(\varphi(\tau^{\mu-1}(s))) \equiv 0 \pmod{(m^\mu + \mathcal{I}_{S,o}) \otimes A^{0,2}(\bar{\Omega}, T'X)[[s_1, \dots, s_m]]}.$$

Let $\eta_{i|\mu}(s)$ and $\lambda_{ij|\mu}(s)$ be the canonical moduli of

$$(\bar{\partial} - \varphi(\tau^{\mu-1}(s))) \tilde{z}_i(\tau^{\mu-1}(s)) \pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A^{0,1}(U_i, T'X)[[s_1, \dots, s_m]]}$$

and of

$$\begin{aligned}\tilde{z}_i(f_{ij}(z_j), \tau^{\mu-1}(s)) - F_{ij}(\tilde{z}_j(z_j, \tau^{\mu-1}(s)), \tau^{\mu-1}(s)) \\ \pmod{(m^{\mu+1} + \mathcal{I}_{S,o}) \otimes A^{0,0}(U_i \cap U_j, T'X)[[s_1, \dots, s_m]]},\end{aligned}$$

respectively. Note that they are all homogeneous of order μ , by (4.6)~(4.8). By the same argument as the proof of Sublemma 4.6, we infer from (4.6)~(4.8) the following Sublemma 4.7 which implies Lemma 4.5 (2).

SUBLEMMA 4.7. (1) $\bar{\partial} \eta_{i|\mu}(s) = p_\mu(s)|_{U_i}$,

(2) $\bar{\partial} \lambda_{ij|\mu}(s) = -\eta_{j|\mu}(s) + \eta_{i|\mu}(s)$,

$$(3) \quad \Xi_{ijk|\mu}(s) = \lambda_{jk|\mu}(s) - \lambda_{ik|\mu}(s) + \lambda_{ij|\mu}(s).$$

Thus we have constructed $\{g_i^\mu(s)\}_{i \in A_0}$ and $\{\tau^\mu(s)\}$ satisfying (4.1) and $(4.2)_\mu \sim (4.5)_\mu$ for all $\mu \geq 0$.

The convergences of $g_i(s) = \lim_{\mu \rightarrow +\infty} g_i^\mu(s)$ and $\tau(s) = \lim_{\mu \rightarrow +\infty} \tau^\mu(s)$, with respect to the Folland Stein norm $\|\cdot\|_k$ in $A_b^{0,0}(U_i, T'X|_M)$ and the euclidean norm in C^r respectively, follow by the same arguments as in §3 (II) of [A-M1]. We will omit it.

§5. Uniqueness of versal families of strongly pseudo-convex CR structures.

The family of strongly pseudo-convex CR structures $\omega(s)$ (parametrized by (S, o)) constructed in [Ak2] has the following property, which is called “*versal in the sense of Kuranishi*” in [Ak2]:

- (1) For any family of complex manifolds $\pi' : \mathcal{X}' \rightarrow (T', o)$ such that $\pi'^{-1}(o)$ is a neighbourhood of M in X , there exists a holomorphic map $\tau' : (T', o) \rightarrow (S, o)$ and a family of CR-embeddings $G : M \times (T', o) \rightarrow \mathcal{X}'$ with respect to $\omega(\tau'(t))$ over (T', o) .
- (2) The infinitesimal deformation map $T_o S \rightarrow H_{\bar{\partial}}^1(M, T')$ is bijective.

As a corollary of the Main Theorem, we have a uniqueness of the versal family of strongly pseudo-convex CR structures, in the following sense.

COROLLARY 5.1. *Let M be as in the Main Theorem. Then any versal family of CR structures on M (in the sense of Kuranishi) is realized as a real hypersurface of the (formally) versal family of deformations of X near M . In particular, their parameter spaces coincide with each other.*

PROOF. Let $\varphi(s)$, parametrized by (S, o) , be a versal family of CR structures on M (in the sense of Kuranishi), and let $\pi : \mathcal{X} \rightarrow (T, o)$ be the (formally) versal family of deformations of X near M (cf. §3).

From the versality (in the sense of Kuranishi) of $\varphi(s)$, we have a holomorphic map $\sigma : (T, o) \rightarrow (S, o)$ and family of CR embeddings $G : M \times (T, o) \rightarrow \mathcal{X}$ with respect to the induced family $\omega(\sigma(t))$. G is given by a collection $\{g_i(t)\}_{i \in A_0}$ satisfying (1)~(4) of Definition 2.2 with respect to $\omega(\sigma(t))$. Differentiating (3) and (4) of them, we have a commutative diagram

$$\begin{array}{ccc} T_o T & \xrightarrow{d\sigma_o} & T_o S \\ \downarrow & & \downarrow \\ \check{H}^1(\mathcal{U}, \Theta) & \xrightarrow{\beta^1 \circ \check{r}^1} & H_{\bar{\partial}_b}^1(M, T'X|_M) \xleftarrow{\rho'} H_{\bar{\partial}_b}^1(M, T'), \end{array}$$

where ρ' is an isomorphism induced from the projection $\rho'|_{T'} : T' \rightarrow T'X|_M$ (cf. Remark 1.2).

On the other hand, from the argument in §4, we have a formal map $\hat{\tau} : \hat{S} \rightarrow \hat{T}$ and

a formal powerseries of CR-embeddings $\hat{H} : M \times \hat{S} \rightarrow \hat{X} \times \hat{\tau} \hat{S}$ with respect to $\omega(s)$, that is, \hat{H} is given by a collection $\{\hat{h}_i(s)\}_{i \in \Lambda_0}$ of formal powerseries of s satisfying (1), (2) and the formal version of (3) and (4) of Definition 2.2, with respect to $\{F_{ij}^\alpha(\zeta_j, \hat{\tau}(s))\}_{i, j \in \Lambda_0}$ and $\omega(s)$. Differentiating (3) and (4) of them, we have that $d\hat{\tau}_o : T_o S \rightarrow T_o T$ commutes with the above diagram.

Since the infinitesimal deformation maps $T_o T \rightarrow \check{H}^1(\mathcal{U}, \Theta)$ and $T_o S \rightarrow H_{\bar{\partial}_b}^1(M, T')$ are isomorphisms, and since $\beta^1 \circ \check{r}^1 = r^1 \circ \alpha^1$ is an isomorphism by Proposition 4.2, we have that $d\sigma_o \circ d\hat{\tau}_o = \text{id}$ and $d\hat{\tau}_o \circ d\sigma_o = \text{id}$. Hence, by Annex, Note 1 of [Ar2], we have that $\hat{\sigma}$ and $\hat{\tau}$ are isomorphisms. Then we have that σ is isomorphism by Corollary (1.6) of [Ar1]. Q.E.D.

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Note added in proof.

In the proof of Main Theorem, the convergence of $g_i(s) = \lim_{\mu \rightarrow +\infty} g_i^\mu(s)$ and $\tau(s) = \lim_{\mu \rightarrow +\infty} \tau^\mu(s)$ does not follow by the *same* argument as in §3 (II) of [A-M1]. In this note, we give a proof. It relies on estimates with respect to a pseudo-norm $\| \cdot \|_\rho$ ($\rho \in \mathbf{R}_+^m$). We first follow the expression of the pseudo-norm by [F-K1]. Refer to [F-K1] §1 for more details. Let $\sum_{v \in N^m} \gamma_v s^v$ be the Taylor series expansion of $M(s) = \prod_{i=1}^m (\frac{1}{2} - \frac{1}{4} \sqrt{1 - s_i})$. Let \mathbf{K} be a Banach space with its norm $|\cdot|_{\mathbf{K}}$ (in the following argument, $(\mathbf{K}, |\cdot|_{\mathbf{K}})$ will be $(\mathcal{A}'_k{}^{0,0}(U_i \cap M, T'X|_M), \| \cdot \|_k)$, $(\mathcal{A}'_k{}^{0,1}(U_i \cap M, T'X|_M), \| \cdot \|_k)$ or $(\mathbf{C}^r, | \cdot |)$). For $\rho = (\rho_1, \dots, \rho_m) \in \mathbf{R}_+^m$ and $f(s) \in \mathbf{K}\{s\}$, we define a pseudo-norm $\|f(s)\|_\rho := \sup_v \{(\rho^v / \gamma_v) |f_v|_{\mathbf{K}}\}$.

The following properties for $f(s), g(s), f_i(s) \in \mathbf{K}\{s\}$ are essential in our argument.

(N-1) $f(s) \in \mathbf{K}\{s\}$ if and only if $\|f(s)\|_\rho < +\infty$ for some ρ .

We denote $\rho' < \rho$ (resp. $\rho' \leq \rho$) if $\rho'_i < \rho_i$ (resp. $\rho'_i \leq \rho_i$) holds for all $i = 1, 2, \dots, m$, and denote $\rho' / \rho := \max_{1 \leq i \leq m} \{\rho'_i / \rho_i\}$.

(N-2) $\|f(s)\|_{\rho'} \leq \|f(s)\|_\rho$ if $\rho' \leq \rho$.

(N-3) If $f(0) = 0$ then $\|f(s)\|_{\rho'} \leq (\rho' / \rho) \|f(s)\|_\rho$ holds for $\rho' \leq \rho$.

(N-4) $\|f(s)g(s)\|_\rho \leq d \|f(s)\|_\rho \|g(s)\|_\rho$,
if $|\alpha\beta|_{\mathbf{K}} \leq d |\alpha|_{\mathbf{K}} |\beta|_{\mathbf{K}}$ holds for all α and β in \mathbf{K} .

(N-5) $\|f(s) + g(s)\|_\rho \leq \|f(s)\|_\rho + \|g(s)\|_\rho$.

(N-6) If $\{f_i(s)\}_{i \in I}$ are disjoint (i.e. for each $v \in N^m$, $(f_i)_v = 0$ except one index $i = i(v)$), then $\|\sum_{i \in I} f_i(s)\|_\rho = \sup_{i \in I} \|f_i(s)\|_\rho$.

We need the following two facts in our argument.

LEMMA N.1. Let $U \subset\subset W \subset\subset \mathbf{C}^N$ be open domains, $V \subset U$ a real closed submanifold and k a positive integer. Then there exist constants $\varepsilon > 0$ and $C_1 > 0$, depending only on V, U, W and k , such that

$$\|h(z + \psi(s)) - h(z) - \frac{\partial h}{\partial z}(z)\psi(s)\|_{\rho} \leq C_1 |h|_W \|\psi(s)\|_{\rho}^2$$

holds for $h \in \Gamma(W, \mathcal{O})$, $\psi(s) \in C^{\infty}(V, \|\cdot\|_k)^N\{s\}$ such that $|h|_W := \sup_{z \in W} |h(z)| < +\infty$, $\psi(z, 0) = 0$ and $\|\psi\|_{\rho} < \varepsilon$, where we consider $h(z + \psi(s)) - h(z) - (\partial h / \partial z)(z)\psi(s)$ as an element of $C^{\infty}(V, \|\cdot\|_k)\{s\}$.

We can prove Lemma N.1 along the way of [F-K2] Lemma 2.17.

Let S be a germ of an analytic subset of $(\mathbb{C}^m, 0)$ defined by an ideal $\mathcal{I}_S \subset \mathbb{C}\{s\}$. We denote by $red_{\mathcal{I}_S} f(s)$ the canonical modulus of $f(s) \in \mathbb{K}\{s\}$ modulo $\mathcal{I}_S \mathbb{K}\{s\}$.

LEMMA N.2 ([G] Satz 4, [F-K1] Satz 1.14). *There exists a constant $C_2 > 0$ such that $\|red_{\mathcal{I}_S} f(s)\|_{\rho} \leq C_2 \|f(s)\|_{\rho}$ holds for all ρ .*

Let k be the positive integer in the Main Theorem such that $\omega(s)$ is convergent in $\|\cdot\|_k$ -norm. Fix $\rho_0 \in \mathbb{R}_+^m$ such that $\sigma := \|\omega(s)\|_{\rho_0} < +\infty$.

We collect some constants.

(N-7) $\|\phi\psi\|_k \leq d \|\phi\|_k \|\psi\|_k$ for $\phi, \psi \in C_0^{\infty}(U_i \cap M)$ and for all $i \in \Lambda_0$.

(N-8) $\|\bar{\partial}_b g\|_k \leq e \|g\|_k'$ for $g \in C_0^{\infty}(U_i \cap M, T')$ and for all $i \in \Lambda_0$.

(N-9) $|H_b \xi| \leq f \|\xi\|_k'$ for $\xi \in A_b^{0,1}(T')$.

(N-10) $|\{[\Theta_{ij}|_M]\}| \leq h |\eta|$ for $\{\Theta_{ij}\} \in Z^1(\mathcal{U}, \Theta_X)$ with $\{\Theta_{ij}|_M\}$ being cohomologous to $\eta \in H_b^1(T')$.

(N-11) $\|\sum_j \rho_j \sum_{\sigma} \Theta_{ji}^{\sigma}|_M \tau^{\sigma}\|_k' \leq b |[\sum_{\sigma} \Theta_{ji}^{\sigma}|_M \tau^{\sigma}]|$.

(N-12) $\|\mathfrak{D}_b N_b \xi\|_k' \leq a \|\xi\|_k$.

We may assume that $(U_i \cap U_j) \times 0 \subset\subset W_{ij}$.

(N-13) $F := \sup_{(z_j, t) \in W_{ij}} |F_{ij}(z_j, t)|$.

By (N-1) and (N-6), it is enough to show that the following estimates hold for some $0 < \rho \leq \rho_0$, $0 < Z < \sigma^{-1}\varepsilon$ and for all $\mu = 1, 2, \dots$:

(1 $_{\mu}$) $\|g'_{i\mu}(s) + g''_{i\mu}(s) + g'''_{\mu}(s)\|_{\rho} \leq Z\sigma$,

(2 $_{\mu}$) $\|\tau_{\mu}(s)\|_{\rho} \leq Z\sigma$.

PROPOSITION N.3. *Suppose (1 $_v$) and (2 $_v$) hold for some $0 < \rho \leq \rho_0$, $0 < Z < \sigma^{-1}\varepsilon$ and $v = 1, 2, \dots, \mu - 1$ ($\mu \leq 2$), then the following holds:*

(1) $\|g'_{i\mu}(s) + g''_{i\mu}(s) + g'''_{\mu}(s)\|_{\rho} \leq C^{(1)} Z^2 \sigma^2 + C^{(2)} Z \left(\frac{\rho}{\rho_0}\right) \sigma^2 + C^{(3)} \left(\frac{\rho}{\rho_0}\right) \sigma$,

$$(2) \quad \|\tau_\mu(s)\|_\rho \leq C^{(4)}Z^2\sigma^2 + C^{(5)}Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C^{(6)}\left(\frac{\rho}{\rho_0}\right)\sigma,$$

where $C^{(1)} \sim C^{(6)}$ are constants independent of μ, ρ, σ and Z .

The following lemma implies Proposition N.3.

LEMMA N.4. *Under the assumption of Proposition N.3, we have*

$$(1) \quad \|\sigma_{ij|\mu}(s)\|_\rho \leq C_3Z^2\sigma^2,$$

$$(2) \quad \|g'_{i|\mu}(s)\|_\rho \leq C_4Z^2\sigma^2,$$

$$(3) \quad \|\xi_{i|\mu}(s)\|_\rho \leq C_5Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C_6\left(\frac{\rho}{\rho_0}\right)\sigma,$$

$$(4) \quad \|\xi'_\mu(s)\|_\rho \leq C_7Z^2\sigma^2,$$

$$(5) \quad \|\tau_\mu(s)\|_\rho \leq C_8Z^2\sigma^2 + C_9Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C_{10}\left(\frac{\rho}{\rho_0}\right)\sigma,$$

$$(6) \quad \|g''_{i|\mu}(s)\|_\rho \leq C_{11}Z^2\sigma^2 + C_{12}Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C_{13}\left(\frac{\rho}{\rho_0}\right)\sigma,$$

$$(7) \quad \|\xi''_\mu(s)\|_\rho \leq C_{14}Z^2\sigma^2 + C_{15}Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C_{16}\left(\frac{\rho}{\rho_0}\right)\sigma,$$

$$(8) \quad \|g'''_\mu(s)\|_\rho \leq C_{17}Z^2\sigma^2 + C_{18}Z\left(\frac{\rho}{\rho_0}\right)\sigma^2 + C_{19}\left(\frac{\rho}{\rho_0}\right)\sigma,$$

where $C_3 \sim C_{19}$ are constants independent of μ, ρ, σ and Z .

PROOF. (1) Set

$$\Phi_{ij}(\psi, \tau) := F_{ij}(z_j + \psi, \tau) - F_{ij}(z_j, 0) - \frac{\partial F_{ij}}{\partial z_j}(z_j, 0)\psi - \frac{\partial F_{ij}}{\partial t}(z_j, 0)\tau.$$

Let $\psi = g_j^{\mu-1}(s) - g_j^0$ and $\tau = \tau^{\mu-1}(s)$. Since $g_i^{\mu-1}(s) - g_i^0$, $g_j^{\mu-1}(s) - g_j^0$ and $\tau^{\mu-1}(s)$ are $(\mu-1)$ -th order polynomials coinciding with their canonical modulus respectively, $\sigma_{ij|\mu}(s)$ is the homogeneous part of order μ of $-\text{red}_{\mathcal{S}_s}\Phi_{ij}(\psi, \tau)$. And since $\|\psi(s)\|_\rho \leq Z\sigma < \varepsilon$ and $\|\tau(s)\|_\rho \leq Z\sigma < \varepsilon$, we have $\|\sigma_{ij|\mu}(s)\|_\rho \leq \|\text{red}_{\mathcal{S}_s}\Phi_{ij}(\psi, \tau)\|_\rho \leq C_2\|\Phi_{ij}(\psi, \tau)\|_\rho \leq C_1C_2FZ^2\sigma^2$, by Lemmas N.1 and N.2.

(2) follows from (1), since $g'_{i|\mu}(s) = \sum_j \rho_j \sigma_{ji|\mu}(s)$.

(3) Since $(\bar{\partial}_b - \omega(s))g_i^{\mu-1}(s) = \bar{\partial}_b(g_i^{\mu-1}(s) - g_i^0) - \omega(s)g_i^{\mu-1}(s)$ and $g_i^{\mu-1}(s) - g_i^0$ is a $(\mu-1)$ -th order polynomial satisfying $\text{red}_{\mathcal{S}_s}(g_i^{\mu-1}(s) - g_i^0) = g_i^{\mu-1}(s) - g_i^0$, $\xi_{i|\mu}(s)$ is the homogeneous part of order μ of $\text{red}_{\mathcal{S}_s}\omega(s)g_i^{\mu-1}(s)$. Hence, $\|\xi_{i|\mu}(s)\|_\rho \leq \|\text{red}_{\mathcal{S}_s}\omega(s)g_i^{\mu-1}(s)\|_\rho \leq C_2\|\omega(s)g_i^{\mu-1}(s)\|_\rho \leq C_2\{\|\omega(s)(g_i^{\mu-1}(s) - g_i^0)\|_\rho + \|\omega(s)\|_\rho\} \leq C_2\{d(\rho/\rho_0)Z\sigma^2 + (\rho/\rho_0)\sigma\}$, by

(N-3).

(4) follows from (2) and (N-8), since $\xi'_\mu(s) = \bar{\partial}_b g'_{i|\mu}(s)$.(5) follows from (3), (4), (N-9) and (N-10), since $[\sum_j \rho_j \sum_\sigma \Theta_{ji}^\sigma |_{\mathcal{M}} \tau^\sigma(s)]$ is cohomologous to $H_b(\xi_\mu(s) + \xi'_\mu(s))$.(6) follows from (5) and (N-11), since $g''_{i|\mu}(s) = -(\sum_j \rho_j \sum_\sigma \Theta_{ji}^\sigma |_{\mathcal{M}} \tau^\sigma(s))$.(7) follows from (6) and (N-8), since $\xi''_\mu(s) = \bar{\partial}_b g''_{i|\mu}(s)$.(8) follows from (3), (4), (7) and (N-12), since $g'''_{i|\mu}(s) = -\mathfrak{D}_b N_b(\xi_\mu(s) + \xi'_\mu(s) + \xi''_\mu(s))$.

Q.E.D.

PROOF of convergence. Let $Z > 0$ be such that $C^{(1)}Z\sigma < 1/3$ and $C^{(4)}Z\sigma < 1/3$ hold. Let $\rho_1 \in \mathbf{R}_+^m$ be such that $\rho_1 < \rho_0$, $\|g'_{i|1}(s) + g''_{i|1}(s) + g'''_{i|1}(s)\|_{\rho_1} \leq Z\sigma$ and $\|\tau_1(s)\|_{\rho_1} \leq Z\sigma$ hold. Next, choose $0 < \rho < \rho_1$ such that $C^{(2)}(\rho/\rho_0)\sigma < 1/3$, $C^{(3)}(\rho/\rho_0) < Z/3$, $C^{(5)}(\rho/\rho_0)\sigma < 1/3$ and $C^{(6)}(\rho/\rho_0) < Z/3$ hold. Then, by Proposition N.3, (1_μ) and (2_μ) hold for $\mu = 1, 2, \dots$. This completes the proof of convergence.

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Present Address:

MATHEMATICAL INSTITUTE, COLLEGE OF LIBERAL ARTS, KAGOSHIMA UNIVERSITY
 KAGOSHIMA 890, JAPAN