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ON THE NORMAL FORM OF THE SYMMETRIC HYPERBOLIC-PARABOLIC SYSTEMS ASSOCIATED WITH THE CONSERVATION LAWS

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1. Introduction. First-order nonlinear systems of conservation laws are the equations of the form

$$w_t + \sum_{j=1}^{n} f^j(w)_{x_j} = 0 \ . \tag{1.1}$$

Here f^j, $j = 1, \cdots, n$, w denote m-dimensional vectors, with f^j smooth functions of w, and w a functions of the time t and the space coordinate $x = (x_1, \cdots x_n)$. The subscripts t and x_j refer to the partial derivatives with respect to t and x_j, respectively. As is well known, the existence of an entropy function for (1.1) is characterized by the property that (1.1) can be symmetrized by introducing a new dependent variable. We owe these results to Godunov [5] and Friedrichs-Lax [3]. See, in this connection, [6], [7], [11].

The primary objective of this paper is the initial value problem for the second-order nonlinear systems associated with (1.1), i.e.,

$$w_t + \sum_{j=1}^{n} f^j(w)_{x_j} = \sum_{i,j=1}^{n} \{G^{ij}(w) w_{x_j}\}_{x_i} \ . \tag{1.2}$$

Here $G^{ij}(w)$, i, $j = 1, \cdots, n$, denote $m \times m$ matrices depending smoothly on w. Both $f^j(w)$ and $G^{ij}(w)$ are defined on an open set $\Omega \subset \boldsymbol{R}^m$. Such systems arise, for example, as the equations of viscous compressible fluid. The notion of the entropy function has a natural extension to the second-order systems (1.2). The definition was given previously by one of the authors in [8]. That the symmetrizability of the system can be characterized by the existence of an entropy function remains valid for (1.2). We give a brief review of these observations in §2. In §3, we consider the system (1.2) by assuming the symmetrizability. We formulate a sufficient condition under which (1.2) can be put in a normal form of the hyperbolic-parabolic system. This means that the resulting system is expressed as a coupled system of a hyperbolic system and a parabolic system. Consequently, the local existence of solutions of the initial value problem can be shown by using iteration. The equations of the hydrodynamics are treated as an application in §4. In §5, we discuss an analogous system of equations arising in the discrete kinetic theory. This is the Navier-Stokes equation derived from the discrete velocity models of the Boltzmann equation by the Chapman-Enskog method.

2. Entropy function and the symmetrizable systems. First of all we define the entropy function for the system (1.2).

DEFINITION 2.1. Let $\eta=\eta(w)$ be a real-valued smooth function defined on a convex open set $\mathcal{O}_w\subset\Omega$. Then η is called an entropy function for (1.2), if the following properties hold:

(a) η is a strictly convex function on $\mathcal{O}_w$ in the sense that the Hessian $D_w^2\eta$ is positive definite on $\mathcal{O}_w$.

(b) There exist real-valued smooth functions $q^j = q^j(w)$, $j = 1, \cdots, n$, such that $D_w\eta(w)D_wf^j(w) = D_wq^j(w)$, $j = 1, \cdots, n$, for $w\in\mathcal{O}_w$.

(c) $(D_w^2\eta(w))^{-1}G^{ij}(w)^T = G^{ji}(w)(D_w^2\eta(w))^{-1}$, $i, j = 1, \cdots, n$, for $w\in\mathcal{O}_w$, where the superscript T denotes the transpose.

(d) $\sum_{i,j=1}^n G^{ij}(w)(D_w^2\eta(w))^{-1}\omega_i\omega_j$ is real symmetric and non-negative definite for $w\in\mathcal{O}_w$ and $\omega = (\omega_1, \cdots, \omega_n)\in S^{n-1}$.

REMARK. The definition we adopted seems to be somewhat stringent, because the entropy function is required to have a convex domain of definition. The convexity is needed in the proofs of Theorems 2.1 and 3.1. In concrete problems, the domain of definition of an entropy function may eventually be non-convex. We shall return to this point in §4.

Now we rewrite (1.2) by introducing a new dependent variable u in place of w. It is assumed that there is a diffeomorphism $u\to w$ from an open set $\mathcal{O}_u$ onto $\mathcal{O}_w$. Let us set $w = w(u)$ in (1.2). Then (1.2) becomes of the form

$$A^0(u)u_t + \sum_{j=1}^n A^j(u)u_{x_j} = \sum_{i,j=1}^n \{B^{ij}(u)u_{x_j}\}_{x_i}\,, \tag{2.1}$$

where

$$(2.2)_1 \qquad A^0(u) = D_uw(u)\,.$$

$$(2.2)_2 \qquad A^j(u) = D_uf^j(w(u)) = (D_wf^j)(w(u))D_uw(u)\,, \quad j = 1, \cdots, n\,.$$

$$(2.2)_3 \qquad B^{ij}(u) = G^{ij}(w(u))D_uw(u)\,, \quad i, j = 1, \cdots, n\,.$$

We give the definition of the symmetric form as follows.

DEFINITION 2.2. The system (2.1) is said to be of the symmetric form if the coefficient matrices satisfy the following properties:

(a) $A^0(u)$ is real symmetric and positive definite for $u \in \mathcal{O}_u$.

(b) $A^j(u)$, $j = 1, \cdots, n$, are real symmetric for $u \in \mathcal{O}_u$.

(c) $B^{ij}(u)^T = B^{ji}(u)$, $i, j = 1, \cdots, n$, for $u \in \mathcal{O}_u$.

(d) $B(u, \omega) = \sum_{i,j=1}^n B^{ij}(u)\omega_i\omega_j$ is real symmetric and non-negative definite for $u \in \mathcal{O}_u$ and $\omega = (\omega_1, \cdots, \omega_n) \in S^{n-1}$.

As we noted in §1, the following theorem is a direct generalization of the known results for the first-order system (1.1) of the nonlinear conservation laws. The second part of the theorem was given in [8] in a slightly different form.

THEOREM 2.1. *Let $\mathcal{O}_w \subset \Omega$ be a convex open set. Suppose that the system* (1.2) *can be symmetrized on $\mathcal{O}_w$. In other words, let the following hold: there exists a diffeomorphism $u \to w$ from an open set $\mathcal{O}_u$ onto $\mathcal{O}_w$ such that* (2.1), *obtained from* (1.2) *by setting $w = w(u)$, is of the symmetric form. Then* (1.2) *has an entropy function defined on $\mathcal{O}_w$. Conversely, suppose that there is an entropy function for* (1.2) *defined on a convex open set $\mathcal{O}_w \subset \Omega$. Then* (1.2) *can be symmetrized on $\mathcal{O}_w$.*

PROOF. Let the properties (a), (b), (c), (d) of Definition 2.2 hold. Since $\mathcal{O}_w$ is assumed to be convex, $\mathcal{O}_u$ must be simply connected. We note that the mapping $u \to w$ is by definition a diffeomorphism from $\mathcal{O}_u$ onto $\mathcal{O}_w$. Then, by Poincaré's lemma, there exist smooth real-valued functions $\tilde{\eta}$ and $\tilde{q}^j$, $j = 1, \cdots, n$, defined on $\mathcal{O}_u$ such that

$$(2.3)_1 \qquad D_u\tilde{\eta}(u) = w(u)^T ,$$

$$(2.3)_2 \qquad D_u\tilde{q}^j(u) = f^j(w(u))^T , \quad j = 1, \cdots, n .$$

Here we used the fact that $D_uw(u)$ and $D_uf^j(w(u))$, $j = 1, \cdots, n$, are real symmetric matrices. We set

$$(2.4)_1 \qquad \eta(w) = \langle w, u(w)\rangle - \tilde{\eta}(u(w)) ,$$

$$(2.4)_2 \qquad q^j(w) = \langle f^j(w), u(w)\rangle - \tilde{q}^j(u(w)) , \quad j = 1, \cdots, n ,$$

where $\langle\ ,\ \rangle$ denotes the standard inner product in $\boldsymbol{R}^m$. Then it turns out that $\eta(w)$ is an entropy function and $q^j(w)$, $j = 1, \cdots, n$, are the corresponding functions defined in (b) of Definition 2.1. In fact, differentiating $(2.4)_1$ with respect to w and using $(2.3)_1$ we find that

$$(2.5)_1 \qquad D_w\eta(w) = u(w)^T .$$

Similarly, we get from $(2.4)_2$ and $(2.3)_2$

$$(2.5)_2 \qquad D_wq^j(w) = u(w)^TD_wf^j(w) , \quad j = 1, \cdots, n .$$

These equalities imply (b) of Definition 2.1. It follows from $(2.5)_1$ that

$D_w^2\eta(w) = D_w u(w)$. Hence $(D_u w)(u(w)) = (D_w^2\eta(w))^{-1}$. Substituting this into $(2.2)_{1-3}$ yields

$$A^0(u(w)) = (D_w^2\eta(w))^{-1}\ , \tag{2.6$_1$}$$

$$A^j(u(w)) = (D_w f^j(w))(D_w^2\eta(w))^{-1}\ , \quad j = 1, \cdots, n\ , \tag{2.6$_2$}$$

$$B^{ij}(u(w)) = G^{ij}(w)(D_w^2\eta(w))^{-1}\ , \quad i, j = 1, \cdots, n\ . \tag{2.6$_3$}$$

Combining $(2.6)_1$ with (a) of Definition 2.2, we deduce that $D_w^2\eta(w)$ is positive definite. This implies (a) of Definition 2.1. Since (a), (c) of Definition 2.2 are assumed to hold, (c) of Definition 2.1 is verified by using $(2.6)_3$. Finally, (d) of Definition 2.1 follows from (d) of Definition 2.2 combined with $(2.6)_3$. Thus the proof of the first part of Theorem 2.1 is complete.

We turn to the proof of the second part of the theorem. Let us assume the existence of an entropy function $\eta(w)$ defined on a convex open set $\mathscr{O}_w$. We define the mapping $w \to u$ by

$$u(w) = (D_w\eta(w))^T\ . \tag{2.7}$$

Then $D_w u(w) = D_w^2\eta(w)$ is real symmetric and positive definite by (a) of Definition 2.1. Hence the mapping $w \to u$ is one-to-one on $\mathscr{O}_w$, because $\mathscr{O}_w$ is a convex set. On the other hand, $D_w u(w)$ is nonsingular on $\mathscr{O}_w$ as we noticed above. Therefore, by the inverse function theorem, $w \to u$ defines a local diffeomorphism at any point of $\mathscr{O}_w$. It follows from these observations that the mapping $w \to u$ is a diffeormorphism from $\mathscr{O}_w$ onto an open set $\mathscr{O}_u$. Consequently, the inverse mapping $u \to w$ is defined on $\mathscr{O}_u$. The system (1.2) can be written as (2.1) by change of the dependent variable, i.e., by setting $w = w(u)$. We shall show that the coefficient matrices given by $(2.2)_{1-3}$ satisfy the properties (a), (b), (c), (d) of Definition 2.2. Since $(D_u w)(u(w)) = (D_w^2\eta(w))^{-1}$. We see that $(2.6)_{1-3}$ hold. Hence (a) of Definition 2.2 follows from (a) of Definition 2.1. To show (b) of Definition 2.2, we employ the arguments given in [3]. By applying D_w to the equality in (b) of Definition 2.1, we obtain

$$D_w^2 q^j(w) = D_w^2\eta(w) D_w f^j(w) + D_w\eta(w) D_w^2 f^j(w)\ , \quad j = 1, \cdots, n\ . \tag{2.8}$$

The left side of (2.8) is clearly symmetric. The fact that $D_w^2 f^j(w)$ is a symmetric bilinear mapping from $\boldsymbol{R}^m \times \boldsymbol{R}^m$ into $\boldsymbol{R}^m$ implies that the second term on the right side of (2.8) is also symmetric. Consequently, the first term on the right side of (2.8) must be symmetric. Thus, $D_w^2\eta(w) D_w f^j(w) = (D_w f^j(w))^T D_w^2\eta(w)$. Combining this with $(2.6)_2$, we see that (b) of Definition 2.2 is satisfied. The other properties, i.e., (c), (d) of Definition 2.2 follow from (c), (d) of Definition 2.1 and $(2.6)_3$. The proof of Theorem

2.1 is completed.

We give in passing the equation satisfied by an entropy function $\eta(w)$. Let $q^j(w)$, $j = 1, \cdots, n$, be the corresponding functions defined in (b) of Definition 2.1. A straightforword computation yields

$$\eta(w)_t + \sum_{j=1}^{n} q^j(w)_{x_j} = \sum_{i,j=1}^{n} \{\langle u(w), G^{ij}(w)u(w)_{x_j}\rangle\}_{x_i} - \sum_{i,j=1}^{n} \langle u(w)_{x_i}, B^{ij}(u(w))u(w)_{x_j}\rangle . \tag{2.9}$$

Here $u(w)$ and $B^{ij}(u(w))$, $i, j = 1, \cdots, n$, are given by (2.7) and $(2.6)_3$.

3. Normal form of the symmetric hyperbolic-parabolic systems. Let us assume that (1.2) is symmetrizable on a convex open set $\mathcal{O}_w$ and consider the symmetric system (2.1) on $\mathcal{O}_u$ derived from (1.2). When the right side is identically zero, (2.1) is a symmetric hyperbolic system. When $B(u, \omega)$ defined in (d) of Definition 2.2 is positive definite for $u \in \mathcal{O}_u$ and $\omega \in S^{n-1}$, (2.1) is a symmetric strongly parabolic system. Between these two limit cases, there are intermediate cases where $B(u, \omega)$ does not vanish identically but has a nontrivial null space. Our aim in this section is to give a simple sufficient condition such that, by introducing a new dependent variable again, (2.1) can be rewritten in the normal form, i.e., a coupled system of a symmetric hyperbolic system and a symmetric strongly parabolic system.

We consider a diffeomorphism $v \to u$ from an open set $\mathcal{O}_v$ onto $\mathcal{O}_u$. By substituting $u = u(v)$ into (2.1) and then multiplying by the transpose of the Jacobian matrix $D_v u(v)$, we obtain the transformed system with v as the dependent variable

$$\bar{A}^0(v)v_t + \sum_{j=1}^{n} \bar{A}^j(v)v_{x_j} = \sum_{i,j=1}^{n} \bar{B}^{ij}(v)v_{x_i x_j} + \bar{g}(v, D_x v) . \tag{3.1}$$

Here,

$$(3.2)_1 \qquad \bar{A}^0(v) = (D_v u(v))^T A^0(u(v)) D_v u(v) ,$$

$$(3.2)_2 \qquad \bar{A}^j(v) = (D_v u(v))^T A^j(u(v)) D_v u(v) , \quad j = 1, \cdots, n ,$$

$$(3.2)_3 \qquad \bar{B}^{ij}(v) = \frac{1}{2}(D_v u(v))^T \{B^{ij}(u(v)) + B^{ji}(u(v))\} D_v u(v) , \quad i, j = 1, \cdots, n ,$$

$$(3.2)_4 \qquad \bar{g}(v, D_x v) = \sum_{i,j=1}^{n} (D_v u(v))^T \{B^{ij}(u(v)) D_v u(v)\}_{x_i} v_{x_j} .$$

Since we started with the assumption that (2.1) is of the symmetric form, it is obvious that

$(3.3)_1$ $\bar{A}^0(v)$ is real symmetric and positive definite for $v \in \mathcal{O}_v$

$(3.3)_2$ $\bar{A}^j(v)$, $j = 1, \cdots, n$, are real symmetric for $v \in \mathcal{O}_v$,

$(3.3)_3$ $\bar{B}^{ij}(v)$, $i, j = 1, \cdots, n$, are real symmetric for $v \in \mathcal{O}_v$ and in addition $\bar{B}^{ij}(v) = \bar{B}^{ji}(v)$ for $v \in \mathcal{O}_v$ and $i, j = 1, \cdots, n$.

$(3.3)_4$ $\bar{B}(v, \omega) \equiv \sum_{i,j=1}^n \bar{B}^{ij}(v)\omega_i\omega_j$ is real symmetric and non-negative definite for $v \in \mathcal{O}_v$ and $\omega = (\omega_1, \cdots, \omega_n) \in S^{n-1}$.

We introduce here the definition of what we call the normal form.

DEFINITION 3.1. The system (3.1) is said to be of the normal form of the symmetric hyperbolic-parabolic system, if $(3.3)_{1-4}$ are satisfied and in addition there exists a partition $\{I, II\}$ of the standard basis $\{e_1, \cdots, e_m\}$ of $\boldsymbol{R}^m$ such that, $\bar{A}^0(v)$, $\bar{B}^{ij}(v)$, $i, j = 1, \cdots n$, and $\bar{g}(v, D_x v)$ are decomposed as follows:

(a)
$$\bar{A}^0(v) = \begin{pmatrix} \bar{A}^0_I(v) & 0 \\ 0 & \bar{A}^0_{II}(v) \end{pmatrix} .$$

Namely, $\bar{A}^0(v)$ is block diagonalized.

(b)
$$\bar{B}^{ij}(v) = \begin{pmatrix} 0 & 0 \\ 0 & \bar{B}^{ij}_{II}(v) \end{pmatrix} , \quad i, j = 1, \cdots, n ,$$

where, $\sum_{i,j=1}^n \bar{B}^{ij}_{II}(v)\omega_i\omega_j$ is positive definite for $v \in \mathcal{O}_v$ and $\omega = (\omega_1, \cdots, \omega_n) \in S^{n-1}$.

(c)
$$\bar{g}(v, D_x v) = (\bar{g}_I(v, D_x v_{II}), \bar{g}_{II}(v, D_x v))^T ,$$

where $v = (v_I, v_{II})^T$. In other words, $\bar{g}_I$ does not depend on $D_x v_I$.

Let the system (3.1) be of the normal form in the sense defined above. Let

$$\bar{A}^j(v) = \begin{pmatrix} \bar{A}^j_{II}(v) & \bar{A}^j_{I\,II}(v) \\ \bar{A}^j_{II\,I}(v) & \bar{A}^j_{II\,II}(v) \end{pmatrix} , \quad j = 1, \cdots, n .$$

Then (3.1) is written as

$$(3.4)_1 \qquad A^0_I(v)v_{I,t} + \sum_{j=1}^n \bar{A}^j_{I\,I}(v)v_{I,x_j} = \bar{h}_I(v, D_x v_{II}) ,$$

$$(3.4)_2 \qquad A^0_{II}(v)v_{II,t} - \sum_{i,j=1}^n \bar{B}^{ij}_{II}(v)v_{II,x_i x_j} = \bar{h}_{II}(v, D_x v) .$$

Here $v = (v_I, v_{II})^T$ and the right sides of $(3.4)_{1,2}$ are given explicitly by

$$\bar{h}_I(v, D_x v_{II}) = \bar{g}_I(v, D_x v_{II}) - \sum_{j=1}^n \bar{A}^j_{I\,II}(v)v_{II,x_j} ,$$

$$\bar{h}_{II}(v, D_x v) = \bar{g}_{II}(v, D_x v) - \sum_{j=1}^n \{\bar{A}^j_{II\,I}(v)v_{I,x_j} + \bar{A}^j_{II\,II}(v)v_{II,x_j}\} .$$

The system $(3.4)_{1,2}$ can be regarded as a coupled system of a symmetric hyperbolic system for v_I and a symmetric strongly parabolic system for v_{II}. Hence, for the initial value problem, the existence and the uniqueness of solutions local in time are proved in an appropriate function space. (See Theorem 2.9 of [12].) This implies that, if (1.2) is symmetrizable and the corresponding (2.1) can be written in the normal form, then the initial value problem for (1.2) is well posed. For such systems, several sufficient conditions that guarantee the existence of solutions global in time are known also. The reader is referred to [8], [12].

We shall show that the symmetric system (2.1) can be put into a normal form if the following condition is assumed to hold.

Condition N. The null space of $B(u, \omega)=\sum_{i,j}^{n} B^{ij}(u)\omega_i\omega_j$ is independent of $u \in \mathcal{O}_u$ and $\omega \in S^{n-1}$.

It is clear that this condition holds if (2.1) itself is of the normal form.

THEOREM 3.1. *Suppose that the symmetric system* (2.1) *is derived from* (1.2) *which is symmetrizable on a convex open set* $\mathcal{O}_w \subset \Omega$, *by change of the dependent variable. Let Condition* N *hold on* $\mathcal{O}_u$ *that corresponds to* $\mathcal{O}_w$. *Then there exists a diffeomorphism* $v \to u$ *from an open set* $\mathcal{O}_v$ *onto* $\mathcal{O}_u$ *such that, by rewriting* (2.1) *with* v *as the dependent variable and then multiplying by* $(D_v v(u))^T$, *the resulting system* (3.1) *is of the normal form. Furthermore,* (c) *of Definition* 3.1 *is satisfied with* $\bar{g}_I \equiv 0$.

PROOF. We may assume without loss of generality that $B^{ij}(u)$ are of the form

$$B^{ij}(u) = \begin{pmatrix} 0 & 0 \\ 0 & B^{ij}_{II}(u) \end{pmatrix}, \quad i, j = 1, \cdots, n . \tag{3.5}$$

Here we denote by $\{I, II\}$ the partition of the standard basis $\{e_1, \cdots, e_m\}$ of $\boldsymbol{R}^m$ defined as $I = \{e_1, \cdots, e_s\}$, $II = \{e_{s+1}, \cdots, e_m\}$. To see this, suppose for a moment that this is not the case. Let R be an orthogonal matrix with constant elements and let $u' = R^T u$. Then, $u = Ru'$. Substituting this into (2.1) and then multiplying by R^T, we obtain again a symmetric system which is equivalent to (2.1). It is easily seen that, by virtue of Condition N, (3.5) is realized with u' in place of u by a suitable choice of R.

Now we define a smooth mapping $u \to v$ by utilizing the diffeomorphism $u \to w$ that symmetrizes (1.2) on $\mathcal{O}_w$. Namely,

$$v = (w_1(u), \cdots, w_s(u), u_{s+1}, \cdots, u_m)^T . \tag{3.6}$$

Here $w_k(u)$ and u_k denote the k-th components of $w(u)$ and u, respectively.

The domain of definition of the above mapping is $\mathcal{O}_u$. But, to show that this mapping is a diffeomorphism, some care must be taken. Once this is proved, we shall see that the inverse mapping of the mapping (3.6) put the symmetric system (2.1) in a normal form.

We note that $\sum_{i,j=1}^n B_{II}^{ij}(u)\omega_i\omega_j$ is real symmetric and positive definite by Condition N. Let

$$A^0(u) = \begin{pmatrix} A_{I\,I}^0(u) & A_{I\,II}^0(u) \\ A_{II\,I}^0(u) & A_{II\,II}^0(u) \end{pmatrix}. \tag{3.7}$$

Since $A^0(u)=D_u w(u)$, we have $A_{I\,I}^0(u)=\partial w_I/\partial u_I$, $A_{I\,II}^0(u)=\partial w_I/\partial u_{II}$, $A_{II\,I}^0(u)=\partial w_{II}/\partial u_I$, $A_{II\,II}^0(u)=\partial w_{II}/\partial u_{II}$. Here $u=(u_I, u_{II})^T$, $w=(w_I, w_{II})^T$. These partial derivatives may be understood in the sense of Fréchet derivatives. Let us define a smooth mapping $w\to v$ by

$$v_I = w_I\,, \quad v_{II} = u_{II}(w_I, w_{II})\,. \tag{3.8}$$

We show in the following that this mapping is a diffeomorphism. First we note that $D_u w(u) = A^0(u)$ is positive definite. This implies that $D_w u(w)$ is positive definite. Hence, $\partial u_{II}/\partial w_{II}$ is positive definite also. Since $\mathcal{O}_w$ is a convex open set, the mapping $w_{II}\to u_{II}$ is one-to-one for arbitrarily fixed w_I. We deduce from this fact that the mapping $w\to v$ is one-to-one from $\mathcal{O}_w$ onto $\mathcal{O}_u$. To show that $D_w v(w)$ is nonsingular on $\mathcal{O}_w$, we notice that $D_w v(w)$ is lower triangular as a block matrix, i.e., $\partial v_I/\partial w_{II}=0$. Since $\partial v_I/\partial w_I = E_I$, we obtain $\det(D_w v(w)) = \det(\partial u_{II}/\partial w_{II})$. But $\partial u_{II}/\partial w_{II}$ is positive definite as we noted above. Therefore, $\det(D_w v(w))$ does not vanish on $\mathcal{O}_w$. We conclude that the mapping $w\to v$ is a diffeomorphism from $\mathcal{O}_w$ onto an open set $\mathcal{O}_v$.

Let us form the product of the mapping $u\to w$ and the mapping defined by (3.8). Namely,

$$v_I = w_I(u_I, u_{II})\,, \quad v_{II} = u_{II}\,. \tag{3.9}$$

This is a diffeomorphism from $\mathcal{O}_u$ onto $\mathcal{O}_v$. Differentiating (3.9) with respect to u, we get

$$D_u v(u) = \begin{pmatrix} A_{I\,I}^0(u) & A_{I\,II}^0(u) \\ 0 & E_{II} \end{pmatrix}. \tag{3.10}$$

Consequently,

$$D_v u(v) = \begin{pmatrix} A_{I\,I}^0(u)^{-1} & -A_{I\,I}^0(u)^{-1}A_{I\,II}(u) \\ 0 & E_{II} \end{pmatrix}. \tag{3.11}$$

It is easy to see that the mapping defined by (3.9) coincides with that defined by (3.6). Now we rewrite (2.1) with v as the dependent variable.

Substituting $u = u(v)$ into (2.1) and then multiplying the resulting equation by $(D_v u(v))^T$, we obtain (3.1) where $\bar{A}^0(v)$, $\bar{A}^j(v)$, $\bar{B}^{ij}(v)$ and $\bar{g}(v, D_x v)$ are defined as in $(3.2)_{1-4}$. Clearly, $(3.3)_{1-4}$ are satisfied. The properties (a), (b), (c) of Definition 3.1 can be verified by using (3.5), (3.7), (3.11). In particular, (c) of Definition 3.1 holds with $g_I \equiv 0$. The system (3.1) is therefore a symmetric hyperbolic-parabolic system of the normal form. The proof of Theorem 3.1 is completed.

4. **Hydrodynamical equations.** In this section we treat the system of equations for compressible fluids as an application of the general theory. The equations are given as

$$(4.1)\qquad \begin{aligned} &\rho_t + \sum_{j=1}^{3} (\rho u)^j_{x_j} = 0 \\ &(\rho u^i)_t + \sum_{j=1}^{3} (\rho u^i u^j + p\delta_{ij})_{x_j} \\ &\qquad = \sum_{j=1}^{3} \left\{\mu(u^i_{x_j} + u^j_{x_i}) + \mu'\Big(\sum_{k=1}^{3} u^k_{x_k}\Big)\delta_{ij}\right\}_{x_j}, \quad i = 1, 2, 3\,. \\ &\{\rho(e + |u|^2/2)\}_t + \sum_{j=1}^{3} \{\rho u^j(e + |u|^2/2) + pu^j\}_{x_j} \\ &\qquad = \sum_{j=1}^{3} \left\{\mu \sum_{i=1}^{3} u^i(u^i_{x_j} + u^j_{x_i}) + \mu' u^j \sum_{k=1}^{3} u^k_{x_k} + \kappa\theta_{x_j}\right\}_{x_j}. \end{aligned}$$

Here, ρ, $u = (u^1, u^2, u^3)$ and θ are the mass density, the fluid velocity and the absolute temparature, respectively. The pressure p and the internal energy e are given smooth functions of $\rho > 0$ and $\theta > 0$ satisfying

$$(4.2)\qquad p_\rho = \partial p(\rho, \theta)/\partial\rho > 0\,, \quad e_\theta = \partial e(\rho, \theta)/\partial\theta > 0\,.$$

The viscosity coefficients μ, μ' and the heat conduction coefficient κ are also given functions depending smoothly on $\rho > 0$ and $\theta > 0$. We consider in the following the four cases listed below.

(4.3)
- (i) $\mu,\ 2\mu + \mu',\ \kappa > 0$.
- (ii) $\mu = \mu' = 0,\ \kappa > 0$.
- (iii) $\mu,\ 2\mu + \mu' > 0,\ \kappa = 0$.
- (iv) $\mu = \mu' = \kappa = 0$.

We take ρ and θ as the two independent thermodynamical variables and set

$$(4.4)\qquad V = (\rho, u, \theta)^T\,.$$

V ranges over the open convex set $\mathscr{D}_V = (0, \infty)\times \boldsymbol{R}^3 \times (0, \infty)$ in $\boldsymbol{R}^5$. Next we set

$$(4.5)\qquad W = (\rho, \rho u, \rho(e + |u|^2/2))^T\,.$$

We regard W as a function of V defined on $\mathscr{D}_V$. The range of the mapping $V \to W$ is denoted by Ω. A direct computation shows that the Jacobian matrix of the mapping $V \to W$ is given as follows.

$$
(4.6) \qquad D_V W = \begin{pmatrix} 1 & 0 & 0 \\ u^T & \rho I_3 & 0 \\ e + |u|^2/2 + \rho e_\rho & \rho u & \rho e_\theta \end{pmatrix}.
$$

Here, $e_\rho = \partial e(\rho, \theta)/\partial\rho$ and I_3 denotes the 3×3 unit matrix. We note that $\partial\rho(e + |u|^2/2)/\partial\theta = \rho e_\theta > 0$ by (4.2). Hence $\rho(e + |u|^2/2)$ is a monotone increasing function of θ for arbitrarily fixed ρ and u. It is evident that the mapping $(\rho, u) \to (\rho, \rho u)$ is one-to-one. Combining these observations we see that the mapping $V \to W$ is one-to-one on $\mathscr{D}_V$. On the other hand, $D_V W$ is nonsingular on $\mathscr{D}_V$ as is seen from (4.6). Hence, $V \to W$ defines a local diffeomorphism at every point of the domain of definition. We conclude therefore that the mapping $V \to W$ is a diffeomorphism from $\mathscr{D}_V$ onto Ω. (It follows from this fact that Ω is a simply connected open set in $\boldsymbol{R}^5$.) We rewrite (4.1) with W as the dependent variable and obtain

$$
(4.7) \qquad W_t + \sum_{j=1}^{3} f^j(W)_{x_j} = \sum_{j,k=1}^{3} \{G^{jk}(W) W_{x_k}\}_{x_j} ,
$$

where

$$
(4.8) \quad \sum_{j=1}^{3} f^j(W)\xi_j = (\rho(u\cdot\xi),\ \rho u(u\cdot\xi) + p\xi,\ \rho(e + |u|^2/2)(u\cdot\xi) + p(u\cdot\xi))^T
$$

for $\xi = (\xi_1, \xi_2, \xi_3) \in \boldsymbol{R}^3$. Note that $u\cdot\xi$ stands for the standard inner product of u and ξ in $\boldsymbol{R}^3$. We omit the explicit forms of $G^{jk}(W)$.

Let us denote by s the entropy of the fluid. It is assumed that s is a smooth function of $\rho > 0$ and $\theta > 0$. We recall here the identity $de = \theta ds - pd(1/\rho)$ which is an expression of the first law of the thermodynamics. From this follow the relations

$$
(4.9) \qquad e_\rho = (p - \theta p_\theta)/\rho^2 , \quad s_\rho = -p_\theta/\rho^2 , \quad s_\theta = e_\theta/\theta .
$$

Here the subscripts ρ and θ refer to the partial differentiations with respect to these variables. We set

$$
(4.10) \qquad \eta = -\rho s .
$$

Then η can be regarded as a function of V, and hence, of W. We shall show that the Hessian $D_W^2\eta$ is symmetric and positive definite on Ω. The gradient of η with respect to W, i.e., $\nabla_W\eta$ can be computed by using the relation $\nabla_W\eta = \nabla_V\eta(D_V W)^{-1}$ and (4.6), (4.9). We obtain

$$
(4.11) \qquad U \equiv (\nabla_W\eta)^T = (-s + (e - |u|^2/2 + p/\rho)/\theta,\ u/\theta,\ -1/\theta)^T .
$$

We regard (4.11) as the definition of the mapping $V \to U$. The domain of this mapping is $\mathscr{D}_V$. We denote the range by $\mathscr{D}_U$. The Jacobian matrix can be computed by using (4.9) as follows.

$$D_V U = \frac{1}{\theta}\begin{pmatrix} p_\rho/\rho & -u & -(e - |u|^2/2 + \rho e_\rho)/\theta \\ 0 & I_3 & -u^T/\theta \\ 0 & 0 & 1/\theta \end{pmatrix}. \tag{4.12}$$

We observe that $\partial\{-s + (e - |u|^2/2 + p/\rho)/\theta\}/\partial\rho = p_\rho/\rho\theta > 0$ by (4.2). Hence $-s + (e - |u|^2/2 + p/\rho)/\theta$ is a monotone increasing function of ρ for arbitrarily fixed u and θ. It is readily seen that the mapping $(u, \theta) \to (u/\theta, -1/\theta)$ is one-to-one. From these observations follows that the mapping $V \to U$ is one-to-one on $\mathscr{D}_V$. On the other hand, $V \to U$ defines a local diffeomorphism at every point of the domain of definition, because $D_V U$ is nonsingular on $\mathscr{D}_V$ as is seen from (4.12). Therefore, the mapping $V \to U$ is a diffeomorphism from $\mathscr{D}_V$ onto $\mathscr{D}_U$. As a consequence the mapping $W \to U$ defined also by (4.11) is a diffeomorphism from Ω onto $\mathscr{D}_U$. Since $D_W^2\eta = D_W U = D_V U(D_V W)^{-1}$, we get $(D_W^2\eta)^{-1} = D_V W(D_V U)^{-1}$. Let us set

$$\bar{A}^0(V) = (D_V U)^T (D_W^2\eta)^{-1} D_V U\,. \tag{4.13}$$

Then, $\bar{A}^0(V) = (D_V U)^T D_V W$. Hence the explicit form of $\bar{A}^0(V)$ is obtained by using (4.12), (4.6) and (4.9). Namely,

$$\bar{A}^0(V) = \frac{1}{\theta}\begin{pmatrix} p_\rho/\rho & 0 & 0 \\ 0 & \rho I_3 & 0 \\ 0 & 0 & \rho e_\theta/\theta \end{pmatrix}. \tag{4.14}$$

It is clear by inspection that $\bar{A}^0(V)$ is symmetric and positive definite on $\mathscr{D}_V$. Hence by (4.13) $D_W^2\eta$ is symmetric and positive definite on Ω. In other words, η is a strictly convex function on Ω in the sense defined in §2.

Since the inverse mapping $U \to W$ of the mapping $W \to U = \nabla_W \eta$ is a diffeomorphism from $\mathscr{D}_U$ onto Ω, we may substitute $W = W(U)$ into (4.7) and rewrite the equation. Then we obtain the following equation with U as the dependent variable.

$$A^0(U)U_t + \sum_{j=1}^{3} A^j(U)U_{x_j} = \sum_{j,k=1}^{3} \{B^{jk}(U)U_{x_k}\}_{x_j}\,. \tag{4.15}$$

Taking into account of the relation $D_U W = (D_W^2\eta)^{-1}$, we see that the coefficient matrices are given by

$$A^0(U) = (D_W^2\eta)^{-1}\,,$$
$$(4.16)\qquad A^j(U) = D_W f^j(W)(D_W^2\eta)^{-1}\,,\quad j=1,2,3\,,$$
$$B^{jk}(U) = G^{jk}(W)(D_W^2\eta)^{-1}\,,\qquad j,k=1,2,3\,.$$

Now we shall prove that (4.15) is of the symmetric form in the sense of Definition 2.2. The positive definiteness of $A^0(U)$ is easily seen. We set

$$(4.17)\qquad \tilde{\eta} = p/\theta,\ \tilde{q}^j = pu^j/\theta\,,\quad j=1,2,3\,,$$

and regard these quantities as functions of V, and hence, of U. A direct computation using (4.12), (4.9) shows that

$$(4.18)\qquad \nabla_U\tilde{\eta} = W^T,\ \nabla_U\tilde{q}^j = f^j(W)^T\,,\quad j=1,2,3\,.$$

In particular, it follows from the latter relation that $A^j(U) = D_U^2\tilde{q}^j$ for $j=1,2,3$. This implies that $A^j(U)$ are real symmetric matrices. (We omit the concrete forms of $A^j(U)$.) Let us consider the case (iv) of (4.3). Since the right side of (4.15) is identically zero in this case, we may conclude that (4.15) is a symmetric hyperbolic system. We owe this result to Godunov [5]. We consider next the other three cases of (4.3). The right side of (4.15) can be computed by using the expressions of the right side of (4.1) in terms of u/θ, $-1/\theta$, and their space derivatives. We omit the explicit forms and only note that

$$(4.19)\qquad B^{jk}(U)^T = B^{kj}(U)\,,\quad j,k=1,2,3\,.$$

$$(4.20)\qquad B(U,\xi) = \sum_{j,k=1}^{n} B^{jk}(U)\xi_j\xi_k$$
$$= \theta\begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu|\xi|^2 I_3 + (\mu+\mu')\xi^T\xi & \mu u^T|\xi|^2 + (\mu+\mu')(u\cdot\xi)\xi^T \\ 0 & \mu u|\xi|^2 + (\mu+\mu')(u\cdot\xi)\xi & (\kappa\theta + \mu|u|^2)|\xi|^2 + (\mu+\mu')(u\cdot\xi)^2 \end{pmatrix}.$$

Here, $\xi = (\xi_1, \xi_2, \xi_3) \in \boldsymbol{R}^3$. Let $z \in \boldsymbol{R}^3$ and let $Z = (z_0, z, z_4) \in \boldsymbol{R}^5$. Then it follows from (4.20) that

$$(4.21)\qquad \langle (1/\theta)B(U,\xi)Z, Z\rangle = \mu|\xi|^2|z+uz_4|^2 + (\mu+\mu')\{(z+uz_4)\cdot\xi\}^2 + \kappa\theta|\xi|^2 z_4^2\,,$$

where $\langle\ ,\ \rangle$ denotes the inner product in $\boldsymbol{R}^5$. Note that the inner product appearing in the second term of the right side is that of $\boldsymbol{R}^3$. By inspection of (4.21) it is seen that $B(U,\xi)$ is nonnegative definite for any $U \in \mathscr{D}_U$ and $\xi \in \boldsymbol{R}^3$ in all cases of (4.3). We conclude therefore that (4.15) is of a symmetric form. This means that η can be regarded as an entropy function despite of the fact that we are unable to show the convexity of Ω. Now we recall the arguments in § 2 and set

$$\eta = \langle W, U\rangle - \tilde{\eta}$$

by abuse of notation. We define also

$$q^j = \langle f^j(W), U\rangle - \tilde{q}^j\,, \quad j = 1, 2, 3\,.$$

Then it turns out that this η coincides with that defined by (4.10). We obtain

$$q^j = -\rho u^j s\,. \quad j = 1, 2, 3\,. \tag{4.22}$$

These assertions are verified by making use of (4.5), (4.8), (4.11) and (4.17).

Finally we examine whether Condition N is satisfied by the symmetric form (4.15) or not. Let $\omega \in S^2$ and let $\mathcal{N}(B(U, \omega))$ be the null space of $B(U, \omega)$. In the case (i) of (4.3), $\mathcal{N}(B(U, \omega))$ consists of vectors Z such that $z = z_4 = 0$ and hence has one dimension. Condition N is satisfied in this case. By the arguments in § 3, (4.15) can be put into a normal form. In fact this is realized by using the diffeomorphism $V \to U$. Let us set $U = U(V)$ in (4.15). Then we obtain the following equation.

$$\bar{A}^0(V)V_t + \sum_{j=1}^{3} \bar{A}^j(V)V_{x_j} = \sum_{j,k=1}^{3} \bar{B}^{jk}(V)V_{x_j x_k} + \bar{g}(V, D_x V)\,. \tag{4.23}$$

Here, $\bar{A}^0(V)$ is given by (4.14) and

$$\sum_{j=1}^{3} \bar{A}^j(V)\xi_j = \frac{1}{\theta}\begin{pmatrix} (p_\rho/\rho)(u\cdot\xi) & p_\rho\xi & 0 \\ p_\rho\xi^T & \rho(u\cdot\xi)I_3 & p_\theta\xi^T \\ 0 & p_\theta\xi & (\rho e_\theta/\theta)(u\cdot\xi) \end{pmatrix}, \tag{4.24}$$

$$\sum_{j,k=1}^{3} \bar{B}^{jk}(V)\xi_j\xi_k = \frac{1}{\theta}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu|\xi|^2 I_3 + (\mu + \mu')\xi^T\xi & 0 \\ 0 & 0 & (\kappa/\theta)|\xi|^2 \end{pmatrix}, \tag{4.25}$$

for $\xi = (\xi_1, \xi_2, \xi_3) \in \boldsymbol{R}^3$. Furthermore,

$$\begin{cases} \bar{g}(V, D_x V) = \dfrac{1}{\theta}(0, h, h_4)^T\,, \\ h = \left\{\displaystyle\sum_{j=1}^{3} \mu_{x_j}(u^i_{x_j} + u^j_{x_i}) + \mu'_{x_i}\Big(\sum_{j=1}^{3} u^j_{x_j}\Big)\right\}_{i=1,2,3} \\ h_4 = \dfrac{1}{\theta}\left\{\dfrac{\mu}{2}\displaystyle\sum_{i,j=1}^{3}(u^i_{x_j} + u^j_{x_i})^2 + \mu'\Big(\sum_{j=1}^{3} u^j_{x_j}\Big)^2 + \sum_{j=1}^{3}\kappa_{x_j}\theta_{x_j}\right\}\,. \end{cases} \tag{4.26}$$

It is easily seen from (4.26) that $\overline{g_I} \equiv 0$. We turn to the case (ii) of (4.3). By (4.21), $\mathcal{N}(B(U, \omega))$ consists of vectors Z such that $z_4 = 0$. Hence $\mathcal{N}(B(U, \omega))$ is a four dimensional subspace. Condition N is satisfied. We can get the corresponding normal form by setting $\mu = \mu' = 0$ in (4.23). Since $h = 0$ in (4.26), we see that $\overline{g_I} \equiv 0$. Now we treat the case (iv) of (4.3). We have $\mathcal{N}(B(U, \omega)) = \boldsymbol{R}^5$ in this case. Hence it is evident that

Condition N holds true. The corresponding normal form is obtained by setting $\mu = \mu' = \kappa = 0$ in (4.15), or equivalently, in (4.23). Briefly, the right side vanishes in this case. Let us consider the case (iii) of (4.3) that remains. $\mathcal{N}(B(U, \omega))$ is a two dimensional subspace consisting of vectors Z satisfying $z + uz_4 = 0$. Hence, in this case, $\mathcal{N}(B(U, \omega))$ depends on U. (It does not depend on $\omega \in S^2$.) Consequently, Condition N does not hold. Despite of this fact, we see that (4.23) with $\kappa = 0$ gives the normal form for this case. We note that $\bar{g}_I \equiv 0$ is violated in this normal form.

PROPOSITION 4.1. *Suppose that the conditions* (4.2) *and* (4.3) *are satisfied for the system of equations* (4.1) *for compressible fluids. Let W be defined as in* (4.5). *Then, with W as the dependent variable,* (4.1) *can be written as* (4.7). *This system of equations has an entropy function* η *defined by* (4.10). *The corresponding* q^j, $j = 1, 2, 3$, *are given by* (4.22). *Let U be defined as in* (4.11). *Then, if we rewrite* (4.7) *with U as the dependent variable, the resulting system of equations is of the symmetric form. Condition* N *holds except for the case* (iii) *of* (4.3). *Furthermore,* (4.15) *can be reduced to the normal form* (4.23) *for all cases of* (4.3). *This is realized by using V as the dependent variable, where V is defined by* (4.4). (*In the case* (iv) *of* (4.3), (4.23) *is a symmetric hyperbolic system. This case can be regarded as a limit case.*)

5. Application to the discrete kinetic theory. We discuss the Navier-Stokes equation derived from the discrete velocity models of the Boltzmann equation. Let us consider an n-dimensional N-velocity model. Let $v_1, \cdots, v_N \in \boldsymbol{R}^n$ be the velocity vectors. The discrete Boltzmann equation for this model is written in the following form:

$$\frac{\partial F_i}{\partial t} + v_i \cdot \nabla_x F_i = Q_i(F, F) \quad i = 1, \cdots, N . \tag{5.1}$$

Here each unknown $F_i (i = 1, \cdots, N)$ is a function of the time $t \geqq 0$ and the space coordinate $x \in \boldsymbol{R}^n$, and denotes the density distribution of particles with the velocity v_i. The right side of (5.1) is given as

$$Q_i(F, F) = \frac{1}{2} \sum_{j,k,l} (A_{kl}^{ij} F_k F_l - A_{ij}^{kl} F_i F_j) , \quad i = 1, \cdots, N . \tag{5.2}$$

A_{kl}^{ij} is a positive constant if the quadruplet i, j, k, l corresponds to a collision and, if otherwise, A_{kl}^{ij} is zero. It is assumed also that $A_{kl}^{ij} = A_{lk}^{ij} = A_{kl}^{ji}$ and that $A_{kl}^{ij} = A_{ij}^{kl}$.

We recall here two basic concepts in the discrete kinetic theory. One is summational invariant and the other is Maxwellian. An element $\phi = (\phi_1, \cdots, \phi_N)$ of $\boldsymbol{R}^N$ such that

$$A_{ij}^{kl}(\phi_i + \phi_j - \phi_k - \phi_l) = 0 \tag{5.3}$$

for any i, j, k, l is called a summational invariant. The set of all summational invariants forms a subspace of $\boldsymbol{R}^N$ and is denoted by $\mathscr{M}$. We write $\dim \mathscr{M} = m$. Let $F = (F_1, \cdots, F_N)^T \in \boldsymbol{R}^N$ satisfy $F > 0$, i.e., $F_i > 0$ for $i = 1, \cdots, N$. If

$$A_{ij}^{kl}(F_iF_j - F_kF_l) = 0 \tag{5.4}$$

holds for any i, j, k, l, F is called a Maxwellian. F is a Maxwellian if and only if $Q_i(F, F) = 0$ for $i = 1, \cdots, N$, provided that $F > 0$. The set of all Maxwellians will be denoted by Γ. Then Γ is a m-dimensional open manifold in $\boldsymbol{R}^N$.

We apply the method of Chapmann-Enskog to (5.1) and obtain the Euler and the Navier-Stokes equations. These equations are the equations of hydrodynamics derived from the discrete kinetic theory. Let ψ_k, $k = 1, \cdots, m$, be a basis of $\mathscr{M}$ and let $F \in \boldsymbol{R}^N$. We set

$$w = (w_1, \cdots, w_m), \ w_k = \langle \psi_k, F \rangle , \quad k = 1, \cdots, m , \tag{5.5}$$

where $\langle \ , \ \rangle$ denotes the standard inner product in $\boldsymbol{R}^N$. The w_k are called the hydrodynamical moments or the macroscopic variables. The unknowns of the Euler and the Navier-Stokes equations obtained by using the Chapman-Enskog expansion are the w_k. The Euler equation takes the form of (1.1), while the Navier-Stokes equation is written in the form of (1.2).

Now let us regard (5.5) as the definition of a mapping from $(\boldsymbol{R}_+)^N$ into $\boldsymbol{R}^m$ which sends F to w. We set

$$\Omega = \{w = (w_1, \cdots, w_m)^T; \ w_k = \langle \psi_k, F \rangle, \ k = 1, \cdots, m, \text{ for some } F > 0\} . \tag{5.6}$$

It is known that the mapping $F \to w$ defined by (5.5) can be regarded as a diffeomorphism from Γ onto Ω. This result is due to Gatignol. (See, for the proof, Appendix 2 of [4]). As a consequence, $\Omega \subset \boldsymbol{R}^m$ is an open set. We shall see that Ω is convex. Let w, $w' \in \Omega$. Then there exist $F > 0$ and $F' > 0$ such that $\langle \psi_k, F \rangle = w_k$, $\langle \psi_k, F' \rangle = w'_k$, $k = 1, \cdots, m$. It follows that for $0 \leqq \lambda \leqq 1$, $\langle \psi_k, \lambda F + (1 - \lambda)F' \rangle = \lambda w_k + (1 - \lambda)w'_k$, $k = 1, \cdots, m$. Since $\lambda F + (1 - \lambda)F' > 0$, this implies that $\lambda w + (1 - \lambda)w' \in \Omega$. Hence, Ω is convex. Briefly, Ω is a convex open set. The unknown w of the system of hydrodynamical equations takes values in Ω.

It can be shown that the system of the Navier-Stokes equations with w as the dependent variable is symmetrizable on $\mathcal{O}_w = \Omega$. In other words, there exists an entropy function defined on $\mathscr{O}_w = \Omega$. Hence we can transform the system in a symmetric form by introducing a new dependent variable u in place of w. Let us consider Condition N for the symmetric system with u as the dependent variable. This condition can be verified under an additional assumption, i.e., the existence of a particular basis of $\mathscr{M}$. Therefore, applying Theorem 3.1 and a result of Kawashima (Theorem 2.9 of [8]), the initial value problem for the system of Navier-Stokes equations is solved locally in time. The additional assumption can be checked for various concrete models among which we mention the 14-velocity and the 32-velocity models introduced by Cabannes [1], [2]. (See in this respect [10], [12].) The detailed proofs of the results presented in this section as well as the global existence theorems will be given in the forthcoming paper [9].

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