

Global output-feedback stabilization for a class of nonlinear systems with unknown output function

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Abstract

This paper investigates the problem of output feedback stabilization for a class of non-linear systems satisfying some relaxed triangular-type condition whose output function and nonlinear terms are unknown. The dual-domination approach enables us to develop a state observer and an output feedback control law that stabilize nonlinear systems globally asymptotically. The novelty of this work, is to solve the global problem of stabilizing the output feedback of non-linear systems with unknown measurement sensitivity. The investigated system is significantly different from the existing results and highlights the main contributions of the paper. With the help of a numerical, we demonstrate the effectiveness of the proposed method.

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1 Introduction

The problem of output feedback stabilization for systems with unmeasured state-dependent growth, as a rather difficult problem in nonlinear control. The study of the stabilization of non-linear systems of the system with unknown measurement sensitivity has been the subject of many researchers, see for example [16, 7, 11, 12, 18, 21] and the references contained therein. In the past decades, researchers have paid considerable attention to stabilizing output feedback for non-linear systems whose measurement sensitivity is unknown or whose output function is unknown.

In [3] introduces an output feedback controller for a class of nonlinear systems satisfying the linear growth condition with an unknown growth rate. Using high gain observers, the

stabilization of nonlinear systems with growth dependent on unmeasured states is studied in [9, 10].

The problem of global adaptive stabilization by output-feedback for a class of nonlinear systems with unknown output function is addressed in [18]. For unknown measurement, [4] studied the event-triggered adaptive neural tracking control of non strict feedback nonlinear systems. In [19] the authors studied semi-global output feedback stabilization for the uncertain non-triangular system in the presence of unknown output gains. The work [25] investigated the problem of output tracking for nonlinear systems in the presence of uncertain parameters under adaptive control. Under high gain dynamic observer, the stabilization of global output feedback was investigated in [17] for a class of uncertain non-linear systems with dependent growth of unmeasured states. For the systems subject to stochastic noise, [15] studies the output-feedback control for stochastic nonlinear systems with unknown measurement sensitivity. Recently, the growth rate of the nonlinear systems is described as polynomial-of-output multiplying a constant [11] investigated the global output-feedback stabilization for a class of nonlinear systems. In [16], the authors suggested a control system for output feedback that can be applied to nonlinear systems with measurement sensitivity that is not known nor differentiable. For a class of uncertain nonlinear systems with unmeasured states dependent growth the global asymptotic stability of the closed-loop system is examined for a nonlinear system in [17]. By developing an adaptive output feedback controller based on a high-gain dynamic technique, the global asymptotic stability of the closed-loop system is investigated in [6, 13], for switched nonlinear systems with unknown parameters, unknown output uncertainties. In [14], the global stabilisation of output feedback is studied for a class of high order non-linear systems with an unknown output function.

In this paper, we investigate the problem of global stabilisation by output-feedback of a class of nonlinear systems with some relaxed triangular-type condition and unknown output function. We impose a generalised condition on the nonlinearity to cover the systems considered by [17] and [16]. Motivated by literature and [17], we are building a suitable Lyapunov function to establish global stability of the closed-loop systems. First, the output feedback design is being proposed with a new design approach. Then, a new observer whose gains from the observer can be suitably enlarged to make the mistake arbitrarily small. It is possible to carefully select the observer and control law to dominate the possible sensor sensitivity error and unknown nonlinearities. Finally, a dynamic output compensator is created to ensure the global asymptotic stability of the closed-loop system.

The following summarizes the main contributions of this work:

1. We propose an output feedback design under a generalized condition to deal nonlinear systems with unknown output function and we demonstrate that existing conditions

such as the triangular condition or the feedforward condition are special cases of our new condition. The nonlinearities that satisfy relaxed from the linear growth condition.

2. There is a proposal for a new dynamic high-order observer design method. The observer's gains can be appropriately enlarged to make the error arbitrarily small.
3. The suggested controller is linear-like. The stabilization problem is simpler to implement. The closed-loop system converges to the equilibrium point quickly thanks to the dynamic output compensator's advantage.

The rest of the paper is organized as follows. The observer design problem and the main assumptions are discussed in Section 2. Section 3 and Section 4 presents the main results of the paper, while Section 5 examines numerical examples. Section 6 contains the conclusion remarks.

2 Problem formulation and system description

In this paper, we consider the problem of global output feedback stabilization for a particular family of nonlinear systems with unknown continuous output function as follows:

$$\begin{cases} \dot{x}_1(t) &= x_2 + f_1(t, x, u), \\ \dot{x}_2(t) &= x_3 + f_2(t, x, u), \\ &\vdots \\ \dot{x}_i(t) &= x_{i+1} + f_i(t, x, u), \\ &\vdots \\ \dot{x}_n(t) &= u(t) + f_n(t, x, u), \\ y(t) &= \theta(t)x_1(t). \end{cases} \quad (1)$$

where, for $i = 1, \dots, n$, $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input of the system and $y \in \mathbb{R}$ is the measured output. The time-varying measurement error $\theta(t)$ is a bounded unknown continuous function. The mappings $f_i : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$, $i = 1, \dots, n$ are continuous functions.

Remark 1 *For system (1), it has been recognized that the accuracy of the measurement output y dependent on the sensor sensitivity $\theta(t)$ plays a critical role in feedback control design. The literature, most of existing results on the output feedback control of system (1) have been achieved for the ideal case when $\theta(t) = 1$ [23]. However, in practice, the sensor sensitivity $\theta(t)$ is not always a constant as discussed.*

To complete the description of the nonlinear system the following assumptions are taken into consideration.

Assumption 1. There exists a function $\gamma(\varepsilon) > 0$ such that for $\varepsilon > 0$,

$$\sum_{i=1}^n \varepsilon^{i-1} |f_i(t, x, u)| \leq \gamma(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |x_i|. \quad (2)$$

Assumption 2. The sensor sensitivity $\theta(t)$ is an unknown continuous function and there is an allowable sensitivity error $\bar{\theta}$ such that:

$$\theta(t) \in [1 - \bar{\theta}, 1 + \bar{\theta}]; \quad \forall t \in \mathbb{R}_+. \quad (3)$$

Remark 2 It is easy to see that if a linear growth condition is satisfied by $f_i(t, x, u)$ [16], there exists $c > 0$ such that,

$$|f_i(t, x, u)| \leq c \sum_{j=1}^i |x_j| \quad (4)$$

then **Assumption 1** is satisfied with $\gamma(\varepsilon) = c(1 + \varepsilon + \dots + \varepsilon^{n-1})$. This generalized condition is reduced to the generalized condition introduced by [16] and [18]. For, $y = x_1$ is measurable, the global exponential stabilization by output feedback stabilization problem was solved in [8] under the same condition (2).

Remark 3 **Assumption 1** is a newly imposed condition on the perturbed term $f(t, x, u)$ for systems whose measurement output y depends on the sensitivity of the sensor. For comparison, we present a condition which is a special case of (2) which generalizes the condition given by Eq (4) imposed in [20].

Remark 4 Under **Assumption 1**, the problem of global stabilization of nonlinear system (1) is achieved under a linear output feedback, where the output y is a linear function of the real x_1 in [8]. However, the output y is a linear output $\theta(t)x_1(t)$ with an unknown constant where the upper-bound and lower-bounder of are known is a natural extension of the system given by [8]. The control law in [8] cannot be used directly in this paper because of the unknown output function coefficients.

3 Output-feedback stabilizing control design

The following technical lemma will play a key role in the proof of the in proving the main result of this paper.

Lemma 1 Let a and b be positive real numbers. For any $x \in \mathbb{R}$ and $y \in \mathbb{R}$, the following inequality holds:

$$a|x||y| \leq b|x|^2 + \frac{a^2b^{-1}}{4}|y|^2$$

3.1 Observer design

In this paragraph, we study the designing of an observer with a variable domination gain designed without using the details of the measured output and system nonlinearities.

Remark 5 *Due to the existence of unknown output functions, the control law in [8] cannot be applied directly to this work.*

A high gain candidate nonlinear observer for the above class of systems (1) is given by:

$$\begin{cases} \dot{\hat{x}}_1 &= \hat{x}_2 - \frac{1}{\varepsilon}a_1\hat{x}_1 \\ \dot{\hat{x}}_2 &= \hat{x}_3 - \frac{1}{\varepsilon^2}a_2\hat{x}_1 \\ &\vdots \\ \dot{\hat{x}}_n &= u - \frac{1}{\varepsilon^n}a_n\hat{x}_1 \end{cases} \quad (5)$$

where $0 < \varepsilon < 1$ is a domination gain to be specified later, and $a_i > 0$, for $i = 1, \dots, n$, are the coefficients of any Hurwitz $s^n + a_1s^{n-1} + \dots + a_{n-1}s + a_n$. Next, treat (5) as an observer for system (1), and consider the estimation error:

$$e_i = \varepsilon^{i-1}(x_i - \hat{x}_i), \quad 1 \leq i \leq n \quad (6)$$

Let $D(\varepsilon) = \text{diag}[1, \varepsilon, \dots, \varepsilon^{n-1}]$ and using (1) and (5) a simple computing shows that:

$$\dot{e} = \frac{1}{\varepsilon}A_1e + \frac{1}{\varepsilon}Kx_1 + D(\varepsilon)f(t, x, u), \quad (7)$$

where

$$A_1 = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad e(t) = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, \quad K = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Since A_1 is a Hurwitz matrix, then there exists a positive definite symmetric matrix P which satisfies solution of the Lyapunov equation:

$$A_1^T P + P A_1 = -I_n \quad (8)$$

To press ahead with the analysis of estimation error, we consider the following Lyapunov function candidate:

$$V_1(e) = e^T P e \quad (9)$$

Using (8), the time derivative of $V_1(e)$ along the trajectories of (7) is given by:

$$\begin{aligned} \dot{V}_1(e) &= -\frac{1}{\varepsilon}\|e\|^2 + \frac{2}{\varepsilon}x_1 e^T P K + 2e^T P D(\varepsilon)f(t, x, u) \\ &\leq -\frac{1}{\varepsilon}\|e\|^2 + \frac{2}{\varepsilon}|x_1| \cdot \|P\| \cdot \|K\| \cdot \|e\| + 2\|e\| \cdot \|P\| \cdot \|D(\varepsilon)f(t, x, u)\| \end{aligned} \quad (10)$$

So by Assumption 1, we get:

$$\begin{aligned}
\| D(\varepsilon)f(t, x, u) \| &\leq \sum_{i=1}^n \varepsilon^{i-1} |f_i(t, x, u)| \\
&\leq \gamma(\varepsilon) \sum_{i=1}^n \varepsilon^{i-1} |x_i| \\
&\leq n\gamma(\varepsilon) \| D(\varepsilon)x \|
\end{aligned} \tag{11}$$

Based on (10) and (11), one can get:

$$\dot{V}_1(e) \leq -\frac{1}{\varepsilon} \|e\|^2 + \frac{2}{\varepsilon} |x_1| \cdot \|P\| \cdot \|K\| \cdot \|e\| + 2n\gamma(\varepsilon) \|e\| \cdot \|P\| \cdot \|D(\varepsilon)x\| \tag{12}$$

Remark 6 *Compared to design observers in the literature, the observers studied in this work are different. The information of the measurement output was used for the observer design in see [24]. However, the exact information of non-linearities and the measurement output are not used in (5) for the observers design.*

Remark 7 *Unlike that in [10] and [9], the observer (5) introduced in this paper is dynamic, not a constant. This is essential to overcome the technical obstacles caused by the unknowns in the system growth rate.*

3.2 Control design

In this section, with high gain control, we construct a Lyapunov function, which is sufficient for the existence of a stabilizing feedback control law. Motivated by the high-gain observer construction in [16] and [18] we are given the augmented systems:

$$\begin{cases} \dot{x}_1 &= x_2 + f_1(t, x, u), \\ \dot{\hat{x}}_2 &= \hat{x}_3 + \frac{1}{\varepsilon^2} a_2 (e_1 - x_1), \\ \dot{\hat{x}}_3 &= \hat{x}_4 + \frac{1}{\varepsilon^3} a_3 (e_1 - x_1), \\ &\vdots \\ \dot{\hat{x}}_n &= u + \frac{1}{\varepsilon^n} a_n (e_1 - x_1). \end{cases} \tag{13}$$

Introduce the following change of coordinates:

$$\xi_1 = x_1, \quad \xi_i = (\varepsilon\varepsilon_1)^{i-1} \hat{x}_i, \quad v = (\varepsilon\varepsilon_1)^n u, \quad i = 2, \dots, n; \tag{14}$$

where $0 < \varepsilon_1 < 1$ is a constant gain to be determined later. Under the new coordinates $\xi = [\xi_1, \dots, \xi_n]^T \in \mathbb{R}^n$, the augmented system (14) can be rewritten as:

$$\begin{cases} \dot{\xi}_1 &= \frac{1}{\varepsilon_1} \xi_2 + \frac{1}{\varepsilon} e_2 + f_1(t, x, u), \\ \dot{\xi}_2 &= \frac{1}{\varepsilon_1} \xi_3 + \frac{\varepsilon_1}{\varepsilon} a_2 (e_1 - \xi_1), \\ \dot{\xi}_3 &= \frac{1}{\varepsilon_1} \xi_4 + \frac{\varepsilon_1^2}{\varepsilon} a_3 (e_1 - \xi_1), \\ &\vdots \\ \dot{\xi}_n &= \frac{1}{\varepsilon_1} v + \frac{\varepsilon_1^{n-1}}{\varepsilon} a_n (e_1 - \xi_1). \end{cases} \quad (15)$$

Now, we design the output feedback control law:

$$v = -b_n y - b_{n-1} \xi_2 - \dots - b_2 \xi_{n-1} - b_1 \xi_n \quad (16)$$

where $b_i > 0$, for $i = 1, \dots, n$, are coefficients of the Hurwitz polynomial $s^n + b_1 s^{n-1} + \dots + b_{n-1} s + b_n$. Substituting the control law (16) into (15) yields:

$$\dot{\xi} = \frac{1}{\varepsilon \varepsilon_1} B_1 \xi + \frac{1}{\varepsilon \varepsilon_1} b_n (1 - \theta(t)) L_3 \xi_1 + \frac{\varepsilon_1}{\varepsilon} L_1 (e_1 - \xi_1) + \frac{1}{\varepsilon} L_2 e_2 + \tilde{f}(t, x, u) \quad (17)$$

where

$$L_1 = \begin{bmatrix} 0 \\ a_2 \\ \vdots \\ \varepsilon_1^{n-2} a_n \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{f}(t, x, u) = \begin{bmatrix} f_1(t, x, u) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and

$$B_1 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_n & -b_{n-1} & \cdots & -b_1 \end{bmatrix}$$

Due to the fact that B_1 is a Hurwitz matrix, there is a corresponding positive definite matrix S satisfying:

$$B_1^T S + S B_1 = -I_n. \quad (18)$$

Construct the Lyapunov function:

$$V_2(\xi) = \xi^T S \xi \quad (19)$$

The time derivative of (19) along the trajectories of (17) is

$$\dot{V}_2(\xi) = -\frac{1}{\varepsilon \varepsilon_1} \|\xi\|^2 + \frac{2}{\varepsilon \varepsilon_1} b_n (1 - \theta(t)) \xi^T S L_3 \xi_1 + 2 \xi^T S \tilde{f}(t, x, u) + 2 \xi^T S \left(\frac{\varepsilon_1}{\varepsilon} L_1 e_1 + \frac{1}{\varepsilon} L_2 e_2 - \frac{\varepsilon_1}{\varepsilon} L_1 \xi_1 \right) \quad (20)$$

Using (11) through (14) and after a few calculations, we obtain:

$$\begin{aligned} \|2\xi^T S \tilde{f}(t, x, u)\| &\leq 2n\gamma(\varepsilon)\|S\|\|\xi\|\|D(\varepsilon)x\| \\ &\leq 2n\gamma(\varepsilon)\|S\|\|\xi\|\left(\|e\| + \frac{1}{\varepsilon_1^{n-1}}\|\xi\|\right) \end{aligned} \quad (21)$$

There exists a bounded constant of the allowable sensitivity error $\bar{\theta}$ given by:

$$\bar{\theta} < \frac{1}{2b_n\|S\|} \quad (22)$$

As we have $1 - \bar{\theta} \leq \theta(t) \leq 1 + \bar{\theta}$, one gets:

$$1 - 2b_n|1 - \theta(t)|\|S\| \geq 1 - 2b_n\bar{\theta}\|S\| := \rho \quad (23)$$

From $\|L_2\| = \|L_3\| = 1$ and with the help of Lemma 1, it is straightforward to verify that the derivative of (19) satisfies:

$$\begin{aligned} \dot{V}_2(\xi) &\leq -\frac{\rho}{\varepsilon\varepsilon_1}\|\xi\|^2 + 2n\gamma(\varepsilon)\|S\|\|\xi\|\|e\| + 2n\varepsilon_1^{1-n}\gamma(\varepsilon)\|S\|\|\xi\|^2 \\ &\quad + \frac{2\varepsilon_1}{\varepsilon}\|L_1\|\|S\|\|\xi\|\|e\| + \frac{2}{\varepsilon}\|S\|\|\xi\|\|e\| + \frac{2\varepsilon_1}{\varepsilon}\|S\|\|L_1\|\|\xi\|^2 \\ &\leq -\frac{\rho}{\varepsilon\varepsilon_1}\|\xi\|^2 + n\gamma(\varepsilon)\|S\|\|\xi\|^2 + n\gamma(\varepsilon)\|S\|\|e\|^2 + \frac{1}{4\varepsilon}\|e\|^2 + M\|S\|\|\xi\|^2 \\ &\quad + \frac{8\varepsilon_1^2}{\varepsilon}\|L_1\|^2\|S\|^2\|\xi\|^2 + \frac{8}{\varepsilon}\|S\|^2\|\xi\|^2 + \frac{2\varepsilon_1}{\varepsilon}\|L_1\|\|S\|\|\xi\|^2 \end{aligned} \quad (24)$$

where $M = 2n\varepsilon_1^{1-n}\gamma(\varepsilon)$.

4 Main results

Through (12) and (24), we can achieve the document's main goal. In particular, we show the dynamic behavior of the closed-loop system via a Lyapunov candidate function, and the intrinsic relationship between the high gain and the other variables, respectively.

Theorem 1 *Suppose that Assumptions 1 and 2 hold, and there exist positive constants ε , ε_1 such that:*

$$\frac{1}{2\varepsilon} - 2n\gamma(\varepsilon)\|P\| - n\gamma(\varepsilon)\|S\| > 0 \quad (25)$$

$$\frac{\rho}{\varepsilon_1} - n\gamma(\varepsilon)\|S\| - \widetilde{M} > 0 \quad (26)$$

where

$$\widetilde{M} = M\|S\| + 8\|L_1\|^2\|S\|^2 + 8\|S\|^2 + 2\|L_1\|\|S\| + 8\|P\|^2\|K\|^2 + 2M^2\|P\|^2$$

then system (1) is globally asymptotically stabilized by the output-feedback control law (5), (15), and (16).

Proof. Since we have $\xi_1 = x_1$, and using Lemma 1, we can obtain:

$$\begin{aligned}\dot{V}_1(e) &\leq -\frac{1}{\varepsilon}\|e\|^2 + \frac{2}{\varepsilon}\|P\|\|K\|\|e\|\|\xi\| + 2n\varepsilon_1^{1-n}\gamma(\varepsilon)\|P\|\|e\|\|\xi\| + 2n\gamma(\varepsilon)\|P\|\|e\|^2 \\ &\leq -\frac{3}{4\varepsilon}\|e\|^2 + \frac{8}{\varepsilon}\|P\|^2\|K\|^2\|\xi\|^2 + 8n^2\varepsilon\varepsilon_1^{2-2n}\gamma^2(\varepsilon)\|P\|^2\|\xi\|^2 + 2n\gamma(\varepsilon)\|P\|\|e\|^2 \\ &\leq -\frac{3}{4\varepsilon}\|e\|^2 + \frac{8}{\varepsilon}\|P\|^2\|K\|^2\|\xi\|^2 + 2M^2\|P\|^2\|\xi\|^2 + 2n\gamma(\varepsilon)\|P\|\|e\|^2\end{aligned}\quad (27)$$

Choosing a Lyapunov candidate functional:

$$V(e, \xi) = V_1(e) + V_2(\xi) \quad (28)$$

Now, by combining (10) and (27) together, we get:

$$\dot{V}(e, \xi) \leq -\left\{\frac{1}{2\varepsilon} - 2n\gamma(\varepsilon)\|P\| - n\gamma(\varepsilon)\|S\|\right\}\|e\|^2 - \frac{1}{\varepsilon}\left\{\frac{\rho}{\varepsilon_1} - n\gamma(\varepsilon)\|S\| - \widetilde{M}\right\}$$

By the conditions (25), (26) and Lyapunov stability theory, one can conclude that the closed-loop control system consisting of (5), (15), and (16) is asymptotically stable. ■

Remark 8 *Our new criteria given by Theorem 1 are able to overcome some of the main sources of conservatism, unlike [16] and [24]. The criteria in [16] and [24] are considered a special case of our nonlinear system class.*

Remark 9 *The problem of using output feedback to globally control a class of nonlinear systems whose output functions are not precisely known is addressed in [22] by using the homogeneous domination approach. In this paper, we develop global output feedback controllers for nonlinear systems with unknown output functions by developing a dual-domination approach which employs one domination gain in the state observer and another gain in the output feedback control law.*

5 Numerical example

In this section, we give a practical example to illustrate the effectiveness of the proposed theoretical results. Consider the planer system in [8] of the form:

$$\begin{cases} \dot{x}_1(t) &= x_2(t) + \frac{1}{10}x_1(t)\sin x_2^2(t) \\ \dot{x}_2(t) &= u(t) + \frac{1}{10}x_1^{\frac{2}{3}}(t)x_2^{\frac{1}{3}}(t) \\ y(t) &= \theta(t)x_1(t) \end{cases} \quad (29)$$

where $\theta(t) = 0.25 + 0.01|\sin(t)|$. It is easy to check that system (29) satisfies **Assumption 1**, with $\gamma(\varepsilon) = \frac{1}{10}(1 + \varepsilon)$. Due to the presence of $f_1(t, x, u) = \frac{1}{10}x_1(t)\sin x_2^2(t)$ the system (29) is not a lower-triangular form. Moreover $f_2(t, x, u) = \frac{1}{10}x_1^{\frac{2}{3}}(t)x_2^{\frac{1}{3}}(t)$ is not only a non-Lipschitz continuous function. Since the output y depends on $\theta(t)$ and x_1 not equal to one,

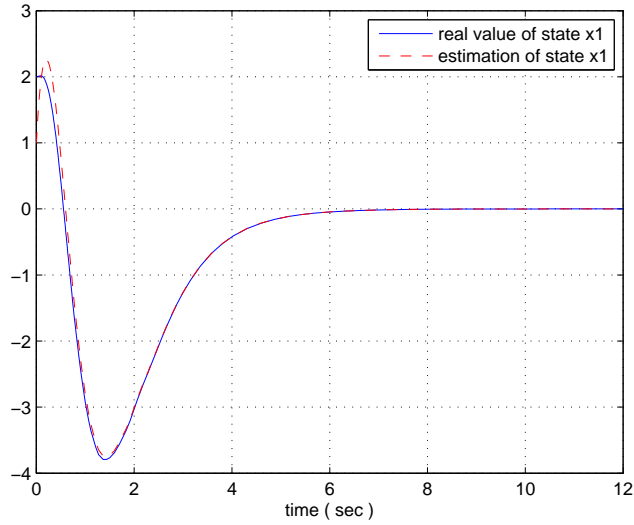


Figure 1: Trajectories of x_1 and its estimate \hat{x}_1

the output feedback scheme in [3, 8, 23] is not applicable. Now, select as $a_1 = a_2 = 1$ and can be chosen $b_1 = b_2 = 5$, A_1 and B_1 are Hurwitz. Using MATLAB, the solution of the Lyapunov equations (8) and (19) are given respectively by:

$$P = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.5 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 0.62 & -0.50 \\ -0.50 & 0.60 \end{bmatrix}$$

So, $\|P\| = 1.8090$ and $\|S\| = 1.1101$. This implies that condition (25) is satisfied for $0 < \varepsilon < 0.3825$. If $\varepsilon_1 = 0.2$, then the condition (26) is satisfied for $\rho > 24.38$. Figs.1 and 2 show the simulation results of states $x_1(t), x_2(t)$.

6 Conclusion

In this paper, we have presented an output feedback controllers scheme for a class of nonlinear systems that cover the class of systems satisfying a linear growth condition with unknown output function. Within this context, a delicate and important case studied here is that the unknown output function but has known upper and lower bounds. By developing a dual-domination approach, we can achieve this by utilizing one domination gain in the state observer and another gain in the output feedback control law. Furthermore, we imposed a generalized condition on nonlinearity to cover the nonlinear systems considered by [2]. As a perspective, a new observer design for time-delay system taking generalised condition on the nonlinearity to cover the time-delay systems considered by [1] and [5] will be a future work.

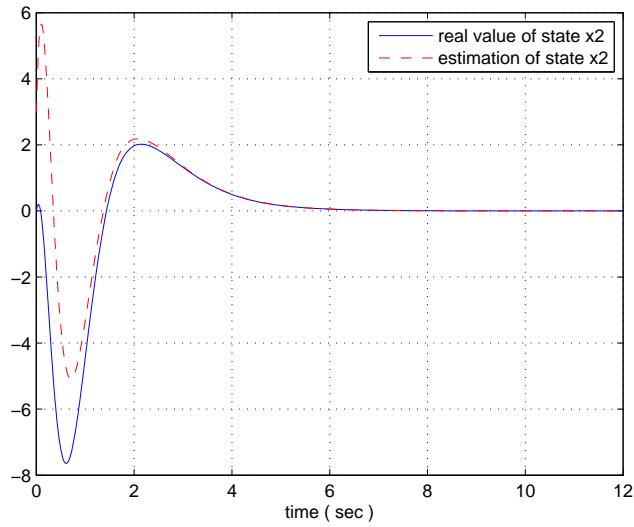


Figure 2: Trajectories of x_2 and its estimate \hat{x}_2

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