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A Conversation with Dick Dudley

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Abstract. Richard Mansfield Dudley (Dick Dudley) was born in 1938. He received the A.B. from Harvard in 1952 and the Ph.D. from Princeton in 1962 (under the supervision of Gilbert Hunt and Edward Nelson). Following an appointment at UC Berkeley as an assistant professor, he joined the Department of Mathematics at MIT in 1967. Dick Dudley has made fundamental contributions to the theory of Gaussian processes and Probability in Banach Spaces. Among his major achievements is the development of a general framework for empirical processes theory, in particular, for uniform central limit theorems. These results have had and continue having tremendous impact in contemporary statistics and in mathematical foundations of machine learning. A more extensive biographical sketch is contained in the preface to the *Selected works of R. M. Dudley* (editors: E. Giné, V. Koltchinskii and R. Norvaisa) published in 2010.

This conversation took place (mostly, via email) in the fall of 2017.

Key words and phrases: Biography, probability, statistics.

1. EARLY YEARS

VK, RN & PR: When did you start being interested in mathematics? Was it during your high school years? Earlier? Later?

RD: I got interested in mathematics during my high school years.

VK, RN & PR: Which other subjects were of interest to you during those years?

RD: I was also interested in physics.

VK, RN & PR: Could you tell us a few words about your family? parents? siblings? Any connections of them to academia? Do you remember which books (in mathematics and beyond) impressed you the most? Any high school teachers that had significant influence on you?

RD: My father, Winston M. Dudley, was a mechanical engineer who knew a fair amount of mathematics and gave me a copy of a calculus text by W. F. Osgood he had learned from. It was somewhat nonrigorous and old-fashioned, but stimulating in showing how maximum-minimum problems could be solved via calculus. I also read books aimed at general readers by Einstein and others on physics.

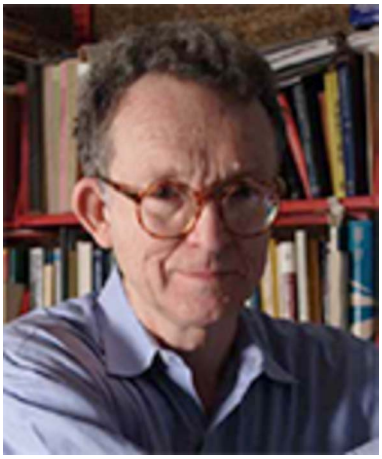

FIG. 1. *Dick Dudley.*

At that time, Advanced Placement Calculus didn't exist in the high school I went to (Chicago Heights).

My father taught at Case School of Applied Science in Cleveland, later merged into Case Western University. Later he taught at Sacramento State College,

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which became California State University at Sacramento.

My sister, Edith Dudley Sylla, is a retired Professor who taught History of Science at North Carolina State University. Her original specialty was physics at Oxford in the mid-14th century. One can find some publications under "Sylla, Edith D." in Web of Science. From a more recent period, she did a translation from Latin of Jakob Bernoulli's book *Ars Conjectandi*.

2. HARVARD

VK, RN & PR: You were at Harvard as an undergraduate. Could you tell us a little bit about Harvard in the 50s? Was mathematics your major? Which other subjects interested you?

RD: As an undergraduate at Harvard, 1955–59, I majored in mathematics. I was also interested in physics. I took only graduate-level courses in math in 1956–59 and did well in them; I also took graduate-level physics courses but did not do as well.

VK, RN & PR: Had you started doing serious research as an undergraduate? Which of Harvard mathematicians taught you during those years? With whom did you interact most? When exactly did you decide to pursue a career in mathematics? When did you decide to specialize in probability?

RD: I took a course in mathematical logic and foundations of mathematics from Prof. Burton Dreben of the Philosophy department. I got interested in recursive functions, a main topic of the course. I and others interested in that started going to a seminar at MIT run by Hartley Rogers. I wrote an undergraduate honors thesis on computable real functions. I wrote a paper showing that such a function must be continuous at each computable real number but may have discontinuities at non-computable real numbers. I submitted the paper for publication but was told that my result was not new, it had been previously proved by a mathematician in Russia.

I was very impressed by a real analysis course given by Lynn Loomis, which had no textbook although it was related to Kelley's General Topology and Loomis's own Abstract Harmonic Analysis. My book *Real Analysis and Probability* owes a lot to Loomis's course.

I was an undergraduate 1955 to 1959, taking mainly graduate courses 1956 to 1959. It was about 1957 that I decided to become a mathematician, as I was getting A's in mathematics courses and B's in graduate-level physics courses.

FIG. 2. *Gilbert Hunt*, 1972 (*from Halmos Collection*).

Around 1959 I decided to specialize in probability, reading Loève's book on my own in the summer of 1959.

Among other students at Harvard, I didn't interact much with classmates, but more with students who would graduate in 1960, Stephen Lichtenbaum (algebra), Robert Solovay (set theory), and Michael Spivak (differential geometry).

3. PRINCETON

VK, RN & PR: You did your Ph.D. at Princeton. Could you tell us about your advisers, Gilbert Hunt and Edward Nelson. With whom of them did you have more interaction? Anything about your classmates at Harvard and at Princeton? Any mathematicians we might know among them?

RD: Gilbert Hunt was supervising first-year graduate students in a seminar. I lectured on Banach algebras, which I was learning about for the first time from notes of Hunt's. There were courses but without grades. I especially liked a course Ed Nelson gave on partial differential equations by way of Fourier analysis and Schwartz distributions. On the side I read Laurent Schwartz's book *Theorie des Distributions*.

Another student at Princeton was Larry Shepp who was working with William Feller. Larry was a year ahead of me.

4. BERKELEY

VK, RN & PR: You moved to Berkeley in the early 60s. Were you in the Department of Statistics and Probability? Could you tell us a few words about this department in those days? With whom did you interact most? (Volker Strassen? Lucien Le Cam? Jack Feldman?)

FIG. 3. *On the right*: *Ed Nelson* (*on the left Bernard Galler*); 1955; *from Halmos Collection*.

RD: I was Assistant Professor of Mathematics in Berkeley, 1962–1966. There was at the time a Statistics department which included some probabilists. Jack Feldman was in the mathematics department, and Lucien Le Cam in statistics. Volker Strassen perhaps visited for only part of 1962–66. I interacted more with the other two. Jack organized a seminar I participated in, and Lucien gave me a lot of manuscripts encouraging joint work, but I preferred to choose my own topics.

VK, RN & PR: Did you start working on weak convergence of measures at Berkeley? On Gaussian processes? On Probability in Banach spaces? Any other important projects during these years?

RD: What I worked on: weak convergence of measures, for example, a paper "Convergence of Baire measures" appeared in 1966. Gaussian processes: the paper "The sizes of compact subsets of Hilbert space and continuity of Gaussian processes" appeared in 1967. Probability in Banach spaces: A paper "On seminorms and probabilities and abstract Wiener spaces" by Dudley, Feldman and LeCam appeared in 1971 with a correction in 1976.

FIG. 4. *Lucien Le Cam in the early* 70*s*.

VK, RN & PR: Which courses did you teach at Berkeley? Did you supervise Ph.D. students there?

RD: Courses taught: each of us Assistant Professors had to teach, each semester, a calculus lecture section of about 200 students, which was divided into 10 smaller sections taught by TAs. The other course we taught was more up to us. Besides that, I substituted for Michel Loève for two weeks in a graduate probability course he was teaching. I think I would not have been qualified to teach statistics at that stage. I had one Ph.D. student while at Berkeley, Chandrakant Mahadeorao Deo, whose thesis was on prediction theory for non-stationary sequences, viewed as sequences in Hilbert space (as in a paper I had in a Berkeley Symposium, 1967).

VK, RN & PR: Could you tell us about probability theory in the mid 60s? Which conferences did you attend? Did you have lots of interactions with probabilists in Europe? In the Soviet Union?

RD: I was not yet connected to the international probability world.

VK, RN & PR: Whom of famous probabilists/statisticians had you met during those years? Wiener? Kolmogorov? Paul Lévy?

RD: I never met Norbert Wiener. I saw him once at the MIT faculty club after one of Hartley Rogers' seminars (not that Wiener came to that seminar). I also never met Paul Lévy. By the time I got to France, Laurent Schwartz had replaced Lévy in the kind of probability I was interested in.

I did meet Kolmogorov. He had a formulation and proof of the law of the iterated logarithm, published

I think in the 1930s. Loève had a formulation in his graduate probability textbook. I thought that Loève's proof was mistaken in that the order of some quantifiers had been reversed. I tried to tell Kolmogorov that when I met him, but he was naturally noncommittal.

VK, RN & PR: Could you tell us about Laurent Schwartz? Have you met him? Where and when?

RD: I met Laurent Schwartz sometime in the 1960s in France, perhaps in Strasbourg.

VK, RN & PR: 1964–65 was quite an interesting time at Berkeley: student protests, free speech movement, etc. Could you tell us about this? Did you participate in these events? Could you tell us about your work on public radio at Berkeley?

RD: Berkeley protests, free speech movement: the protesters were not only students, especially not all undergraduate students, but included some graduate students and some junior faculty. The topics of protests were not only civil rights in the United States and the right of civil rights organizations to enroll members on campus, but also the Vietnam War.

Public Radio at Berkeley: I volunteered to work on the news for radio station KPFA—Berkeley. This was rather a shoestring operation with only one employee paid to work on the news. The sources available while an evening's news was being prepared in an afternoon were (1) the AP (Associated Press) newspaper wire, which came off a teletype machine in a closet, as it was distributed, and (2) the *New York Times* from the preceding day. The two sources complemented each other. Of course, the Internet was invented much later. In one instance, another news volunteer went up on the roof of the building the station's studios were in and saw a lot of large planes (B-52s?) heading West over the Pacific (which would be used to bomb North Vietnam). I worked one afternoon a week. After a while I was entrusted with doing the news on a Saturday night as a solo operation (writing and reading).

At the time there was no national public radio network, but once when out East, public radio station WBAI–New York seemed to me more professional and more interconnected than we were in Berkeley.

5. MIT

VK, RN & PR: Did you move to MIT in 1967? Could you tell us a few words about Mathematics Department in the late 60s?

RD: I moved to MIT in 1967, as a non-tenured Associate Professor. Another member of the department was Daniel Ray. Not too many years later he died suddenly and unexpectedly. Norbert Wiener had passed

FIG. 5. *Dick Dudley in* 1970; *a photo by K. Jacobs from Oberwolfach Collection.*

previously. Henry McKean, a leading probabilist (coauthor of the Itô and McKean book) had departed for NYU-Courant. I thought the department needed a probabilist besides myself and presented a list of who I thought were good candidates, one of whom was Daniel Stroock, who was hired.

VK, RN & PR: Were there other probabilists at MIT? Statisticians? Did you collaborate with someone at MIT or in Boston area at large?

RD: I wrote just one joint paper with Dan Stroock.

There were some statisticians at the Sloan management School, but at the time, none in the mathematics department. Soon the department, specifically the Applied Mathematics Committee (I was in Pure Mathematics, never on the Committee), hired Herman Chernoff from Stanford. Herman set up a program of statistics (and probability) courses. Some instructors and temporary faculty in statistics were hired. There was friction between Herman and the Applied Math Committee. Meanwhile the Harvard Statistics Department hired Peter Huber, but had friction with him. After a while there was a trade in which Herman went from MIT to Harvard, where he remained until his retirement, and Huber from Harvard to MIT, from which he after a year or so went back to Europe.

VK, RN & PR: You did teach statistics courses at MIT (in addition to probability). Could you tell us why,

FIG. 7. *Daniel Ray and Dick Dudley*, 1971; *from Halmos Collection*.

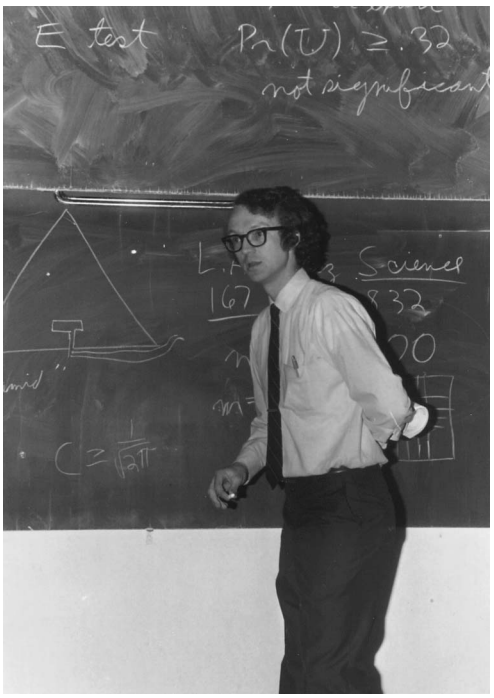

FIG. 6. *Dick Dudley in* 1971; *a photo by K. Jacobs from Oberwolfach Collection*.

in your opinion, there was no statistics department at MIT?

RD: For a while there was a Statistics Center at MIT jointly operated by the Math Department, the Sloan School and the Operations Research Committee. A search for a permanent Director of the Center to replace Herman Chernoff was unsuccessful (if one had been found, the Center might have become a Department, but it didn't happen).

VK, RN & PR: Could you tell us about your students at MIT (Evarist Giné, Ken Alexander, etc)?

RD: I had twenty-some Ph.D. students at MIT and collaborated on papers with several of them. Ken Alexander, a few years after getting his Ph.D. from MIT, changed fields to percolation theory. Evarist Giné did a lot of work on empirical processes, mostly joint with Joel Zinn. I very much like their 1991 *Annals of Probability* paper on uniform Donsker classes, which are useful in respect to bootstrap Donsker classes, among other things.

VK, RN & PR: Empirical processes was one of the main topics of your research at MIT. It also has become a major tool in machine learning. Did you have any interactions with computer scientists at MIT?

RD: I interacted very little with computer scientists at MIT.

6. GAUSSIAN PROCESSES AND PROBABILITY IN BANACH SPACES

VK, RN & PR: Gaussian processes and Probability in Banach spaces were very active areas in the 60s, 70s and 80s, and you were one of the main contributors to these areas. Could you tell us about other important contributors, such as Strassen, Sudakov, Fernique.... Who came up with an idea of entropy bounds on Gaussian processes? Was it Sudakov, Strassen and yourself? What was your most important contribution to Gaussian processes?

FIG. 8. *Evarist Giné*, 1994 *at Yale University*; *picture by courtesy of Roser Giné*.

RD: Gaussian processes and probability in Banach spaces: let C be a subset of Hilbert space H and let L be the isonormal Gaussian process, whose mean is 0 and whose covariance is the inner product. In my 1967 paper "The sizes of compact subsets of Hilbert space and continuity of Gaussian processes", I showed that if a certain integral is finite then the isonormal process restricted to C can be taken to have continuous sample functions. Maybe that was my most important contribution to Gaussian processes? Sudakov proved that $E \sup_{x \in C} L(x)$ is bounded above by a constant times an integral of the metric entropy. Some called this "Dudley's theorem" and there is a Wikipedia article by that title, but I edited it to attribute the theorem to Sudakov. Sudakov also proved a well-known "minoration" for Gaussian processes.

I had useful conversations with Strassen about metric entropy, but I don't know if he had any publications on the topic. At the time he was getting interested in theoretical bounds for speed of computation.

Fernique wrote a highly useful monograph on Gaussian processes, which I relied on in the chapter on such processes in my book *Uniform Central Limit Theorems*.

7. EMPIRICAL PROCESSES

VK, RN & PR: When did you get interested in empirical processes? What was your first work in this area? When did you learn about the work of Vapnik and Chervonenkis? Could you tell us about your work on CLT for empirical processes (Donsker classes)? When did you start and what was your motivation? Was it more a continuation of the line of work on CLT in Banach spaces, or you felt that there might be interesting applications of these ideas in statistics and other areas? What are the most striking results in empirical processes theory in your few? Could you name three of them?

RD: My first work on empirical processes was "Weak convergence of probabilities on nonseparable metric spaces and empirical measures on Euclidean spaces," *Illinois Journal of Math.*, late 1960s, included in *Selected Works of R. M. D.*, Springer, 2010.

I first learned of Vapnik and Chervonenkis' work by being assigned to review for Math. Revs. their announcement and follow-up longer work, because I knew some Russian. "VC" classes became quite important in later work, with extensions from the original classes of sets to classes of functions. A useful hypothesis has been "Koltchinskii–Pollard" entropy of a class of functions, which Evarist Giné called "VC-type".

Work on central limit theorems in separable Banach spaces by Walter Philipp, James Kuelbs, Joel Zinn et al. turned out to have useful nonseparable analogues for empirical processes.

The cover blurb for the second edition of *Uniform Central Limit Theorems* (Cambridge University Press, 2014) lists about a dozen individual theorems by an even larger number of co-authors, and I would not venture to choose three theorems.

8. GENERAL OUTLOOK ON THE DISCIPLINE

VK, RN & PR: How do you see the development of mathematical statistics in the last 20 years? In your article for the memorial volume for Evarist Giné, you wrote that "within statistics", you thought you were "often viewed as a contrarian". Can you elaborate on what you meant there?

RD: About "contrarian": for some period of time, "compact" or "hadamard" differentiability was popular among statisticians. I espoused using different norms to get Frechet differentiability, possibly based on p-variation norms. I wrote a book with Rimas Norvaisa, *Concrete Functional Calculus*. I'm not aware whether either side convinced the other.

VK, RN & PR: How do you see the development of probability theory in the last 20 years?

RD: I have not been following probability in the past 20 years other than the second edition of my book *Uniform Central Limit Theorems.*

9. MISCELLANEA

VK, RN & PR: Do you have any further reflections on your private life throughout your career and now in retirement?

RD: My first wife, now Priscilla C. Grew, is a geologist. I wrote one paper on orientations of crystals jointly with her and Evarist Giné. My second and current wife is Elizabeth A. Martin, who worked in computer science at MIT for 20 years, then various companies for 18 years.

VK, RN & PR: You travelled a lot over the years and visited many universities. In total, how much time have you spent in the following countries: France, Germany, UK, Denmark, Italy, Mexico, Russia (and the former USSR)? Which university campuses do you like most?

RD: I spent one semester at University of Aarhus, Denmark, spring 1976. I went to several one-week meetings at Oberwolfach, Germany. I went to St.-Flour, France to lecture in a probability summer school. I went to summer meetings in Vilnius, Lithuania, at least twice when Lithuania was in the Soviet Union, but then once after independence. I have not been to Russia otherwise. I was in Mexico one week at an institute in Guanajuato. I was in Rome, Italy one week for a meeting. I was in England twice for one-week meetings, once in Durham and once in Cambridge. I was in Paris, France in 2000 for a one-week meeting. I visited Caracas, Venezuela, twice. I prefer rural institutes such as Oberwolfach.

VK, RN & PR: You mentioned that you studied some Russian and one of us heard that you studied German and wrote an essay on Kafka when you were a student. Is it correct?

RD: It was about Kafka's story "Investigations of a Dog" (Forschungen eines Hundes) I conjectured that "Lufthunde" are birds.

VK, RN & PR: What are you favorite authors (not necessarily in mathematics)?

RD: The books I read selections from are mainly those I use in editing Wikipedia, such as Clara Schumann, *An Artist's Life Based on Material Found in Diaries and Letters.*

VK, RN & PR: Could you tell us a bit more about your work for Wikipedia? Which other articles have you contributed to?

RD: I also worked on the articles about violinists Joseph Joachim and Leopold Auer.

VK, RN & PR: What are your favorite painters? Which museums do you like most?

RD: My favorite painter is Claude Monet and museum, the Musée d'Orsay in Paris.

VK, RN & PR: You have always liked classical music and, in the recent years, you contributed articles to Wikipedia about several musicians. What are your favorite composers? Which concerts or recitals impressed you the most (not only in the recent years)?

RD: My favorite composers are J. S. Bach, Mozart (especially piano concertos), and Beethoven (especially late string quartets). I most enjoyed a performance of Bach's St. Matthew Passion in Symphony Hall, Boston with the Tanglewood Festival Chorus.

VK, RN & PR: Could you tell us about your editorial work for the Appalachian Mountain Club *White Mountain Guide* and about your interest in mountain hiking?

RD: I was on the editorial committee for the Appalachian Mountain Club *White Mountain (hiking) Guide* and chief editor for the 1979 edition.