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WEAK AND STRONG CONVERGENCE THEOREMS OF A MANN-TYPE ITERATIVE ALGORITHM FOR k-STRICT PSEUDO-CONTRACTIONS

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Abstract. In this paper, we consider a Mann-type iterative algorithm for nonself strict pseudo-contractions. Weak and strong convergence theorems are established in the framework of Hilbert spaces. The results presented in this paper improve and extend the results announced by many others.

1. Introduction and Preliminaries

Let K be a nonempty subset of a Hilbert space H and T be a nonlinear mapping from K into H. Let $F(T)$ denote the set of fixed points of T, that is, $F(T) = \{x \in K : Tx = x\}$. Recall that the mapping $T : K \to H$ is said to be k-strictly pseudo-contractive if there exists a constant $k \in [0, 1)$ such that

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in K. \tag{1.1}$$

The class of k-strictly pseudo-contractive mappings includes the class of nonexpansive mappings T on K, that is,

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in K. \tag{1.2}$$

Note that T is nonexpansive if and only if T is 0-strictly pseudo-contractive. The mapping $T : K \to H$ is also said to be pseudo-contractive if the coefficient $k = 1$. T is said to be strongly pseudo-contractive if there exists a positive constant $\lambda \in (0, 1)$ such that $T + \lambda I$ is pseudo-contractive. We also remark that the class of strongly pseudo-contractive mappings is independent of the class of k-strictly

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pseudo-contractive mappings (see [1, 4, 21]). Clearly, the class of k-strictly pseudo-contractive mappings falls into the one between classes of nonexpansive mappings and pseudo-contractive mappings.

The class of strict pseudo-contractions is one of the most important classes of mappings among nonlinear mappings. Within the past several decades, many authors have been devoting to the studies on the existence and convergence of fixed points for pseudo-contractions (see [1, 4, 6, 7, 9, 10, 13, 14, 19-22] and the references therein).

Recall that the normal Mann's iterative process was introduced by Mann [12] in 1953. Since then, construction of fixed points for nonexpansive mappings and k-strict pseudo-contractions via Mann's iterative process has been extensively investigated by many authors. The normal Mann's iterative process generates a sequence $\{x_n\}$ in the following manner:

$$(1.3) \qquad \begin{cases} x_1 = x \in K \quad \text{arbitrarily chosen,} \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad \forall n \geq 1, \end{cases}$$

where $\{\alpha_n\}$ is a sequence in $(0,1)$.

If T is a nonexpansive mapping with a fixed point and the control sequence $\{\alpha_n\}$ is chosen so that $\sum_{n=0}^{\infty} \alpha_n(1-\alpha_n) = \infty$, then the sequence $\{x_n\}$ generated by the normal Mann's iterative process (1.3) converges weakly to a fixed point of T (this is also valid in a uniformly convex Banach space with a Fréchet differentiable norm [16]).

In 1967, Browder and Petryshyn [2] established the first convergence result for k-strictly pseudo-contractive self-mappings in real Hilbert spaces. They proved weak and strong convergence theorems by using the algorithm (1.3) with a constant control sequence $\{\alpha_n\} = \alpha$ for all $n \geq 1$. Afterward, Rhoades [15] generalized in part the corresponding results in [2] in the sense that a variable control sequence $\{\alpha_n\}$ was taken into consideration. Under the assumption that the domain of mapping T is compact convex, he established a strong convergence theorem by using the algorithm (1.3) with a control sequence $\{\alpha_n\}$ satisfying the conditions $\alpha_1 = 1$, $0 < \alpha_n < 1$, $\sum_{n=1}^{\infty} \alpha_n = \infty$ and $\limsup_{n\to\infty} \alpha_n = \alpha < 1-k$. However, without the compact assumption on the domain of mapping T, in general, one cannot expect to infer any weak convergence results from Rhoades' convergence theorem.

Recently, Marino and Xu [10] obtained a weak convergence theorem by using the normal Mann iterative algorithm (1.3) with a control sequence $\{\alpha_n\}$ satisfying the conditions $0 < \alpha_n < 1-k$ and $\sum_{n=1}^{\infty}(1-\alpha_n-k)\alpha_n = \infty$. Very recently, Zhou [21] further improved the results of Marino and Xu [10] from the self-mappings to the non-self mappings and also relaxed the restriction on parameters.

Attempts to modify the normal Mann's iteration method for nonexpansive mappings and k-strict pseudo-contractive mappings so that strong convergence is guaranteed have recently been obtained (see [1, 3-5, 8-10, 19, 21] and the references therein).

Kim and Xu [8] introduced the following iteration process:

$$(1.4)\qquad \begin{cases} x_0 = x \in K \quad \text{arbitrarily chosen,} \\ y_n = \beta_n x_n + (1-\beta_n)Tx_n, \\ x_{n+1} = \alpha_n u + (1-\alpha_n)y_n, \quad \forall n \geq 0, \end{cases}$$

where T is a nonexpansive mapping of K into itself, $u \in K$ is a given point. They proved that the sequence $\{x_n\}$ defined by (1.4) converges strongly to a fixed point of T provided that the control sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy appropriate conditions.

Recently, Marino and Xu [11] introduced the following general iterative algorithm:

$$(1.5)\qquad \begin{cases} x_0 \in H \quad \text{arbitrarily chosen,} \\ x_{n+1} = \alpha_n \gamma f(x_n) + (I - \alpha_n A)Tx_n, \quad \forall n \geq 0, \end{cases}$$

where T is a self-nonexpansive mapping on H, A is a strongly positive bounded linear operator on H. They prove that the sequence $\{x_n\}$ defined by above iterative process converges strongly to a fixed point of T which solves uniquely the following variation inequality

$$(1.6)\qquad \langle (A - \gamma f)x^*, x - x^* \rangle \leq 0, \quad \forall x \in H,$$

and is also the optimality condition for some minimization problems.

Very recently, Zhou [21] modified the normal Mann's iterative process (1.3) for non-self k-strict pseudo-contractions to have strong convergence in Hilbert spaces. To be more precise, he proved the following result:

Theorem Z. *Let C be a closed convex subset of a real Hilbert space H and $T : C \to H$ be a k-strictly pseudo-contractive non-self mapping such that $F(T) \neq \emptyset$. For any $u \in C$ and sequences $\{\alpha_n\}$ and $\{\beta_n\}$ in $(0, 1)$, the following control conditions are satisfied:*

(i) $\beta_n \to 0$, $\sum_{n=1}^{\infty} \beta_n = \infty$;

(ii) $k \leq \alpha_n \leq b < 1$ *for all* $n \geq 1$;

(iii) $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$, $\sum_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$ *or* $\frac{\beta_n}{\beta_{n+1}} \to 1$ *as* $n \to \infty$.

Let a sequence $\{x_n\}$ in C be generated in the following manner:

$$\begin{cases} x_1 = x \in C \quad arbitrarily\ chosen, \\ y_n = P_C(\alpha_n x_n + (1-\beta_n)Tx_n), \\ x_{n+1} = \beta_n u + (1-\beta_n)y_n, \quad \forall n \geq 1, \end{cases}$$

where P_C is a projection from H onto C. Then $\{x_n\}$ converges strongly to a fixed point z of T, where $z = P_{F(T)}u$.

In this paper, motivated by Cho, Kang and Qin [4], Kim and Xu [8], Marino and Xu [10, 11], Yao, Chen and Yao [19], Yao, Chen and Zhou [20] and Zhou [21], we prove weak and strong convergence theorems for a finite family of non-self k-strict pseudo-contractions in Hilbert spaces. Our results improve and extend the corresponding ones announced by many others.

In order to prove our main results, we need the following definitions and lemmas:

Throughout this paper, we use P_K to denote the metric projection of H onto its closed convex subset K. Recall that a self mapping $f : K \to K$ is contractive on K if there exists a constant $\alpha \in (0, 1)$ such that

$$\|f(x) - f(y)\| \leq \alpha\|x - y\|, \quad \forall x, y \in K. \tag{1.7}$$

We use Π_K to denote the collection of all contractive mappings on K, that is, $\Pi_K = \{f | f : K \to K \text{ a contractive mapping}\}$.

Recall that an operator A is strongly positive if there exists a constant $\bar{\gamma} > 0$ such that

$$\langle Ax, x\rangle \geq \bar{\gamma}\|x\|^2, \quad \forall x \in H. \tag{1.8}$$

Lemma 1.1. ([21]). *Let K be a nonempty closed convex subset of a Hilbert space H and $T : K \to H$ a k-strict pseudo-contraction. Then we have the following:*

(1) *The set $F(T)$ of fixed points of T is closed convex so that the projection $P_{F(T)}$ is well defined.*

(2) *If $F(T) \neq \emptyset$, then $F(P_K T) = F(T)$.*

(3) *Define a mapping $S : K \to H$ by $Sx = \lambda x + (1 - \lambda)Tx$ for each $x \in K$. Then, as $\lambda \in [k, 1)$, S is a nonexpansive mapping such that $F(S) = F(T)$.*

Lemma 1.2. ([17]). *Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in a Banach space X and $\{\beta_n\}$ be a sequence in $[0, 1]$ with*

$$0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1.$$

Suppose that $x_{n+1} = (1 - \beta_n)y_n + \beta_n x_n$ for all integers $n \geq 0$ and

$$\limsup_{n\to\infty}(\|y_{n+1} - y_n\| - \|x_{n+1} - x_n\|) \leq 0.$$

Then $\lim_{n\to\infty} \|y_n - x_n\| = 0$.

Lemma 1.3. ([18]). *Assume that* $\{\alpha_n\}$ *is a sequence of nonnegative real numbers such that*

$$\alpha_{n+1} \leq (1-\gamma_n)\alpha_n + \delta_n, \quad \forall n \geq 1,$$

where γ_n *is a sequence in* $(0,1)$ *and* $\{\delta_n\}$ *is a sequence such that*

(i) $\sum_{n=1}^{\infty} \gamma_n = \infty$;

(ii) $\limsup_{n\to\infty} \frac{\delta_n}{\gamma_n} \leq 0$ *or* $\sum_{n=1}^{\infty} |\delta_n| < \infty$.

Then $\lim_{n\to\infty} \alpha_n = 0$.

Lemma 1.4. ([11]). *Assume that* A *is a strongly positive linear bounded operator on a Hilbert space* H *with coefficient* $\bar{\gamma} > 0$ *and* $0 < \rho \leq \|A\|^{-1}$. *Then* $\|I - \rho A\| \leq 1 - \rho\bar{\gamma}$.

Lemma 1.5. ([11]). *Let* H *be a Hilbert space and* A *be a strongly positive linear bounded self-adjoint operator on* H *with coefficient* $\bar{\gamma} > 0$. *Assume that* $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$. *Let* $T : H \to H$ *be a nonexpansive mapping with a fixed point* x_t *of the contractive mapping* $x \mapsto t\gamma f(x) + (1 - tA)Tx$. *Then* $\{x_t\}$ *converges strongly as* $t \to 0$ *to a fixed point* $\bar{x}$ *of* T *which solves the variational inequality:*

$$\langle (A - \gamma f)\bar{x}, z - \bar{x}\rangle \leq 0, \ \forall z \in F(T).$$

Lemma 1.6. ([10]). *Let* H *be a real Hilbert space. Then the following equations hold:*

(1) $\|x \pm y\|^2 = \|x\|^2 \pm \langle x, y\rangle + \|y\|^2$ *for all* $x, y \in H$.

(2) $\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2$ *for all* $t \in [0,1]$ *and* $x, y \in H$.

Lemma 1.7. *In a Hilbert space* H, *the inequality holds:*

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, (x+y)\rangle, \quad \forall x, y \in H.$$

The following lemmas can be easily obtained from Proposition 2.6 of Acedo and Xu [1].

Lemma 1.8. *For any integer* $N \geq 1$, *assume that, for each* $1 \leq i \leq N$, $T_i : K \to H$ *is a* k_i*-strictly pseudo-contractive mapping for some* $0 \leq k_i < 1$. *Assume that* $\{\eta_i\}_{i=1}^N$ *is a sequence of positive numbers such that* $\sum_{i=1}^N \eta_i = 1$. *Then* $\sum_{i=1}^N \eta_i T_i$ *is a non-self* k*-strictly pseudo-contractive mapping with* $k = \max\{k_i : 1 \leq i \leq N\}$.

Lemma 1.9. *Let* $\{T_i\}_{i=1}^N$ *and* $\{\eta_i\}_{i=1}^N$ *be given as in Lemma 1.8. Suppose that* $\{T_i\}_{i=1}^N$ *has a common fixed point. Then* $F(\sum_{i=1}^N \eta_i T_i) = \cap_{i=1}^N F(T_i)$.

2. Main Results

Now, we are ready to give our main results in this paper.

Theorem 2.1. *Let K be a nonempty closed convex subset of a real Hilbert space H and $T_i : K \to H$ a k_i-strict pseudo-contraction for some $0 \leq k_i < 1$ $(i = 1, 2, \cdots, N)$. Assume that $\cap_{i=1}^{\infty} F(T_i) \neq \emptyset$. Let $\{x_n\}$ be a sequence in K generated via the following manner:*

$$\begin{cases} x_1 \in K \quad arbitrarily\ chosen, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n) P_K S x_n, \quad \forall n \geq 1, \end{cases}$$

where $S : K \to H$ is defined by $Sx = kx + (1-k)\sum_{i=1}^{N} \eta_i T_i x$, $\sum_{i=1}^{N} \eta_i = 1$ and $\alpha_n = \frac{\beta_n - k}{1-k}$ for all $n \geq 1$. If $\{\alpha_n\}$ is chosen such that $\{\alpha_n\} \in [k, 1)$ and $\sum_{n=1}^{\infty} (\beta_n - k)(1-\beta_n) = \infty$, where $k = \max\{k_i : 1 \leq i \leq N\}$, then $\{x_n\}$ converges weakly to a common fixed point of $\{T_i\}_{i=1}^{N}$.

Proof. Define $Tx = \sum_{i=1}^{N} \eta_i T_i x$. By Lemma 1.8 and Lemma 1.9, we conclude that $T : K \to H$ is a k-strict pseudo-contraction with $k = \max\{k_i : 1 \leq i \leq N\}$ and $F(T) = F(\sum_{i=1}^{N} \eta_i T_i) = \cap_{i=1}^{\infty} F(T_i)$. It follows that $Sx = kx + (1-k)\sum_{i=1}^{N} \eta_i T_i x$ reduces to $Sx = kx + (1-k)Tx$. From Lemma 1.1, we see that $S : K \to H$ is a nonexpansive mapping and $F(S) = F(T)$. Also, we have $\cap_{i=1}^{N} F(T_i) \neq \emptyset$, that is, $F(S) \neq \emptyset$. Since $P_K : H \to K$ is nonexpansive, we see that $P_K S : K \to K$ is also nonexpansive.

On the other hand, we see that

$$\sum_{n=1}^{\infty} \alpha_n (1-\alpha_n) = \frac{1}{(1-k)^2} \sum_{n=1}^{\infty} (\beta_n - k)(1-\beta_n) = \infty.$$

At this point, we utilize Theorem 2 given by Reich [16] to conclude that $\{x_n\}$ converges weakly to a fixed point of $P_K S$. Observing that

$$F(P_K S) = F(S) = F(T) = F(\sum_{i=1}^{N} \eta_i T_i) = \cap_{i=1}^{\infty} F(T_i),$$

we can obtain the desired conclusion. This completes the proof.

Remark 2.1. Theorem 2.1 improves and extends Theorem 3.1 of Marino and Xu [10] in the following sense:

(1) from k-strictly pseudo-contractive self-mappings to k-strictly pseudo-contractive nonself-mappings;

(2) the proof method is simpler than the one used in Marino and Xu [10];

(3) relaxing the restrictions on parameters.

Remark 2.2. Theorem 2.1 also improves and extends the results of Zhou [21]. To be more precisely, Theorem 2.1 improves Theorem 3.1 of Zhou [21] from a single mapping to a finite family of mappings.

Next, we modify the normal Mann's iterative process for a finite family of k-strict pseudo-contractions to have strong convergence.

In order to prove our strong convergence theorems, we need the following lemma:

Lemma 2.1. *Let K be a nonempty closed convex subset of a Hilbert space H such that $K \pm K \subset K$. Let A be a strongly positive linear bounded self-adjoint operator with the coefficient $\bar{\gamma} > 0$ such that $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$ and $T : K \to H$ a k-strictly pseudo-contractive non-self mapping with a fixed point. For a fixed $f \in \Pi_K$ with the coefficient $(0 < \alpha < 1)$ and sequences $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in $(0, 1)$, assume that the following control conditions are satisfied:*

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \quad \lim_{n\to\infty} \alpha_n = 0;$

(ii) $0 < \liminf_{n\to\infty} \beta_n \le \limsup_{n\to\infty} \beta_n < 1;$

(iii) $k \le \gamma_n \le \lambda < 1$ *for all* $n \ge 1;$

(iv) $\sum_{n=1}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty.$

Let $\{x_n\}$ be a sequence defined by

$$(\Upsilon) \qquad \begin{cases} x_1 = x \in K \quad arbitrarily\ chosen, \\ y_n = P_C(\gamma_n x_n + (1-\gamma_n)Tx_n), \\ x_{n+1} = \alpha_n \gamma f(x_n) + \beta_n x_n + ((1-\beta_n)I - \alpha_n A)y_n, \quad \forall n \ge 1. \end{cases}$$

Then $\{x_n\}$ converges strongly to some $q \in F(T)$, which also solves the following variational inequality:

$$\langle \gamma f(q) - Aq, p - q \rangle \le 0, \quad \forall p \in F(T). \tag{2.1}$$

Proof. Note that, from the conditions (i) and (ii), we may assume, without loss of generality, that $\alpha_n \le (1-\beta_n)\|A\|^{-1}$. Since A is a strongly positive bounded linear self-adjoint operator on C, we have

$$\|A\| = \sup\{|\langle Ax, x\rangle| : x \in H, \|x\| = 1\}.$$

Observe that

$$\begin{aligned}\langle((1-\beta_n)I-\alpha_nA)x,x\rangle &= 1-\beta_n-\alpha_n\langle Ax,x\rangle\\ &\geq 1-\beta_n-\alpha_n\|A\|\\ &\geq 0,\end{aligned}$$

that is, $(1-\beta_n)I-\alpha_nA$ is positive. It follows that

$$\begin{aligned}\|(1-\beta_n)I-\alpha_nA\| &= \sup\{\langle((1-\beta_n)I-\alpha_nA)x,x\rangle : x\in H, \|x\|=1\}\\ &= \sup\{1-\beta_n-\alpha_n\langle Ax,x\rangle : x\in H, \|x\|=1\}\\ &\leq 1-\beta_n-\alpha_n\bar{\gamma}.\end{aligned}$$

For any $p\in F(T)$, from (1.1) and Lemma 1.6, we see

$$\begin{aligned}\|y_n-p\|^2 &= \|P_C[\gamma_nx_n+(1-\gamma_n)Tx_n]-p\|^2\\ &\leq \|\gamma_n(x_n-p)+(1-\gamma_n)(Tx_n-p)\|^2\\ &= \gamma_n\|x_n-p\|^2+(1-\gamma_n)\|Tx_n-p\|^2-\gamma_n(1-\gamma_n)\|Tx_n-x_n\|^2\\ &\leq \|x_n-p\|^2-(1-\gamma_n)(\gamma_n-k)\|x_n-Tx_n\|^2\\ &\leq \|x_n-p\|^2.\end{aligned}$$

It follows that

$$\begin{aligned}&\|x_{n+1}-p\|\\ &= \|\alpha_n(\gamma f(x_n)-Ap)+\beta_n(x_n-p)+((1-\beta_n)I-\alpha_nA)(y_n-p)\|\\ &\leq (1-\beta_n-\alpha_n\bar{\gamma})\|x_n-p\|+\beta_n\|x_n-p\|+\alpha_n\|\gamma f(x_n)-Ap\|\\ &\leq (1-\alpha_n\bar{\gamma})\|x_n-p\|+\alpha_n\alpha\gamma\|x_n-p\|+\alpha_n\|\gamma f(p)-Ap\|\\ &= [1-(\bar{\gamma}-\gamma\alpha)\alpha_n]\|x_n-p\|+\alpha_n\|\gamma f(p)-Ap\|.\end{aligned}$$

By simple inductions, we have

$$\|x_n-p\|\leq \max\{\|x_0-p\|, \frac{\|Ap-\gamma f(p)\|}{\bar{\gamma}-\gamma\alpha}\}, \quad \forall n\geq 1,$$

which yields that the sequence $\{x_n\}$ is bounded, so is $\{y_n\}$. Define a mapping $T_nx := P_K(\gamma_nx+(1-\gamma_n)Tx)$ for all $x\in K$. Then $T_n : K\to K$ is nonexpansive for each $n\geq 1$. Indeed, by using (1.1) and Lemma 1.6, we have, for all $x,y\in K$,

$$\begin{aligned}\|T_nx-T_ny\|^2 &= \|P_C[\gamma_nI+(1-\gamma_n)T]x-P_C[\gamma_nI+(1-\gamma_n)T]y\|^2\\ &\leq \|[\gamma_nI+(1-\gamma_n)T]x-[\gamma_nI+(1-\gamma_n)T]y\|^2\\ &= \gamma_n\|x-y\|^2+(1-\gamma_n)\|Tx-Ty\|^2\end{aligned}$$

$$
\begin{aligned}
&- \alpha_n(1-\gamma_n)\|(I-T)x-(I-T)y\|^2 \\
&\le \gamma_n\|x-y\|^2 + (1-\gamma_n)[\|x-y\|^2 + k\|(I-T)x-(I-T)y\|^2] \\
&- \gamma_n(1-\gamma_n)\|(I-T)x-(I-T)y\|^2 \\
&\le \|x-y\|^2,
\end{aligned}
$$

which implies that T_n is nonexpansive. It follows that the iterative process (Υ) reduces to

$$
x_{n+1} = \alpha_n\gamma f(x_n) + \beta_n x_n + [(1-\beta_n)I - \alpha_n A]T_n x_n, \quad \forall n \ge 1. \tag{2.2}
$$

Define $l_n = \frac{x_{n+1}-\beta_n x_n}{1-\beta_n}$, that is, $x_{n+1} = (1-\beta_n)l_n + \beta_n x_n$ for all $n \ge 1$. It follows that

$$
\begin{aligned}
l_{n+1} - l_n &= \frac{x_{n+2}-\beta_{n+1}x_{n+1}}{1-\beta_{n+1}} - \frac{x_{n+1}-\beta_n x_n}{1-\beta_n} \\
&= \frac{\alpha_{n+1}\gamma f(x_{n+1}) + [(1-\beta_{n+1})I - \alpha_{n+1}A]T_{n+1}x_{n+1}}{1-\beta_{n+1}} \\
&\quad - \frac{\alpha_n\gamma f(x_n) + [(1-\beta_n)I - \alpha_n A]T_n x_n}{1-\beta_n} \\
&= \frac{\alpha_{n+1}}{1-\beta_{n+1}}\gamma f(x_{n+1}) - \frac{\alpha_n}{1-\beta_n}\gamma f(x_n) + T_{n+1}x_{n+1} - T_n x_n \\
&\quad + \frac{\alpha_n}{1-\beta_n}AT_n x_n - \frac{\alpha_{n+1}}{1-\beta_{n+1}}AT_{n+1}x_{n+1} \\
&= \frac{\alpha_{n+1}}{1-\beta_{n+1}}(\gamma f(x_{n+1}) - AT_{n+1}x_{n+1}) + T_{n+1}x_{n+1} - T_{n+1}x_n \\
&\quad + T_{n+1}x_n - T_n x_n + \frac{\alpha_n}{1-\beta_n}(AT_n x_n - \gamma f(x_n)),
\end{aligned}
$$

which implies that

$$
\begin{aligned}
&\|l_{n+1} - l_n\| - \|x_{n+1} - x_n\| \\
&\le \frac{\alpha_{n+1}}{1-\beta_{n+1}}\|\gamma f(x_{n+1}) - AT_{n+1}x_{n+1}\| + \|T_{n+1}x_n - T_n x_n\| \\
&+ \frac{\alpha_n}{1-\beta_n}\|AT_n x_n - \gamma f(x_n)\|.
\end{aligned} \tag{2.3}
$$

Next, we estimate $\|T_{n+1}x_n - T_n x_n\|$. In fact, we have

$$
\begin{aligned}
&\|T_{n+1}x_n - T_n x_n\| \\
&= \|P_C[\gamma_{n+1}x_n + (1-\gamma_{n+1})Tx_n] - P_C[\gamma_n x_n + (1-\gamma_n)Tx_n]\| \\
&\le \|[\gamma_{n+1}x_n + (1-\gamma_{n+1})Tx_n] - [\gamma_n x_n + (1-\gamma_n)Tx_n]\| \\
&\le \|x_n - Tx_n\||\gamma_{n+1} - \gamma_n|.
\end{aligned} \tag{2.4}
$$

Substituting (2.4) into (2.3), we have

$$\begin{aligned}
&\|l_{n+1}-l_n\|-\|x_{n+1}-x_n\| \\
\le{}& \frac{\alpha_{n+1}}{1-\beta_{n+1}}\|\gamma f(x_{n+1})-AT_{n+1}x_{n+1}\|+\|x_n-Tx_n\||\gamma_{n+1}-\gamma_n| \\
&+\frac{\alpha_n}{1-\beta_n}\|AT_nx_n-\gamma f(x_n)\|.
\end{aligned}$$

It follows from the conditions (i), (ii) and (iv) that

$$\limsup_{n\to\infty}\{\|l_{n+1}-l_n\|-\|x_{n+1}-x_n\|\}\le 0.$$

From Lemma 1.2, we obtain

$$\lim_{n\to\infty}\|l_n-x_n\|=0. \tag{2.5}$$

On the other hand, we have

$$\|x_{n+1}-x_n\|=(1-\beta_n)\|x_n-l_n\|.$$

It follows from the condition (ii) and (2.5) that

$$\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \tag{2.6}$$

Finally, we show that $x_n\to q$ strongly. From (2.2), we can obtain

$$\begin{aligned}
&\|T_nx_n-x_n\| \\
={}& \|x_n-x_{n+1}\|+\|x_{n+1}-T_nx_n\| \\
\le{}& \|x_n-x_{n+1}\|+\beta_n\|x_n-T_nx_n\|+\alpha_n(\|AT_nx_n\|+\gamma\|f(x_n)\|),
\end{aligned}$$

which togethers with the conditions (i), (ii) and (2.6) implies that

$$\lim_{n\to\infty}\|T_nx_n-x_n\|=0. \tag{2.7}$$

On the other hand, by the conditions (iii) and (iv), we have $\gamma_n\to\rho$ as $n\to\infty$, where $\rho\in[k,1)$. Define a mapping $W:K\to H$ by $Wx=\rho x+(1-\rho)Tx$. Then W is nonexpansive with $F(W)=F(T)$ by Lemma 1.1. It also follows from Lemma 1.1 that $F(P_KW)=F(W)=F(T)$. Notice that

$$\begin{aligned}
&\|P_KWx_n-x_n\| \\
\le{}& \|x_n-T_nx_n\|+\|T_nx_n-P_KWx_n\| \\
\le{}& \|x_n-T_nx_n\|+\|\gamma_nx_n+(1-\gamma_n)Tx_n-[\rho x_n+(1-\rho)Tx_n]\| \\
\le{}& \|x_n-T_nx_n\|+|\gamma_n-\rho|\|x_n-Tx_n\|,
\end{aligned}$$

which, combining with (2.7), yields that

$$\lim_{n\to\infty} \|P_K W x_n - x_n\| = 0. \tag{2.8}$$

Now, we claim that

$$\limsup_{n\to\infty} \langle \gamma f(q) - Aq, x_n - q\rangle \le 0, \tag{2.9}$$

where $q = \lim_{t\to 0} x_t$ with x_t being the fixed point of the contractive mapping

$$x \mapsto t\gamma f(x) + (I - tA)P_K W x.$$

That is, $x_t = t\gamma f(x_t) + (I - tA)P_K W x_t$. Thus we have

$$\|x_t - x_n\| = \|(I - tA)(P_K W x_t - x_n) + t(\gamma f(x_t) - Ax_n)\|.$$

It follows from Lemma 1.7 that

$$\begin{aligned} \|x_t - x_n\|^2 &= \|(I - tA)(P_K W x_t - x_n) + t(\gamma f(x_t) - Ax_n)\|^2 \\ &\le (1 - \bar{\gamma}t)^2 \|P_K W x_t - x_n\|^2 + 2t\langle \gamma f(x_t) - Ax_n, x_t - x_n\rangle \\ &\le (1 - 2\bar{\gamma}t + (\bar{\gamma}t)^2)\|x_t - x_n\|^2 + f_n(t) \\ &+ 2t\langle \gamma f(x_t) - Ax_t, x_t - x_n\rangle + 2t\langle Ax_t - Ax_n, x_t - x_n\rangle, \end{aligned} \tag{2.10}$$

where

$$f_n(t) = (2\|x_t - x_n\| + \|x_n - P_K W x_n\|)\|x_n - P_K W x_n\| \to 0 \tag{2.11}$$

as $n \to 0$. Observing that A is linear strongly positive and using (1.8), we have

$$\langle Ax_t - Ax_n, x_t - x_n\rangle = \langle A(x_t - x_n), x_t - x_n\rangle \ge \bar{\gamma}\|x_t - x_n\|^2. \tag{2.12}$$

Thus, from (2.10) and (2.12), we have

$$\begin{aligned} &2t\langle Ax_t - \gamma f(x_t), x_t - x_n\rangle \\ &\le (\bar{\gamma}^2 t^2 - 2\bar{\gamma}t)\|x_t - x_n\|^2 + f_n(t) + 2t\langle Ax_t - Ax_n, x_t - x_n\rangle \\ &\le (\bar{\gamma}t^2 - 2t)\langle A(x_t - x_n), x_t - x_n\rangle + f_n(t) + 2t\langle Ax_t - Ax_n, x_t - x_n\rangle \\ &\le \bar{\gamma}t^2\langle A(x_t - x_n), x_t - x_n\rangle + f_n(t) \end{aligned}$$

and so

$$\langle Ax_t - \gamma f(x_t), x_t - x_n\rangle \le \frac{\bar{\gamma}t}{2}\langle Ax_t - Ax_n, x_t - x_n\rangle + \frac{1}{2t} f_n(t). \tag{2.13}$$

Letting $n \to \infty$ in (2.13) and noting that (2.11), we arrive at

$$\limsup_{n\to\infty}\langle Ax_t - \gamma f(x_t), x_t - x_n\rangle \le \frac{t}{2}M_1, \tag{2.14}$$

where $M_1 > 0$ is a constant such that $M_1 \ge \bar{\gamma}\langle Ax_t - Ax_n, x_t - x_n\rangle$ for all $t \in (0,1)$ and $n \ge 1$. Taking $t \to 0$ from (2.14), we have

$$\limsup_{t\to 0}\limsup_{n\to\infty}\langle Ax_t - \gamma f(x_t), x_t - x_n\rangle \le 0. \tag{2.15}$$

On the other hand, we have

$$\begin{aligned}
&\langle \gamma f(q) - Aq, x_n - q\rangle \\
&= \langle \gamma f(q) - Aq, x_n - q\rangle - \langle \gamma f(q) - Aq, x_n - x_t\rangle \\
&\quad + \langle \gamma f(q) - Aq, x_n - x_t\rangle - \langle \gamma f(q) - Ax_t, x_n - x_t\rangle \\
&\quad + \langle \gamma f(q) - Ax_t, x_n - x_t\rangle - \langle \gamma f(x_t) - Ax_t, x_n - x_t\rangle \\
&\quad + \langle \gamma f(x_t) - Ax_t, x_n - x_t\rangle.
\end{aligned}$$

It follows that

$$\begin{aligned}
&\limsup_{n\to\infty}\langle \gamma f(q) - Aq, x_n - q\rangle \\
&\le \|\gamma f(q) - Aq\|\|x_t - q\| + \|A\|\|x_t - q\|\lim_{n\to\infty}\|x_n - x_t\| \\
&\quad + \gamma\alpha\|q - x_t\|\lim_{n\to\infty}\|x_n - x_t\| + \limsup_{n\to\infty}\langle \gamma f(x_t) - Ax_t, x_n - x_t\rangle.
\end{aligned}$$

Therefore, from (2.15), we have

$$\begin{aligned}
&\limsup_{n\to\infty}\langle \gamma f(q) - Aq, x_n - q\rangle \\
&= \limsup_{t\to 0}\limsup_{n\to\infty}\langle \gamma f(q) - Aq, x_n - q\rangle \\
&\le \limsup_{t\to 0}\|\gamma f(q) - Aq\|\|x_t - q\| + \limsup_{t\to 0}\|A\|\|x_t - q\|\lim_{n\to\infty}\|x_n - x_t\| \\
&\quad + \limsup_{t\to 0}\gamma\alpha\|q - x_t\|\lim_{n\to\infty}\|x_n - x_t\| \\
&\quad + \limsup_{t\to 0}\limsup_{n\to\infty}\langle \gamma f(x_t) - Ax_t, x_n - x_t\rangle \\
&\le 0.
\end{aligned}$$

Hence, (2.9) holds. Now, from Lemma 1.7, we have

$$
\begin{aligned}
&\|x_{n+1}-q\|^2\\
&=\|((1-\beta_n)I-\alpha_nA)(T_nx_n-q)+\beta_n(x_n-q)+\alpha_n(\gamma f(x_n)-Aq)\|^2\\
&\le\|((1-\beta_n)I-\alpha_nA)(T_nx_n-q)+\beta_n(x_n-q)\|^2\\
&\quad+2\alpha_n\langle\gamma f(x_n)-Aq,x_{n+1}-q\rangle\\
&\le[(1-\beta_n-\alpha_n\bar{\gamma})\|x_n-q\|+\beta_n\|x_n-q\|]^2\\
&\quad+\alpha_n\gamma\alpha(\|x_n-q\|^2+\|x_{n+1}-q\|^2)+2\alpha_n\langle\gamma f(q)-Aq,x_{n+1}-q\rangle\\
&\le(1-\alpha_n\bar{\gamma})^2\|x_n-q\|^2+\alpha_n\gamma\alpha(\|x_n-q\|^2+\|x_{n+1}-q\|^2)\\
&\quad+2\alpha_n\langle\gamma f(q)-Aq,x_{n+1}-q\rangle,
\end{aligned}
\tag{2.16}
$$

which implies that

$$
\begin{aligned}
&\|x_{n+1}-q\|^2\\
&\le\frac{(1-\alpha_n\bar{\gamma})^2+\alpha_n\gamma\alpha}{1-\alpha_n\gamma\alpha}\|x_n-q\|^2+\frac{2\alpha_n}{1-\alpha_n\gamma\alpha}\langle\gamma f(q)-Aq,x_{n+1}-q\rangle\\
&\le[1-\frac{2\alpha_n(\bar{\gamma}-\alpha\gamma)}{1-\alpha_n\gamma\alpha}]\|x_n-q\|^2\\
&\quad+\frac{2\alpha_n(\bar{\gamma}-\alpha\gamma)}{1-\alpha_n\gamma\alpha}[\frac{1}{\bar{\gamma}-\alpha\gamma}\langle\gamma f(q)-Aq,x_{n+1}-q\rangle+\frac{\alpha_n\bar{\gamma}^2}{2(\bar{\gamma}-\alpha\gamma)}M_2],
\end{aligned}
\tag{2.17}
$$

where M_2 is an appropriate constant such that $M_2\ge\sup_{n\ge1}\{\|x_n-q\|^2\}$. Put

$$j_n=\frac{2\alpha_n(\bar{\gamma}-\alpha\gamma)}{1-\alpha_n\alpha\gamma}$$

and

$$t_n=\frac{1}{\bar{\gamma}-\alpha\gamma}\langle\gamma f(q)-Aq,x_{n+1}-q\rangle+\frac{\alpha_n\bar{\gamma}^2}{2(\bar{\gamma}-\alpha\gamma)}M_2.$$

Then, from (2.17), we have

$$\|x_{n+1}-q\|^2\le(1-j_n)\|x_n-q\|^2+j_nt_n. \tag{2.18}$$

It follows from the condition (i) and (2.9) that

$$\lim_{n\to\infty}j_n=0,\quad\sum_{n=1}^{\infty}j_n=\infty,\quad\limsup_{n\to\infty}t_n\le0.$$

Therefore, applying Lemma 1.3 to (2.18), we obtain $x_n\to q$ as $n\to\infty$. This completes the proof.

Taking $\gamma = 1$ and $A = I$ (: the identity mapping) in Lemma 2.1, we have the following result.

Theorem 2.2. *Let K be a nonempty closed convex subset of a Hilbert space H and $T : K \to H$ a k-strictly pseudo-contractive non-self mapping such that $F(T) \neq \emptyset$. For any fixed $f \in \Pi_C$ with the coefficient $(0 < \alpha < 1)$ and sequences $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}$ in $(0, 1)$, assume that the following control conditions are satisfied:*

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \quad \lim_{n\to\infty} \alpha_n = 0;$

(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1;$

(iii) $k \leq \gamma_n \leq \lambda < 1$ *for all* $n \geq 1$;

(iv) $\sum_{n=1}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty.$

Let $\{x_n\}$ be the composite process defined by

$$(2.19) \qquad \begin{cases} x_1 = x \in C \quad arbitrarily\ chosen, \\ y_n = P_C[\gamma_n x_n + (1 - \gamma_n)Tx_n], \\ x_{n+1} = \alpha_n f(x_n) + \beta_n x_n + (1 - \beta_n - \alpha_n)y_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ converges strongly to some $q \in F(T)$, which also solves the following variational inequality:

$$(2.20) \qquad \langle f(q) - q, p - q\rangle \leq 0, \quad \forall p \in F(T).$$

Remark 2.3. Theorem 2.2 improves Cho, Kang and Qin [5], Kim and Xu [8], Yao, Chen and Yao [19], Yao, Chen and Zhou [20], respectively. To be more precise, we extend above results from nonexpansive self-mappings to k-strictly pseudo-contractive non-self mappings.

If $T : K \to H$ is a nonexpansive mapping, that is, T is 0-strict pseudo-contractive, we have the following theorem:

Theorem 2.3. *Let K be a nonempty closed convex subset of a Hilbert space H and $T : K \to H$ a nonexpansive non-self mapping such that $F(T) \neq \emptyset$. For any fixed $f \in \Pi_C$ with coefficient $(0 < \alpha < 1)$ and sequences $\{\alpha_n\}$, $\{\beta_n\}$ in $(0, 1)$, assume that the following control conditions are satisfied:*

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \quad \lim_{n\to\infty} \alpha_n = 0;$

(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1;$

(iii) $\sum_{n=1}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty$.

Let $\{x_n\}$ *be a sequence defined by* (2.19). *Then* $\{x_n\}$ *converges strongly to a point* $q \in F(T)$ *which also solves the variational inequality* (2.20).

Remark 2.4. Theorem 2.3 includes the result of Yao, Chen and Zhou [20] as a special case. To be more precise, Theorem 2.3 reduces to Theorem 3.1 of Yao, Chen and Zhou [20] when T is a self-mapping, $\{\gamma_n\} = 0$ for all $n \geq 1$ and, for all $x \in K$, $f(x) = u \in K$.

Now, we are in a position to prove a strong convergence theorem for a finite family of k-strictly pseudo-contractive non-self mappings.

Theorem 2.4. *Let* K *be a nonempty closed convex subset of a Hilbert space* H *such that* $K \pm K \subset K$. *Let* A *be a strongly positive linear bounded self-adjoint operator with the coefficient* $\bar{\gamma} > 0$ *such that* $0 < \gamma < \frac{\bar{\gamma}}{\alpha}$ *and* $\{T\}_{i=1}^{N} : K \to H$ *a finite family of* k*-strictly pseudo-contractive non-self mappings. Assume* $\cap_{i=1}^{N} F(T_i) \neq \emptyset$. *Let* $\{\eta_i\} \subset (0,1)$ *be* N *real numbers with* $\sum_{i=1}^{N} \eta_i = 1$. *For any fixed* $f \in \Pi_K$ *with the coefficient* $(0 < \alpha < 1)$ *and sequences* $\{\alpha_n\}$, $\{\beta_n\}$ *and* $\{\gamma_n\}$ *in* $(0,1)$, *assume that the following control conditions are satisfied:*

(i) $\sum_{n=0}^{\infty} \alpha_n = \infty, \quad \lim_{n\to\infty} \alpha_n = 0$;

(ii) $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$;

(iii) $k \leq \gamma_n \leq \lambda < 1$ *for all* $n \geq 1$;

(iv) $\sum_{n=1}^{\infty} |\gamma_{n+1} - \gamma_n| < \infty$.

Let $\{x_n\}$ *be the composite process defined by*

$$\begin{cases} x_1 = x \in C \quad arbitrarily\ chosen, \\ y_n = P_C[\gamma_n x_n + (1-\gamma_n)\sum_{i=1}^{N} \eta_i T_i x_n], \\ x_{n+1} = \alpha_n \gamma f(x_n) + \beta_n x_n + [(1-\beta_n)I - \alpha_n A]y_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *converges strongly to somw* $q \in \cap_{i=1}^{N} F(T_i)$, *which also solves the following variational inequality:*

$$\langle \gamma f(q) - Aq, p - q \rangle \leq 0, \quad \forall p \in \cap_{i=1}^{N} F(T_i).$$

Proof. Define $Tx = \sum_{i=1}^{N} \eta_i T_i x$. By Lemma 1.8 and Lemma 1.9, we conclude that $T : K \to H$ is a k-strictly pseudo-contractive mapping with $k = \max\{k_i : 1 \leq i \leq N\}$ and $F(T) = F(\sum_{i=1}^{N} \eta_i T_i) = \cap_{i=1}^{\infty} F(T_i)$. From Lemma 2.1, we can easily obtain desired conclusion.

REFERENCES

1. G. L. Acedo and H. K. Xu, Iterative methods for strict pseudo-contractions in Hilbert spaces, *Nonlinear Anal.*, **67** (2007), 2258-2271.
2. F. E. Browder and W. V. Petryshyn, Construction of fixed points of nonlinear mappings in Hilbert space, *J. Math. Anal. Appl.*, **20** (1967), 197-228.
3. Y. J. Cho, S. M. Kang and X. Qin, Approximation of common fixed points of an infinite family of nonexpansive mappings in Banach spaces, *Comput. Math. Appl.*, **56** (2008), 2058-2064.
4. Y. J. Cho, S. M. Kang and X. Qin, *Some results on k-strictly pseudo-contractive mappings in Hilbert spaces,* Nonlinear Anal., (2008), doi:10.1016/j.na.2008.02.094.
5. Y. J. Cho, S. M. Kang and X. Qin, *Convergence theorems of fixed points for a finite family of nonexpansive mappings in Banach spaces,* Fixed Point Theory Appl. 2008 (2008), Article ID 856145.
6. S. S. Chang, Y. J. Cho, B. S. Lee, J. S. Jung and S. M. Kang, Iterative approximations of fixed points and solutions for strongly accretive and strongly pseudo-contractive mappings in Banach spaces, *J. Math. Anal. Appl.*, **224** (1998), 149-165.
7. L. C. Ceng, S. Al-Homidan, Q. H. Ansari and J. C. Yao, *An iterative scheme for equilibrium problems and fixed point problems of strict pseudo-contraction mappings*, J. Comput. Appl. Math., (2008), doi:10.1016/j.cam.2008.03.032.
8. T. H. Kim and H. K. Xu, Strong convergence of modified Mann iterations, *Nonlinear Anal.*, **61** (2005), 51-60.
9. T. H. Kim and H. K. Xu, Convergence of the modified Manns iteration method for asymptotically strict pseudo-contractions, *Nonlinear Anal.*, **68** (2008), 2828-2836.
10. G. Marino and H. K. Xu, Weak and strong convergence theorems for k-strict pseudo-contractions in Hilbert spaces, *J. Math. Anal. Appl.*, **329** (2007), 336-349.
11. G. Marino and H. K. Xu, A general iterative method for nonexpansive mappings in Hilbert spaces, *J. Math. Anal. Appl.,* **318** (2006), 43-52.
12. W. R. Mann, Mean value methods in iteration, *Proc. Amer. Math. Soc.*, **4** (1953), 506-510.
13. J. W. Peng and J. C. Yao, Some new iterative algorithms for generalized mixed equilibrium problems with strict pseudo-contractions and monotone mappings, to appear in *Taiwanese J. Math.*.
14. X. Qin and Y. Su, Approximation of a zero point of accretive operator in Banach spaces, *J. Math. Anal. Appl.*, **329** (2007), 415-424.
15. B. E. Rhoades, Fixed point iterations using infinite matrices, *Trans. Amer. Math. Soc.*, **196** (1974), 162-176.
16. S. Reich, Weak convergence theorems for nonexpansive mappings in Banach spaces, *J. Math. Anal. Appl.*, **67** (1979), 274-276.

17. T. Suzuki, Strong convergence of Krasnoselskii and Mann's type sequences for one-parameter nonexpansive semigroups without bochner integrals, *J. Math. Anal. Appl.*, **305** (2005), 227-239.

18. H. K. Xu, Iterative algorithms for nonlinear operators, *J. London Math. Soc.*, **66** (2002), 240-256.

19. Y. Yao, R. Chen and J. C. Yao, Strong convergence and certain control conditions for modified Mann iteration, *Nonlinear Anal.*, **68** (2008), 1687-1693.

20. Y. Yao, R. Chen and H. Zhou, Iterative algorithms to fixed point of nonexpansive mapping, *Acta Math. Sinica* (*Chin. Ser.*), **50** (2007), 139-144.

21. H. Y. Zhou, Convergence theorems of fixed points for k-strict pseudo-contractions in Hilbert space, *Nonliner Anal.*, **69** (2008), 456-462.

22. H. Y. Zhou, Convergence theorems of fixed points for Lipschitz pseudo-contractions in Hilbert spaces, *J. Math. Anal. Appl.*, **343** (2008), 546-556.

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