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BASKAKOV TYPE OPERATORS

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ABSTRACT. The actual construction of Baskakov operators and its various modifications requires estimations of infinite series which in a certain sense restrict their usefulness from the computational point of view. Thus the question arises whether the Baskakov operators cannot be replaced by a finite sum. In connection with this question we propose a new family of linear positive operators.

1. Introduction. Approximation properties of the Baskakov operators

$$V_n(f;x) := \sum_{k=0}^{\infty} \binom{n-1+k}{k} x^k (1+x)^{-n-k} f\left(\frac{k}{n}\right), \tag{1}$$

$x \in R_0 := [0, +\infty)$, $n \in N := \{1, 2, \dots\}$ in polynomial weighted spaces C_p were examined in [**2**]. The space C_p, $p \in N_0 := \{0, 1, 2, \dots\}$, considered in [**2**] is associated with the weight function

$$w_0(x) := 1, \quad w_p(x) := (1+x^p)^{-1}, \quad \text{if } p \geq 1, \tag{2}$$

and consists of all real-valued functions f, continuous on R_0 and such that $w_p f$ is uniformly continuous and bounded on R_0. The norm on C_p is defined by the formula

$$\|f\|_p \equiv \|f(\cdot)\|_p := \sup_{x \in R_0} w_p(x)\, |f(x)|. \tag{3}$$

For the reader's convenience we will summarize here the properties of $V_n f$ and related formulae which will be needed later. Most of these can be found in [**2**].

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A. $V_n f$ is a positive linear operator $C_p \to C_p$.

B. $V_n(1; x) = 1$, $V_n f$ preserves constants.

C. For $f \in C_p$, $p \in N_0$,

$$w_p(x)|V_n(f;x) - f(x)| \le K_1(p)\omega_2\left(f; C_p; \sqrt{\frac{x(1+x)}{n}}\right), \quad x \in R_0, \quad n \in N, \tag{4}$$

where $\omega_2(f;\cdot)$ is the modulus of smoothness of order 2 and $K_1(p)$ is a positive constant.

D. For $f \in C_p^2 := \{f \in C_p : f', f'' \in C_p\}$, $p \in N_0$,

$$\lim_{n\to\infty} n(V_n(f;x) - f(x)) = \frac{x(1+x)}{2} f''(x), \quad x \in R_0.$$

E. For every fixed $2 \le q \in N$ there exist algebraic polynomials $P_{j,q}$, $0 \le j \le q$, on the order $m \le q$ and with coefficients depending only on j and q such that

$$V_n\left((t-x)^q; x\right) = \sum_{j=0}^{[q/2]} \frac{P_{j,q}(x)}{n^{q-j}}, \quad x \in R_0, \quad n \in N, \tag{5}$$

where $[q/2]$ denotes the integral part of $q/2$. Moreover, $V_n(t^q; 0) = 0$ for all $n \in N$ and $q \in N$, see [**10**, page 125].

From (4) it was deduced that

$$\lim_{n\to\infty} V_n(f;x) = f(x), \tag{6}$$

for every $f \in C_p$, $p \in N_0$ and $x \in R_0$. Moreover, the above convergence is uniform on every interval $[x_1, x_2]$, $x_1 \ge 0$.

Very recently Gupta [**3**] and Zeng and Gupta [**13**] estimated the rate of convergence of functions of bounded variation for the Bézier variant of the Baskakov operators for the cases $0 < \alpha < 1$ and $\alpha \ge 1$, respectively. Their results improve other related results in the literature. We refer the readers also to Ispir and Atakut [**1**], Gupta and Maheshwari [**4**], Rempulska and Walczak [**6**], Walczak [**10**] and Xie and Zhang [**12**].

Our paper proposes to replace the infinite sum in the Baskakov operators by a truncated sum. It shows that the truncated operators give the same approximation properties as operators V_n. This is done in one and two dimensions.

In this paper by $K_i(\alpha, \beta)$, $i = 1, 2, \ldots$, we shall denote suitable positive constants depending only on indicated parameters α and β.

2. Approximation of the function of one variable. This note was inspired by the results given in our previous papers.

We introduce the following class of operators in C_p, $p \in N$.

Definition 1. We define the class of operators A_n by the formula

$$A_n(f; a_n; x) := \sum_{k=0}^{[n(x+a_n)]} \binom{n-1+k}{k} x^k (1+x)^{-n-k} f\left(\frac{k}{n}\right), \qquad x \in R_0, \quad n \in N, \tag{7}$$

where $(a_n)_1^\infty$ is a sequence of positive numbers such that $\lim_{n\to\infty} \sqrt{n} a_n = \infty$ and $[n(x + a_n)]$ denotes the integral part of $n(x + a_n)$.

Observe that the operator A_n is linear and positive.

Now we shall give the approximation theorem for A_n.

Theorem 1. *Fix $p \in N$. Then for A_n defined by* (7) *we have*

$$\lim_{n\to\infty} \{A_n(f; a_n; x) - f(x)\} = 0, \quad f \in C_p, \tag{8}$$

uniformly on every interval $[x_1, x_2]$, $x_2 > x_1 \geq 0$.

Proof. We first suppose that $f \in C_p$, $p \in N$. From (1) and (7) we obtain

$$\begin{aligned} A_n(f;a_n;x) - f(x) &= \sum_{k=0}^{[n(x+a_n)]} \binom{n-1+k}{k} x^k (1+x)^{-n-k} f\left(\frac{k}{n}\right) - f(x) \\ &= \sum_{k=0}^{\infty} \binom{n-1+k}{k} x^k (1+x)^{-n-k} f\left(\frac{k}{n}\right) - f(x) \\ &\quad - \sum_{k=[n(x+a_n)]+1}^{\infty} \binom{n-1+k}{k} x^k (1+x)^{-n-k} f\left(\frac{k}{n}\right) \\ &= V_n(f;x) - f(x) - M_n(f;a_n;x), \quad x \in R_0, \quad n \in N. \end{aligned}$$

By our assumption, using the elementary inequality $(a+b)^k \leq 2^{k-1}(a^k+b^k)$, $a,b>0$, $k \in N_0$, we get

$$\begin{aligned} |f(t)| &\leq K_2(1+t^p) \leq K_2(1+(|t-x|+x)^p) \\ &\leq K_2(1+2^{p-1}(|t-x|^p+x^p)). \end{aligned} \tag{9}$$

From this and by (1) we get

$$\begin{aligned} |M_n(f;a_n;x)| &\leq \sum_{k=[n(x+a_n)]+1}^{\infty} \binom{n-1+k}{k} x^k (1+x)^{-n-k} \left| f\left(\frac{k}{n}\right)\right| \\ &\leq \sum_{k=[n(x+a_n)]+1}^{\infty} \binom{n-1+k}{k} x^k (1+x)^{-n-k} \\ &\qquad \times K_2\left(1+2^{p-1}\left(\left|\frac{k}{n}-x\right|^p + x^p\right)\right) \\ &\leq K_2\Bigg((1+2^{p-1}x^p) \\ &\quad \sum_{k=[n(x+a_n)]+1}^{\infty} \binom{n-1+k}{k} \\ &\qquad \times x^k(1+x)^{-n-k} + 2^{p-1} \sum_{k=0}^{\infty} \binom{n-1+k}{k} \end{aligned}$$

$$\times x^k(1+x)^{-n-k}\left|\frac{k}{n}-x\right|^p\Bigg)$$

$$= K_2\Bigg((1+2^{p-1}x^p)\sum_{k=[n(x+a_n)]+1}^{\infty}\binom{n-1+k}{k}$$

$$\times x^k(1+x)^{-n-k} + 2^{p-1}V_n\left(|t-x|^p;x\right)\Bigg).$$

We remark that

$$\sum_{k=[n(x+a_n)]+1}^{\infty}\binom{n-1+k}{k}x^k(1+x)^{-n-k}$$

$$\leq \sum_{a_n<|k/n-x|}^{\infty}\binom{n-1+k}{k}x^k(1+x)^{-n-k}$$

$$\leq \sum_{a_n<|k/n-x|}^{\infty}\binom{n-1+k}{k}x^k(1+x)^{-n-k}\frac{|(k/n)-x|^p}{a_n^p}$$

$$\leq \frac{1}{a_n^p}\sum_{k=0}^{\infty}\binom{n-1+k}{k}x^k(1+x)^{-n-k}\left|\frac{k}{n}-x\right|^p$$

$$= \frac{1}{a_n^p}V_n\left(|t-x|^p;x\right).$$

This implies that

$$|M_n(f;a_n;x)| \leq K_3\left(\frac{(1+2^{p-1}x^p)}{a_n^p}+2^{p-1}\right)V_n\left(|t-x|^p;x\right).$$

From this and in view of (5), the Hölder inequality and the property $V_n(1;x)=1$, we further have

$$|M_n(f;a_n;x)| \leq K_3\left(\frac{(1+2^{p-1}x^p)}{a_n^p}+2^{p-1}\right)$$

$$\times\left\{V_n\left((t-x)^{2p};x\right)V_n\left(1;x\right)\right\}^{1/2}$$

$$= K_4\left(\frac{(1+2^{p-1}x^p)}{a_n^p}+2^{p-1}\right)\left\{\sum_{j=0}^{p}\frac{P_{j,2p}(x)}{n^{2p-j}}\right\}^{1/2}$$

$$\leq \frac{K_3}{n^{p/2}}\left(\frac{(1+2^{p-1}x^p)}{a_n^p}+2^{p-1}\right)\left\{\sum_{j=0}^{p}P_{j,2p}(x)\right\}^{1/2}.$$

The properties of a_n

$$\lim_{n\to\infty} \sqrt{n}a_n = \infty$$

imply that

$$\lim_{n\to\infty} M_n(f; a_n; x) = 0$$

uniformly on every interval $[x_1, x_2]$, $x_2 > x_1 \geq 0$. From this and by (6) we obtain

$$\lim_{n\to\infty} \{A_n(f; a_n; x) - f(x)\} = 0,$$

uniformly on every interval $[x_1, x_2]$, $x_2 > x_1 \geq 0$. This ends the proof of (8). $\square$

Similar results for the modified Szász-Mirakyan operators are given in [**11**].

3. Approximation of the function of two variables. Now we shall introduce certain linear positive operators in polynomial weighted spaces of the function of two variables.

Let $p, q \in N_0$, and let

$$w_{p,q}(x, y) := w_p(x)w_q(y), \quad (x, y) \in R_0^2 := R_0 \times R_0, \tag{10}$$

where $w_p(\cdot)$ is defined by (2). Denote by $C_{p,q}$ the weighted space of all real-valued functions f continuous on R_0^2 for which $w_{p,q}f$ is uniformly continuous and bounded on R_0^2. The norm on $C_{p,q}$ is defined by

$$\|f\|_{p,q} \equiv \|f(\cdot,\cdot)\|_{p,q} := \sup_{(x,y)\in R_0^2} w_{p,q}(x, y)\,|f(x, y)|. \tag{11}$$

Approximation properties of certain linear positive operators

$$L_{m,n}(f; x, y) = \frac{1}{(1+(x+m^{-1})^2)^m(1+(y+n^{-1})^2)^n} \sum_{j=0}^{m}\sum_{k=0}^{n} \binom{m}{j}\binom{n}{k} \times (x + m^{-1})^{2j}(y + n^{-1})^{2k} f\left(\frac{j(1+(x+m^{-1})^2)}{m(x+m^{-1})}, \frac{k(1+(y+n^{-1})^2)}{n(y+n^{-1})}\right)$$

in polynomial weighted spaces of functions of two variables were examined in [**9**].

The Baskakov operators of the function of two variables are defined by the following formula

$$\begin{aligned} (12)\quad & V_{m,n}(f;x,y) \\ &= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\binom{m-1+j}{j}x^j(1+x)^{-m-j}\binom{n-1+k}{k} \\ &\qquad y^k(1+y)^{-n-k}f\left(\frac{j}{m},\frac{k}{n}\right). \end{aligned}$$

Arguing, for example, as in the proof of Theorem 2 [**9**, pages 28–29] it is easy to prove the following theorem.

Theorem 2. *Suppose that $f \in C_{p,q}$, $p,q \in N_0$. Then there exists a positive constant $K_5(p,q)$ such that for all $(x,y) \in R_0^2$,*

$$\begin{aligned} (13)\quad & w_{p,q}(x,y)\,|V_{m,n}(f;x,y)-f(x,y)| \\ &\le K_5(p,q)\,\omega_1\left(f,C_{p,q};\sqrt{\frac{x(x+1)}{m}},\sqrt{\frac{y(y+1)}{n}}\right), \end{aligned}$$

$m,n \in N$, where

$$(14)\qquad \omega_1(f,C_{p,q};t,s) := \sup_{\substack{0\le h\le t\\ 0\le\delta\le s}} \|\Delta_{h,\delta}f(\cdot,\cdot)\|_{p,q}, \quad t,s\ge 0,$$

$\Delta_{h,\delta}f(x,y) := f(x+h,y+\delta)-f(x,y)$, $(x+h,y+\delta) \in R_0^2$ is the modulus of continuity of $f \in C_{p,q}$.

From (14) it follows that

$$(15)\qquad \lim_{t,s\to 0+}\omega_1(f,C_{p,q};t,s) = 0$$

for every $f \in C_{p,q}$, $p,q \in N_0$. From this it was deduced that

$$(16)\qquad \lim_{m,n\to\infty} V_{m,n}(f;x,y) = f(x,y), \quad (x,y)\in R_0^2$$

uniformly on every rectangle $0 \le x \le x_0$, $0 \le y \le y_0$.

In this section we shall give some properties of the following operators.

Definition 2. Fix $p, q \in N$. We define the class of operators $A_{m,n}$ by the formula

$$
\begin{aligned}
(17)\quad & A_{m,n}(f; a_m, b_n; x, y) \\
&:= \sum_{j=0}^{[m(x+a_m)]} \sum_{k=0}^{[n(y+b_n)]} \binom{m-1+j}{j} x^j (1+x)^{-m-j} \\
&\qquad \times \binom{n-1+k}{k} y^k (1+y)^{-n-k} f\left(\frac{j}{m}, \frac{k}{n}\right), \\
&\qquad f \in C_{p,q}, \quad (x, y) \in R_0^2,
\end{aligned}
$$

where $(a_m)_1^\infty$ and $(b_n)_1^\infty$ are given sequences of positive numbers such that $\lim_{m\to\infty} \sqrt{m} a_m = \infty$ and $\lim_{n\to\infty} \sqrt{n} b_n = \infty$.

Observe that the operator $A_{m,n}$ is linear and positive.

Applying Theorem 2 and (12), we can prove the basic property of $A_{m,n}$.

Theorem 3. *Fix $p, q \in N$. Then for $A_{m,n}$ defined by* (17) *we have*

$$
(18)\qquad \lim_{m,n\to\infty} A_{m,n}(f; a_m, b_n; x, y) = f(x, y), \quad f \in C_{p,q}.
$$

Moreover, (18) *holds uniformly on every rectangle $0 \le x \le x_0$, $0 \le y \le y_0$.*

Proof. Suppose that $f \in C_{p,q}$, $p, q \in N$. This implies that

$$
\begin{aligned}
|f(t, z)| &\le K_6(1 + t^p)(1 + z^q) \\
&\le K_6(1 + 2^{p-1}(|t - x|^p + x^p))(1 + 2^{q-1}(|z - y|^q + y^q)).
\end{aligned}
$$

From (17) and (12) we have

$$
\begin{aligned}
A_{m,n}&(f; a_m, b_n; x, y) - f(x, y) \\
&= V_{m,n}(f; x, y) - f(x, y) - M_{m,n}(f; a_m, b_n; x, y)
\end{aligned}
$$

where

$$M_{m,n}(f;a_m,b_n;x,y) = \sum_{j=[m(x+a_m)]+1}^{\infty} \sum_{k=[n(y+b_n)]+1}^{\infty} \binom{m-1+j}{j} x^j(1+x)^{-m-j} \times \binom{n-1+k}{k} y^k(1+y)^{-n-k} f\left(\frac{j}{m},\frac{k}{n}\right), \quad (x,y)\in R_0^2.$$

Observe that

$$(19)\quad |M_{m,n}(f;a_m,b_n;x,y)| \le \sum_{j=[m(x+a_m)]+1}^{\infty} \sum_{k=[n(y+b_n)]+1}^{\infty} \binom{m-1+j}{j} y^j(1+y)^{-m-j} \times \binom{n-1+k}{k} x^k(1+x)^{-n-k}\left|f\left(\frac{j}{m},\frac{k}{n}\right)\right|$$

$$\le K_6 \sum_{j=[m(x+a_m)]+1}^{\infty} \binom{m-1+j}{j} \times x^j(1+x)^{-m-j}\left(1+2^{p-1}\left(\left|\frac{j}{m}-x\right|^p + x^p\right)\right) \times \sum_{k=[n(y+b_n)]+1}^{\infty} \binom{n-1+k}{k} \times y^k(1+y)^{-n-k}\left(1+2^{q-1}\left(\left|\frac{k}{n}-y\right|^q + y^q\right)\right).$$

Arguing as in the second part of Theorem 1, we derive

$$\sum_{j=[m(x+a_m)]+1}^{\infty} \binom{m-1+j}{j} x^j(1+x)^{-m-j}\left(1+2^{p-1}\left(\left|\frac{j}{m}-x\right|^p + x^p\right)\right) \le \frac{K_7}{m^{p/2}}\left(\frac{(1+2^{p-1}x^p)}{a_m^p} + 2^{p-1}\right)\left\{\sum_{j=0}^{p} P_{j,2p}(x)\right\}^{1/2},$$

$$\sum_{k=[n(y+b_n)]+1}^{\infty} \binom{n-1+k}{k} x^k(1+x)^{-n-k}\left(1+2^{q-1}\left(\left|\frac{k}{n}-y\right|^q + y^q\right)\right) \le \frac{K_8}{n^{q/2}}\left(\frac{(1+2^{q-1}y^q)}{b_n^q} + 2^{q-1}\right)\left\{\sum_{j=0}^{q} P_{j,2q}(y)\right\}^{1/2}.$$

From this and in view of Definition 2, we get

$$\lim_{m,n\to\infty} M_{m,n}(f;a_m,b_n;x,y)=0$$

uniformly on every rectangle $0\le x\le x_0$, $0\le y\le y_0$. Applying (16) and (19), we immediately obtain (18).

4. Remarks. Now we shall give some examples of operators of the $A_n f$ $(A_{m,n}f)$ type defined by (7) and (17).

It is similarly verified that analogous approximation properties hold for the following two operators

$$B_n(f;x):=\sum_{k=0}^{n}\binom{n-1+k}{k}x^k(1+x)^{-n-k}f\left(\frac{k}{n}\right), \tag{20}$$

$f\in C_{[0,1]}$, $x\in[0,1)$, $n\in N$,

$$B_{m,n}(f;x,y):=\sum_{j=0}^{m}\sum_{k=0}^{n}\binom{m-1+j}{j}x^j(1+x)^{-m-j}\times\binom{n-1+k}{k}y^k(1+y)^{-n-k}f\left(\frac{j}{m},\frac{k}{n}\right),$$

$f\in C_{[0,1],[0,1]}$, $(x,y)\in[0,1)\times[0,1)$, $m,n\in N$.

Observe that the operators B_n, $n\in N$, are obtained from (7) for $a_n=1-x$, $x\in[0,1)$.

Analogously, we obtain

$$A_{m,n}(f;1-x,1-y;x,y)=B_{m,n}(f;x,y),\quad (x,y)\in[0,1)\times[0,1),\quad m,n\in N.$$

Note that we constructed, for any function $f\in C_{[0,1]}$, the sequence of operators $(B_n)_1^\infty$ very similar to a sequence of polynomials

$$C_n(f;x)=\sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k}f\left(\frac{k}{n}\right), \tag{21}$$

$x \in [0,1]$, $n \in N$. We mention only that these polynomials (21), called Bernstein polynomials, play an important role in computer-aided geometric design and in the theory of multi-dimensional probability distributions.

Moreover, arguing as in the proof of Theorem 3 and applying some properties of the Szász-Mirakyan operators (see, for example, [2, 7]), it is easy to prove the following

Theorem 4. *Fix* $p, q \in N$. *Then*

$$\lim_{m,n\to\infty} D_{m,n}(f; a_m, b_n; x, y) = f(x,y), \quad f \in C_{p,q}, \quad (x,y) \in R_0^2, \tag{22}$$

where

$$D_{m,n}(f; a_m, b_n; x, y) := e^{-mx}e^{-ny} \sum_{j=0}^{[m(x+a_m)]} \sum_{k=0}^{[n(y+b_n)]} \frac{(mx)^j}{j!}\frac{(ny)^k}{k!} f\left(\frac{j}{m}, \frac{k}{n}\right),$$

$(a_m)_1^\infty$ *and* $(b_n)_1^\infty$ *are given sequences of positive numbers such that* $\lim_{m\to\infty}\sqrt{m}a_m = \infty$ *and* $\lim_{n\to\infty}\sqrt{n}b_n = \infty$. *Moreover,* (22) *holds uniformly on every rectangle* $0 \le x \le x_0$, $0 \le y \le y_0$.

The methods used to prove the theorems are similar to those used in the construction of modified Szász-Mirakyan operators [**5, 8, 9**].

Recently in many papers various modifications of operators V_n were introduced.

In [**3**] and [**13**] the more general Baskakov operators

$$E_{n,\alpha}(f;x) = \sum_{k=0}^{\infty} Q_{n,k}^{(\alpha)}(x) f\left(\frac{k}{n}\right),$$
$$\alpha \ge 1 \text{ or } 0 < \alpha < 1, \quad n \in N, \quad x \in R_0,$$

were considered where $Q_{n,k}^{(\alpha)}(x) = J_{n,k}^{\alpha}(x) - J_{n,k+1}^{\alpha}(x)$ and $J_{n,k}(x) = \sum_{j=k}^{\infty}\binom{n-1+j}{j}x^j(1+x)^{-n-j}$. Thus, the question arises whether the

Baskakov-Bézier operators examined in [**3, 13**] cannot be replaced by a finite sum.

Moreover, we propose the same method to replace the infinite sum in the Durrmeyer variant of V_n

$$F_n(f;x) = (n-1)\sum_{k=0}^{\infty}\binom{n-1+k}{k}x^k(1+x)^{-n-k} \times \int_0^{\infty}\binom{n-1+k}{k}t^k(1+t)^{-n-k}f(t)\,dt, \quad x\in R_0, \quad n\in N,$$

by a truncated sum.

These questions are open problems for the readers.

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