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EXTENSIONS OF THE BIG PICARD'S THEOREM

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1. The purpose of this paper is to improve on the n-dimensional extension of the big Picard's theorem given by H. Wu in his paper [7]. We shall prove the following Theorems A and B.

THEOREM A. *Let M be a complex manifold, S a regular thin analytic subset of M and f a holomorphic map of $M - S$ into the N-dimensional complex projective space $P_N(C)$. If f is of rank r somewhere and if $f(M-S)$ omits $2N - r + 2$ hyperplanes in general position, then f can be extended to a holomorphic map of M into $P_N(C)$, where the rank of f at a point $x \in M - S$ means the rank of the Jacobian matrix of f at x.*

This is a generalization of Theorem 5.1 in [3]. Indeed, putting $r = 1$ in Theorem A, we see that every non-constant holomorphic map of $M - S$ into $P_N(C)$ excluding $2N + 1$ hyperplanes in general position can be holomorphically extended to M.

THEOREM B. *Let f be a holomorphic map of the n-dimensional complex euclidean space C^n into $P_N(C)$ excluding h hyperplanes in general position ($h \geqq N + 1$). Then $f(C^n)$ is included in a linear subvariety of dimension $[N/(h - N)]$ in $P_N(C)$, where $[N/(h - N)]$ denotes the largest integer which does not exceed $N/(h - N)$.*

Consider the special case $h = 2N + 1$. If $f(C^n)$ omits $2N + 1$ hyperplanes in general position; then f reduces to a constant (c.f., [2], Theorem IV). This is an improvement of the result of H. Wu in [7]. Moreover, Theorem B implies that the image of any non-degenerate holomorphic map f of C^N into $P_N(C)$ cannot omit $N + 2$ hyperplanes in general position, because $f(C^N)$ includes a non-empty open subset of $P_N(C)$ which is of dimension $N(> N/((N + 2) - N) = N/2)$. This gives an affirmative answer to the conjecture of H. Wu in [7].

2. The proofs of Theorems A and B are based on the following

THEOREM 1. *Let $f_0(z), f_1(z), \cdots, f_{N+1}(z)$ be $N + 2$ nowhere zero holomorphic functions on $\{r_0 < |z| < \infty\}$ in C^1, where r_0 is a non-negative real number. If $\sum_{i=0}^{N+1} f_i(z) \equiv 0$, then there is a partition of indices $I =$*

$\{0, 1, \cdots, N+1\} = I_1 \cup \cdots \cup I_k$ $(I_l \cap I_m = \emptyset,\ l \neq m)$ *with the property that for each* l $(1 \leqq l \leqq k)$ (i) $\sum_{i \in I_l} f_i(z) \equiv 0$ *and* (ii) *any* $f_i f_j^{-1}(i, j \in I_l)$ *can be meromorphically extended to* $\{r_0 < |z| \leqq \infty\}$.

It was firstly stated by H. Cartan in [1] that Theorem 1 is shown by the same argument as in the proof of the classical theorem of E. Borel (c.f., for example, Proposition 5.15 in [8]) given by R. Nevanlinna in [5]. But, the H. Cartan's statements seem to be incomplete, so we describe here the outline of the proof. E. Borel's theorem asserts that for any nowhere zero holomorphic functions $f_i(z)$ $(0 \leqq i \leqq N+1)$ on C^1 with $\sum_{i=0}^{N+1} f_i(z) \equiv 0$ there is a (non-trivial) linear relation over C among any $N+1$ of them, where $N \geqq 1$. To prove this, R. Nevanlinna showed that, if there is no linear relation among the functions $f_1 f_0^{-1}, \cdots, f_{N+1} f_0^{-1}$, then

$$\text{(I)} \qquad \varliminf_{r \to \infty} \frac{T(r, f_i f_0^{-1})}{\log r} < \infty$$

for any i $(1 \leqq i \leqq N+1)$, where $T(r, f_i f_0^{-1})$ denotes the Nevanlinna's characteristic function of $f_i f_0^{-1}$. According to this fact, he concluded that any $f_i f_0^{-1}$ $(1 \leqq i \leqq N+1)$ reduces to a constant, which contradicts the assumption. The arguments used there can be also applied to nowhere zero holomorphic functions $f_i(z)$ $(0 \leqq i \leqq N+1)$ on $\{r_0 < |z| < \infty\}$ with $\sum_{i=0}^{N+1} f_i(z) \equiv 0$ and we can conclude the inequality (I) under the assumption that $f_i f_0^{-1}$ $(1 \leqq i \leqq N+1)$ are linearly independent. Thus, it is not difficult to show that each $f_i f_0^{-1}$ has a meromorphic extension to $\{r_0 < |z| \leqq \infty\}$.

To complete the proof of Theorem 1, it suffices to take the partition $I = I_I \cup \cdots \cup I_k$ such that for each I_l (i) $f_i f_j^{-1}(i, j \in I_l)$ can be meromorphically extended to a neighborhood of ∞ and (ii) $f_i f_j^{-1}(i \in I_l, j \in I_m, l \neq m)$ has an essential singularity at ∞. We may assume that $I_1 = \{0, 1, \cdots, i_1\}$, $I_2 = \{i_1+1, i_1+2, \cdots, i_2\}, \cdots, I_k = \{i_{k-1}+1, i_{k-1}+2, \cdots, i_k\}$, where $i_k = N+1$. Then we have $\sum_{l=1}^{k} g_l(z) f_{i_l}(z) \equiv 0$ with the functions $g_l = \sum_{i \in I_l} f_i f_{i_l}^{-1}$. Since each g_l has no essential singularity at ∞, each $\Phi_l(z) = g_l(z) f_{i_l}(z)$ is an identically vanishing or nowhere zero holomorphic functions on $\{r_0' < |z| < \infty\}$ for a suitable r_0' $(r_0 \leqq r_0' < \infty)$. Assume that $\Phi_l \not\equiv 0$ for some l. Changing indices, we may assume that $\Phi_l \not\equiv 0$ if $1 \leqq l \leqq k'$ and $\Phi_l \equiv 0$ if $k'+1 \leqq l \leqq k$. Obviously, each $\Phi_l \Phi_m^{-1}$ $(1 \leqq l < m \leqq k')$ has an essential singularity at ∞. By the above argument, there is a linear relation among any $k'-1$ of Φ_l $(1 \leqq l \leqq k')$. Applying this repeatedly to obtained linear relations, we obtain an absurd conclusion $\Phi_1 \equiv 0$. Thus we have Theorem 1.

3. Now, we generalize Theorem 1 to the case of holomorphic functions of several variables.

THEOREM 2. *Let* $f_0(z), f_1(z), \cdots, f_{N+1}(z)$ *be nowhere zero holomorphic functions on the domain* D_0 *obtained from the unit polydisc* $D = \{|z_i| < 1, 1 \leqq i \leqq n\}$ *by deleting the set* $S = \{z_1 = 0\} \cap D$ *in* C^n *and suppose that* $\sum_{i=0}^{N+1} f_i(z) \equiv 0$ *in* D_0. *Then, there is a partition of indices* $I = I_1 \cup \cdots \cup I_k$ $(I_l \cap I_m = \emptyset, l \neq m)$ *with the property that for each* l (i) $\sum_{i \in I_l} f_i(z) \equiv 0$ *and* (ii) $f_i f_{i_l}^{-1} (i \in I_l)$ *has a holomorphic extension to* D *with a suitable* $i_l \in I_l$.

PROOF. In virtue of Theorem 1, we may assume $n \geqq 2$. The set I of indices is divided into subclasses $I_l (1 \leqq l \leqq k)$ such that in D_0 $\sum_{i \in I_l} f_i(z) \equiv 0$ and $\sum_{i \in I'} f_i(z) \not\equiv 0$ for any proper subset I' of I_l. Without loss of generality, we may assume that $k = 1$, i.e., $\sum_{i \in I'} f_i(z) \not\equiv 0$ for any $I' \subsetneqq I$. For each $I' \subsetneqq I$, we consider the set

$$V_{I'} = \{z' \in D'; \textstyle\sum_{i \in I'} f_i(z_1, z') \equiv 0 \text{ as a function of } z_1\},$$

where $D' = \{z' = (z_2, \cdots, z_n); |z_i| < 1, 2 \leqq i \leqq n\}$. By the assumption, each $V_{I'}$ and so the union V of all $V_{I'} (I' \subsetneqq I)$ are thin analytic subsets of D'. Take an arbitrary $z' \in D' - V$. As a function of z_1, $\sum_{i \in I'} f_i(z_1, z') \not\equiv 0$ for any $I' \subsetneqq I$. By Theorem 1, each holomorphic function

$$g_{ij}(z_1, z') = f_i(z_1, z') f_j(z_1, z')^{-1} \qquad (i, j \in I)$$

can be meromorphically extended to the unit disc $\{|z_1| < 1\}$ as a function of z_1. As is easily seen by the theorem of Rouché, the order m_{ij} of zero of each meromorphic function $g_{ij}(z_1, z')$ at $z_1 = 0$ is a constant which is independent of each $z' \in D' - V$. This means that $h_{ij}(z_1, z') = z_1^{-m_{ij}} g_{ij}(z_1, z')$ is a nowhere zero holomorphic function of z_1 on $\{|z_1| < 1\}$ for each fixed $z' \in D' - V$. It is easily shown by the Cauchy integral formula that h_{ij} is holomorphic on $(D - S) \cap \{|z_1| < 1, z' \in D' - V\}$. Moreover, since codim $(\{z_1 = 0\} \times V) \geqq 2$ in D, each h_{ij} has a nowhere zero holomorphic extension to D by Riemann's theorem on removable singularities. If $m_{i_0 1} = \min (m_{11}, m_{21}, \cdots, m_{N1})$, each $f_i f_{i_0}^{-1} (i \in I)$ is obviously holomorphically extended to D.

COROLLARY 3. *Let* $f_0(z), f_1(z), \cdots, f_{N+1}(z)$ *be nowhere zero holomorphic functions on* C^n *such that* $\sum_{i=0}^{N+1} f_i(z) \equiv 0$. *Then there is a partition of indices* $I = I_1 \cup \cdots \cup I_k (I_l \cap I_m = \emptyset, l \neq m)$ *such that for each* l (i) $\sum_{i \in I_l} f_i(z) \equiv 0$ *and* (ii) *any* $f_i f_j^{-1} (i, j \in I_l)$ *reduces to a constant.*

PROOF. We may assume that $\sum_{i \in I'} f_i(z) \not\equiv 0$ for any $I' \subsetneqq I$. Applying Theorem 2 to the holomorphic functions $g_i(z_1, z') = f_i(1/z_1, z')$ on $\{0 < |z_1| < \infty\} \times C^{n-1}$, $g_i g_{i_0}^{-1}$ is bounded holomorphic on C^1 for a suitable $i_0 \in I$ and for any fixed z' in C^{n-1}. Therefore, each $f_i f_j^{-1}$ is a constant function of z_1.

The similar assertions are valid for the other coordinates. Thus we have Corollary 3.

4. Consider the unit polydisc D and the subset $S=\{z_1=0\}\cap D$ of D. For non-zero holomorphic functions $f_0(z), f_1(z), \cdots, f_N(z)$ on $D-S$, we have the uniquely determined partition of indices $J=\{0, 1, \cdots, N\}=J_1\cup\cdots\cup J_p$ with the following property:

(*) Each $f_i f_j^{-1}$ has a meromorphic extension to D if $i, j\in J_q$ $(1\leqq q\leqq p)$ and has essential singularities on S if $i\in J_q$, $j\in J_{q'}(q\neq q')$.

LEMMA 4. *In the above situation, if* $f_i(z)\neq 0$ $(0\leqq i\leqq N)$ *and if* $\sum_{i=0}^N f_i(z)\neq 0$ *everywhere on* $D-S$, *then* $\sum_{i\in J_q} f_i(z)\equiv 0$ *for any* q $(1\leqq q\leqq p)$ *except exactly one index* q_0.

PROOF. Put $f_{N+1}=-(f_0+f_1+\cdots+f_N)$, which vanishes nowhere on $D-S$. Applying Theorem 2 to the identity $\sum_{i=0}^{N+1} f_i(z)\equiv 0$, we have a partition of indices $\{0, 1, \cdots, N+1\}=I_1\cup\cdots\cup I_k$ such that $\sum_{i\in I_l} f_i(z)\equiv 0$ and $f_i f_j^{-1}$ is meromorphic on D for each $i, j\in I_l (1\leqq l\leqq k)$. It may be assumed that $N+1\in I_k$. Then, by the property of J_q, we have $I_l\subset J_q$ whenever $I_l\cap J_q\neq\emptyset$ and $1\leqq l\leqq k-1$. Moreover, we can take the index q_0 with $I_k-\{N+1\}\subset J_{q_0}$. As is easily seen, $\sum_{i\in J_q} f_i(z)\equiv 0$ for any $q\neq q_0$ and $\sum_{i\in J_{q_0}} f_i(z)\not\equiv 0$.

Let $f_0(z), f_1(z), \cdots, f_N(z)$ be non-zero holomorphic functions on C^n. In this case, we consider the partition of indices $J=J_1\cup\cdots\cup J_p$ with the following property:

(**) Each $f_i f_j^{-1}$ is a constant function if $i, j\in J_q$ and does not reduce to a constant if $i\in J_q$, $j\in J_{q'}(q\neq q')$.

By the similar argument to that of the proof of Lemma 4, and by using Corollary 3 instead of Theorem 2, we have the following lemma.

LEMMA 5. *In the above situation, if* $f_i(z)\neq 0$ $(0\leqq i\leqq N)$ *and* $\sum_{i=0}^N f_i(z)\neq 0$ *everywhere on* C^n, *then* $\sum_{i\in J_q} f_i(z)\equiv 0$ *for any* q *except exactly one index* q_0.

5. Now, we start to prove Theorem A. The argument we use is essentially the same as in the proof of Theorem IV and Theorem V in [2]. Since our problem is of local character, it may be assumed that $M=D=\{|z_i|<1, 1\leqq i\leqq n\}$ and $S=\{z_1=0\}\cap D$ in C^n. We can choose a system of homogeneous coordinates $w_0: w_1: \cdots: w_N$ in $P_N(C)$ such that the omitted hyperplanes $H_0, H_1, \cdots, H_{h-1}$ $(h=2N-r+2)$ can be written as follows:

H_i: $w_i = 0$, $(0 \leqq i \leqq N)$,

H_{N+s}: $\alpha_s^0 w_0 + \alpha_s^1 w_1 + \cdots + \alpha_s^N w_N = 0$, $(1 \leqq s \leqq t = h - N - 1)$

where any minor of degree $\leqq \min(t, N+1)$ of the matrix (α_s^i) $(1 \leqq s \leqq t, 0 \leqq i \leqq N)$ does not vanish. Then, the well-defined holomorphic functions $f_i = (w_i \circ f)(w_0 \circ f)^{-1}$ $(0 \leqq i \leqq N)$ vanish nowhere and

$$\alpha_s^0 f_0 + \alpha_s^1 f_1 + \cdots + \alpha_s^N f_N \neq 0 \qquad (1 \leqq s \leqq t)$$

everywhere on $D - S$. Consider the partition of indices $J = \{0, 1, \cdots, N\} = J_1 \cup \cdots \cup J_p$ with the property (*) in §4 for the holomorphic functions $f_i (0 \leqq i \leqq N)$. It suffices to show that $p = 1$. Indeed, in this case, each $f_i f_{i_0}^{-1} (i \in J)$ is holomorphic on D for a suitable $i_0 \in J$. This shows that f has a holomorphic extension to D.

Assume that $p \geqq 2$. Since $\alpha_s^i \neq 0$ for any s and i, each partition of indices with the property (*) in §4 for the functions $\alpha_s^0 f_0, \alpha_s^1 f_1, \cdots, \alpha_s^N f_N$ is given by the above partition $J = J_1 \cup \cdots \cup J_p$. By Lemma 4, for each $s (1 \leqq s \leqq t)$, there is the uniquely determined $q(s) (1 \leqq q(s) \leqq p)$ such that $\sum_{i \in J_{q(s)}} \alpha_s^i f_i(z) \not\equiv 0$. We put $m_q = \#\{s; q(s) = q, 1 \leqq s \leqq t\}$, where $\#A$ denotes the number of elements in a set A. Obviously, $t = m_1 + m_2 + \cdots + m_p$ and $K_q = \{s; \sum_{i \in J_q} \alpha_s^i f_i \equiv 0, 1 \leqq s \leqq t\}$ consists of $t - m_q$ elements. The image of the map $(f_i)_{i \in J_q}$ of $D - S$ into $C^{N_q} (N_q = \#J_q)$ is included in a linear variety $L = \{\sum_{i \in J_q} \alpha_s^i w_i = 0, s \in K_q\}$ in C^{N_q} which is of dimension $N_q - (t - m_q)$, because the rank of $(\alpha_s^i)(s \in K_q, i \in J_q)$ is equal to $\min(N_q, t - m_q)$, where $L \neq (0)$. So we see $t - m_q \leqq N_q - 1$. Therefore, by the assumption $p \geqq 2$ the image of the map $(f_0, f_1, \cdots, f_N)$ of $D - S$ into C^{N+1} is included in a linear subvariety of dimension

$$\sum_{q=1}^p N_q - (t - m_q) = N + 1 - pt + t = N + 1 - (p-1)t \leqq N + 1 - t .$$

Thus, $f(D - S)$ is included in a subvariety of dimension $\leqq (N + 1 - t) - 1 = N - t = N - (N - r + 1) = r - 1$ in $P_N(C)$. On the other hand, since f is of rank r somewhere, $f(D - S)$ includes an r-dimensional set in $P_N(C)$. This is a contradiction. The proof of Theorem A is complete.

6. In Theorem A, we cannot omit the assumption of the regularity of a thin analytic set S in M (c.f., [3], §4). For an arbitrary thin analytic set S, we can prove

THEOREM 6. *Under the same condition as in Theorem A, if the assumption on the regularity of S is omitted, then f can be extended to a meromorphic map of M into $P_N(C)$, i.e., the closure of the graph $G_f = \{(z, f(z)): z \in M - S\}$ of f is an analytic subset of $M \times P_N(C)$.*

PROOF. Take a system of homogeneous coordinates $w_0: w_1: \cdots: w_N$ such that $f(M-S) \cap \{w_0=0\} = \emptyset$ and put $f_i = (w_i \circ f) \cdot (w_0 \circ f)^{-1} (1 \leqq i \leqq N)$. By Theorem A, each f_i can be meromorphically extended to a neighborhood of the set S_{reg} of all regularities of S. On the other hand, since $S - S_{\text{reg}}$ is an analytic subset of codimension $\geqq 2$ in M, each f_i has a meromorphic extension to the whole space M. This leads to Theorem 6.

COROLLARY 7. *Let M be a complex manifold, S a thin analytic subset of M and f a holomorphic map of $M - S$ into $P_N(C)$ which is of rank r somewhere. If there are hyperplanes $H_1, \cdots, H_h (h = 2N - r + 2)$ in general position such that each $\overline{f^{-1}(H_i)}$ is a thin analytic subset of M, then f has a meromorphic extension to M.*

PROOF. Put $S' = S \cup (\bigcup_{i=1}^{h} \overline{f^{-1}(H_i)})$. Then $f' = f \mid M - S'$ has the image in $P_N(C) - \bigcup_{i=1}^{h} H_i$, so has a meromorphic extension to M by Theorem 6. This gives Corollary 7.

7. It remains to prove Theorem B. We shall show this by some simple modifications of the proof of Theorem A (c.f., [2]). We use the same notations as in §5. Choosing a suitable system of homogeneous coordinates $w_0: w_1: \cdots: w_N$ in $P_N(C)$, we have nowhere zero holomorphic functions $f_i(z) = (w_i \circ f) \cdot (w_0 \circ f)^{-1}$ $(0 \leqq i \leqq N)$ on C^n such that $\sum_{i=0}^{N} \alpha_s^i f_i(z) \neq 0$ everywhere for any $s (1 \leqq s \leqq t)$, where any minor of the matrix (α_s^i) does not vanish. Consider the partition $J = \{0, 1, \cdots, N\} = J_1 \cup \cdots \cup J_p$ with the property (**) in §4. Using Lemma 5, for each s $(1 \leqq s \leqq t)$ we have just only one $q(s)$ $(1 \leqq q(s) \leqq p)$ such that $\sum_{i \in J_q} \alpha_s^i f_i \not\equiv 0$. Then, putting $m_q = \#\{s; q(s) = q\}$ and $N_q = \# J_q (1 \leqq q \leqq p)$, we have $t = \sum_q m_q$ and $t - m_q \leqq N_q - 1$. It follows that

$$\sum_q (t - m_q) = pt - t \leqq \sum_q (N_q - 1) = N + 1 - p ,$$

so $p \leqq (N + t + 1)/(t + 1)$. On the other hand, since $f_i f_j^{-1} (i, j \in J_q)$ is a constant function, the image of the map $(f_i)_{i \in J_q}$ of C^n into C^{N_q} is included in a subvariety of dimension one. Therefore, $f(C^n) (\subset P_N(C))$ is a subset of a linear subvariety of dimension

$$p - 1 \leqq \frac{N + t + 1}{t + 1} - 1 = \frac{N}{h - N} .$$

Thus we have Theorem B.

8. Finally, we note that, in the conclusion of Theorem B, we cannot replace the number $n_0 = [N/(h - N)]$ by smaller ones. Indeed, for an arbitrarily given h hyperplanes $H_0, H_1, \cdots, H_{h-1}$ $(N + 1 \leqq h \leqq 2N)$ in general position in $P_N(C)$, we can construct a holomorphic map f of C^{n_0} into $P_N(C) -$

$\bigcup_{i=0}^{h-1} H_i$ such that f is of rank n_0 everywhere. As in §5, for a suitable system of homogeneous coordinates $w_0 : \cdots : w_N$ we have

$$H_i\colon w_i = 0 \qquad (0 \leqq i \leqq N)$$
$$H_{N+s}\colon \alpha_s^0 w_0 + \alpha_s^1 w_1 + \cdots + \alpha_s^N w_N = 0 \qquad (1 \leqq s \leqq t = h - N - 1)\ ,$$

where we may assume $\alpha_s^0 = 1$ $(1 \leqq s \leqq t)$. Consider n_0 systems of t linear equations in $t + 1(=u)$ unknowns:

$$\textstyle\sum_1\colon \alpha_s^1 w_1 + \cdots + \alpha_s^u w_u = 0,\ (1 \leqq s \leqq t)\ ,$$
$$\textstyle\sum_2\colon \alpha_s^{u+1} w_{u+1} + \cdots + \alpha_s^{2u} w_{2u} = 0,\ (1 \leqq s \leqq t)\ ,$$
$$\cdots\cdots$$
$$\textstyle\sum_{n_0}\colon \alpha_s^{(n_0-1)u+1} w_{(n_0-1)u+1} + \cdots + \alpha_s^{n_0 u} w_{n_0 u} = 0\ , \qquad (1 \leqq s \leqq t)\ .$$

Obviously, each system $\sum_\nu$ $(1 \leqq \nu \leqq n_0)$ has a non-zero vector $(a_{(\nu-1)u+1}, \cdots, a_{\nu u})$ as a solution. Then, any a_i $(1 \leqq i \leqq n_0 u)$ is not equal to zero because any minor of degree t of the matrix (α_s^i) $(1 \leqq s \leqq t, 1 \leqq i \leqq N)$ does not vanish. If $n_0 u < N$, we take furthermore non-zero real numbers $a_{n_0 u+1}, \cdots, a_N$ such that

$$\alpha_s^{n_0 u+1} a_{n_0 u+1} + \cdots + \alpha_s^N a_N + 1 \neq 0$$

for any s $(1 \leqq s \leqq t)$. Now, we put $f_0(z_1, \cdots, z_{n_0}) \equiv 1$,

$$f_i(z_1, \cdots, z_{n_0}) = \begin{cases} a_i e^{z_1}\ , & 1 \leqq i \leqq u \\ a_i e^{z_2}\ , & u + 1 \leqq i \leqq 2u \\ \cdots\cdots & \\ a_i e^{z_{n_0}}\ , & (n_0 - 1)u + 1 \leqq i \leqq n_0 u \end{cases}$$

and $f_i(z_1, \cdots, z_{n_0}) \equiv a_i$, $n_0 u + 1 \leqq i \leqq N$, if $n_0 u < N$. Then the map $f = f_0\colon f_1\colon \cdots\colon f_N$ of C^{n_0} into $P_N(C)$ has the image in $P_N(C) - \bigcup_{i=0}^{h-1} H_i$ and is of rank n_0 everywhere.

In particular, considering the case $h = 2N$, we see that there is a non-constant holomorphic map of C into $P_N(C)$ excluding $2N$ arbitrarily pre-assigned hyperplanes in general position. This gives another proof of Theorem 6 in [4] (c.f., W. Stoll, [6]).

ADDENDUM. After submitting the manuscript to the editor, the author received from Dr. M. L. Green the preprint entitled "Holomorphic maps into the complex projective space omitting hyperplanes" to appear in Trans. Amer. Math. Soc.. One of his results is the same as our Theorem B and he informed us in his letter that a result quite similar to our Theorem A has also been obtained.

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