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<https://doi.org/rmj.YEAR..PAGE>**SYNCHRONIZATION CONTROL DESIGN OF FRACTIONAL-ORDER JAFARI-SPROTT  
CHAOTIC SYSTEM**

MIN SHI AND ZHOUIJIN CUI

ABSTRACT. The synchronization problem for fractional-order Jafari-Sprott chaotic system is investigated in this paper. Two primary synchronization control laws are proposed to achieve the chaotic synchronization of the system without/with uncertainties. For the simple fractional-order Jafari-Sprott chaotic system without considering uncertainties, active control is proposed to achieve chaotic synchronization. In addition, the robust synchronization control is realized for the corresponding chaotic system with uncertainties, by adopting fractional-order sliding mode control strategy. The controller parameters could be adjusted based on the stability theory of fractional-order system and fractional-order sliding mode theory respectively. As illustrated by the numerical examples, the proposed active control law and slide mode control strategy could work well and accurately.

Keywords. Synchronization control, fractional-order Jafari-Sprott chaotic system, active control, sliding mode control.

**1. Introduction**

Chaotic systems as a special nonlinear system, exists in natural and social science. From the perspective of mathematical theory, researchers are interested in complex dynamic behaviors analysis of the system, including periodic solutions, chaotic phenomenon, route from period doubling to chaos and so on [1–5]. On the other hand, synchronization of chaotic systems has attracted significant interest among multidisciplinary researchers due to its potential applications in communication encryption, image encryption, edge detection, image processing and other fields [6–10].

However, the results mentioned above are mainly focused on integer-order systems. Actually, fractional-order system model are more suitable to describe complex dynamics in real natural environment. Compared with the traditional calculus, fractional calculus reflects time memory and non-local properties in mathematical theory. These special features contribute to characterize some

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1 actual systems such as fluid flow process, circuits, electromagnetic wave and viscoelastic material-  
2 s [11, 12]. From the viewpoint of dynamics and control theory, the dynamic analysis and control  
3 design of fractional-order systems are widely investigated. Fractional-order controller are more flex-  
4 ible to improve the performance index of control systems containing nonlinear factors, uncertainties,  
5 perturbations, etc [13, 14].

7 Many control approaches are proposed to realize chaotic synchronization such as active con-  
8 trol, variable structure control design, self-adaptive method, sliding mode control scheme and so  
9 on [15–19]. Synchronization control of the fractional-order chaotic systems has gained wide attention  
10 in recent decades. In [20], simple and flexible adaptive schemes are proposed for control and syn-  
11 chronization of the fractional-order chaotic system based on the stability theory of fractional-order  
12 dynamic systems. In [21], synchronization of chaotic and uncertain fractional-order Duffing-Holmes  
13 system has been done using the sliding mode control strategy. Simulation results reveal that not  
14 only the performance of the proposed method is satisfying with an acceptable level of control sig-  
15 nal, but also a rather simple stability analysis is performed. In [22], the authors investigated the  
16 fractional-order disturbance observer-based adaptive sliding mode synchronization control for a class  
17 of fractional-order chaotic systems with unknown bounded disturbances. Comprehensive study of  
18 control and synchronization of fractional-order chaotic systems, and shows how chaos is formed in  
19 developing inter-disciplinary researches of fractional-order systems [23]. In addition, synchronization  
20 of incommensurate fractional-order chaotic systems realized based on linear feedback control [24].

25 The main contribution of this work is to focus on synchronization control of fractional-order  
26 Jafari-Sprott chaotic system. Two distinct types of control strategies to realize chaotic synchronization  
27 of the fractional-order system. Firstly, a simple active synchronization control scheme is presented  
28 for the system based on stability theory of fractional-order system. The controller parameters could  
29 be adjusted according to the closed-loop fractional-order system located on the stable region. Sec-  
30 ondly, sliding mode control is proposed to achieve robust synchronization of fractional-order Jafari-  
31 Sprott chaotic system with uncertainties. The corresponding controller parameters design are based  
32 on fractional-order sliding mode theory and Lyapunov stability theory.

36 The rest of this paper is organized as follows. In Sec.2, preliminary knowledge about fractional  
37 calculus and basic stability theory of fractional-order system are given, and then the chaotic phenom-  
38 enon of fractional-order Jafari-Sprott system is described briefly in Sec.3. In Sec.4, we propose a  
39 simple active synchronization control scheme to realize chaotic synchronization of the system. Fur-  
40 ther, in order to achieve robust synchronization of fractional-order Jafari-Sprott chaotic system with  
41

uncertainties, fractional-order sliding mode control is proposed in Sec.5. Numerical examples are given to illustrate the availability of the control effects. Finally, in Sec.6 some concluding remarks are drawn.

## 2. Preliminaries

In this section, some preliminary knowledge about the concepts of fractional calculus and stability of fractional-order autonomous system are introduced. Meanwhile, the relevant important conclusion is presented.

**2.1. Fractional calculus.** Different from integral calculus, fractional calculus has several distinct definitions. The two commonly used definitions of fractional-order derivative are formed by Riemann-Liouville and Caputo. Mathematically, the general fractional-order differential operator  ${}_0D_t^\alpha f(t)$  could be rewritten as  ${}_0^R D_t^\alpha f(t)$  or  ${}_0^C D_t^\alpha f(t)$  in the sense of Riemann-Liouville or Caputo's definition. In the sense of Caputo's fractional calculus, similar to the integral-order case, the autonomous fractional-order systems could be converted to the Initial Value Problem (IVP) and also have definite physical meaning. Hence, we will use the fractional-order derivative with Caputo definition  ${}_0^C D_t^\alpha f(t)$  in this paper.

The Caputo fractional-order derivative [25] is defined as

$$(1) \quad {}_0^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau.$$

where  $n$  is the integer satisfying  $n-1 < \alpha \leq n$ , and  $\Gamma(z)$  is the Gamma function satisfying  $\Gamma(z+1) = z\Gamma(z)$  for  $z > 0$ . In this paper, we consider the case of  $0 < \alpha \leq 1$ . Similarly, the definition of fractional-order integral [25] is

$$(2) \quad {}_0I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau,$$

and the following formula  ${}_0^C D_t^\alpha ({}_0I_t^\alpha f(t)) = f(t)$  holds. As the Caputo fractional-order derivative  $\beta$  close to 1, the property  $\lim_{\beta \rightarrow 1^-} [{}_0^C D_t^\beta f(t)] = \dot{f}(t)$  holds if  $f(t)$  is differentiable [26].

**2.2. Stability theory of fractional-order system.** In this part, stability conditions for fractional-order systems are discussed. Consider the following fractional-order chaotic system

$$(3) \quad {}_0^C D_t^\alpha x = f(x),$$

where  $x \in \mathbb{R}^n$ ,  $f \in \mathbb{R}^n$  represents the system states and nonlinear function containing all the states. Setting  $f(x) = 0$ , we could obtain the equilibrium points of the system (12). Linearization at the

equilibrium, stability conditions are obtained for fractional-order linear systems by the following theorem.

**Theorem 2.1** ([27, 28]) Consider the following fractional-order autonomous system

$$(4) \quad {}^C_0 D_t^\alpha x = Ax, \quad x(0) = x_0$$

where  $x \in \mathbb{R}^n$  denotes the system states with initial condition  $x_0$ ,  $A \in \mathbb{R}^{n \times n}$  is constant matrix. Then the following results hold:

- i) The system is asymptotically stable if and only if all eigenvalues of  $A$  satisfy  $|\arg(\text{eig})A| > \alpha\pi/2$ , as each component of the state variables decays towards 0 with convergence rate  $t^{-\alpha}$ .
- ii) The system is stable if and only if all eigenvalues of  $A$  satisfy  $|\arg(\text{eig})A| \geq \alpha\pi/2$ , and those critical eigenvalues which satisfy  $|\arg(\text{eig})A| = \alpha\pi/2$  have geometric multiplicity one.
- iii) If there exists positive definite matrix  $P$ , such that the following condition  $J = x^T P_0^C D_t^\alpha x \leq 0$  satisfied, then the system is stable.

### 3. Problem statement

It is generally recognized that simple systems modeled as nonlinear differential equations can exhibit chaos. From the perspective of nonlinear dynamics, it is interesting to explore the system parameter variation range through numerical simulation with the goal of finding some desired characteristics such as chaos route of the system.

The Jafari-Sprott chaotic system is one of the typical systems to reveal complexity of nonlinear dynamical systems with simple mathematical model. In [29, 30], the complexity analysis of Jafari-Sprott system is investigated with integer-order and fractional-order model respectively. We can rewrite the unified mathematical model of Jafari-Sprott chaotic system as

$$(5) \quad \begin{cases} {}^C_0 D_t^\alpha x_1 = x_2 \\ {}^C_0 D_t^\alpha x_2 = -x_1 + x_2 x_3 \\ {}^C_0 D_t^\alpha x_3 = -x_1 - ax_1 x_2 - bx_1 x_3 \end{cases}$$

where  $0 < \alpha \leq 1$ . As  $\alpha = 1$ , system (5) is also the integer-order chaotic model, and it will evolved into the fractional-order case as  $0 < \alpha < 1$ . The complex dynamical behavior including bifurcation, chaotic route and so on, has been studied in both integer-order and fractional-order model. Through numerical simulation, we reviewed the complex dynamical behavior in Fig. 1 in fractional-order case, as  $\alpha = 0.98$ . The chaotic attractor appeared in phase space diagram in Fig. 1(a), and the double period bifurcation to chaos illustrated in Fig. 1(b) with the change of system parameter. Few literature

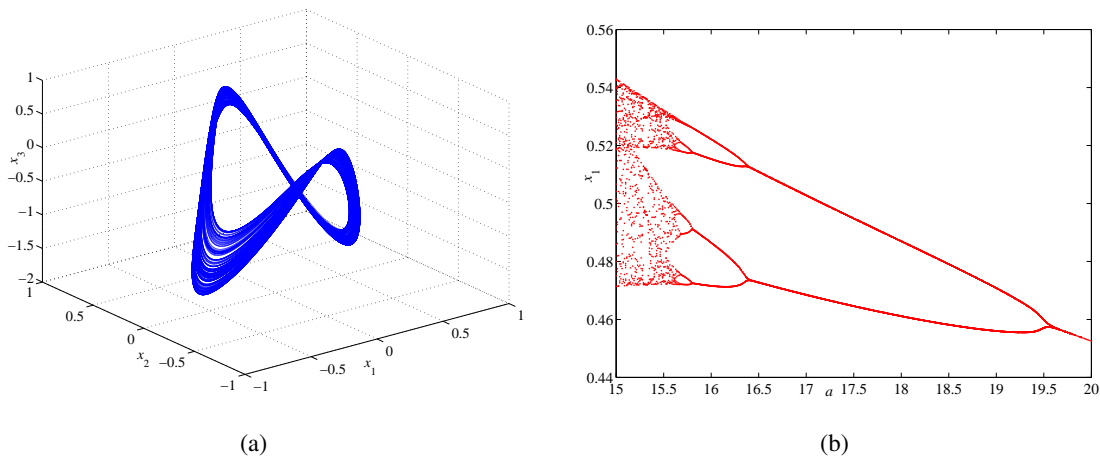


FIGURE 1. Dynamical phenomena of the fractional-order Jafari-Sprott system. (a) phase diagram of the system, (b) bifurcation diagram of the model via system parameter  $a$  with initial values  $(0.1, 0.5, 0.5)$ .

investigate the synchronization control of fractional-order Jafari-Sprott chaotic system. In this paper, we will propose two types of control strategies to realize chaotic synchronization of system (5) as  $0 < \alpha < 1$ . Firstly, a simple active synchronization control scheme is presented for system (5) based on stability theory of fractional-order system. Secondly, sliding mode control is used to achieve synchronization of fractional-order Jafari-Sprott chaotic system. Different from [30], we will consider a more complex chaotic system containing uncertainties.

#### 4. Synchronization of fractional-order Jafari-Sprott chaotic system with active control law

In order to investigate the synchronization control design of fractional-order Jafari-Sprott chaotic system, we construct the corresponding master-slave system. Consider the system (5) be the master system, the corresponding slave system is:

$$(6) \quad \begin{cases} {}^C_0 D_t^\alpha y_1 = y_2 \\ {}^C_0 D_t^\alpha y_2 = -y_1 + y_2 y_3 + u_1 \\ {}^C_0 D_t^\alpha y_3 = -y_1 - a y_1 y_2 - b y_1 y_3 + u_2 \end{cases}$$

The state error between the master system and the slave system is defined as

$$(7) \quad \begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases}$$

then the error dynamics is

$$(8) \quad \begin{cases} {}^C_0D_t^\alpha e_1 = e_2 \\ {}^C_0D_t^\alpha e_2 = -e_1 + y_3 e_2 + x_2 e_3 + u_1 \\ {}^C_0D_t^\alpha e_3 = (-1 - ay_2 - by_3)e_1 - ax_1 e_2 - bx_1 e_3 + u_2 \end{cases}$$

In order to eliminate the nonlinear term, we construct the active control law as follows

$$(9) \quad \begin{cases} u_1 = e_1 - y_3 e_2 + (k_1 - x_2)e_3 \\ u_2 = (ay_2 + by_3)e_1 + (k_2 + ax_1)e_2 - (k_3 + bx_1)e_3 \end{cases}$$

Substituting the control law (9) into the error system (8) leads to

$$(10) \quad \begin{cases} {}^C_0D_t^\alpha e_1 = e_2 \\ {}^C_0D_t^\alpha e_2 = k_1 e_3 \\ {}^C_0D_t^\alpha e_3 = -e_1 - k_2 e_2 - k_3 e_3 \end{cases}$$

The vector form of equation (10) can be rewritten as  ${}^C_0D_t^\alpha \mathbf{e} = A\mathbf{e}$ , specifically

$$(11) \quad \begin{pmatrix} {}^C_0D_t^\alpha e_1 \\ {}^C_0D_t^\alpha e_2 \\ {}^C_0D_t^\alpha e_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & k_1 \\ -1 & -k_2 & -k_3 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

where  $\mathbf{e} = [e_1, e_2, e_3]^T$ . We can choose the control parameters  $k_1, k_2, k_3$  according to theorem 2.1, which means that the controllability of the system equivalent to all the eigenvalues of  $A$  denoted as  $\lambda_i$ , satisfies the condition  $|\arg(\lambda_i)| > \alpha\pi/2$ . The theoretical conclusion can be verified by numerical simulation. We set parameters  $a = 15, b = 1, \alpha = 0.98$  and choose the control parameters  $k_1 = 2, k_2 = 3, k_3 = 4$ , the matrix  $A$  has three eigenvalues  $-0.4563, -1.7718 \pm j1.1151$ . This ensures that the error states will asymptotically converge to zero as  $t \rightarrow \infty$  and therefore the synchronization between master-slave systems (5) and (6) is achieved.

To ensure chaotic motion of the system, simulation results are shown in Fig. 2. Note that at the first time  $t < 5$  the controller is turned off, it is obvious that the state trajectories are disordered and error states grow with time chaotically. Fig. 2 (a)-(c) shows the state trajectories of master and slave.

One can see that the systems are synchronized after a transient state, when the controller is applied at time  $t = 5$ . In Fig. 2 (d), the error states are also illustrated that when the controller is switched after  $t = 5$ , one can see the error states will asymptotically convergence to zero.

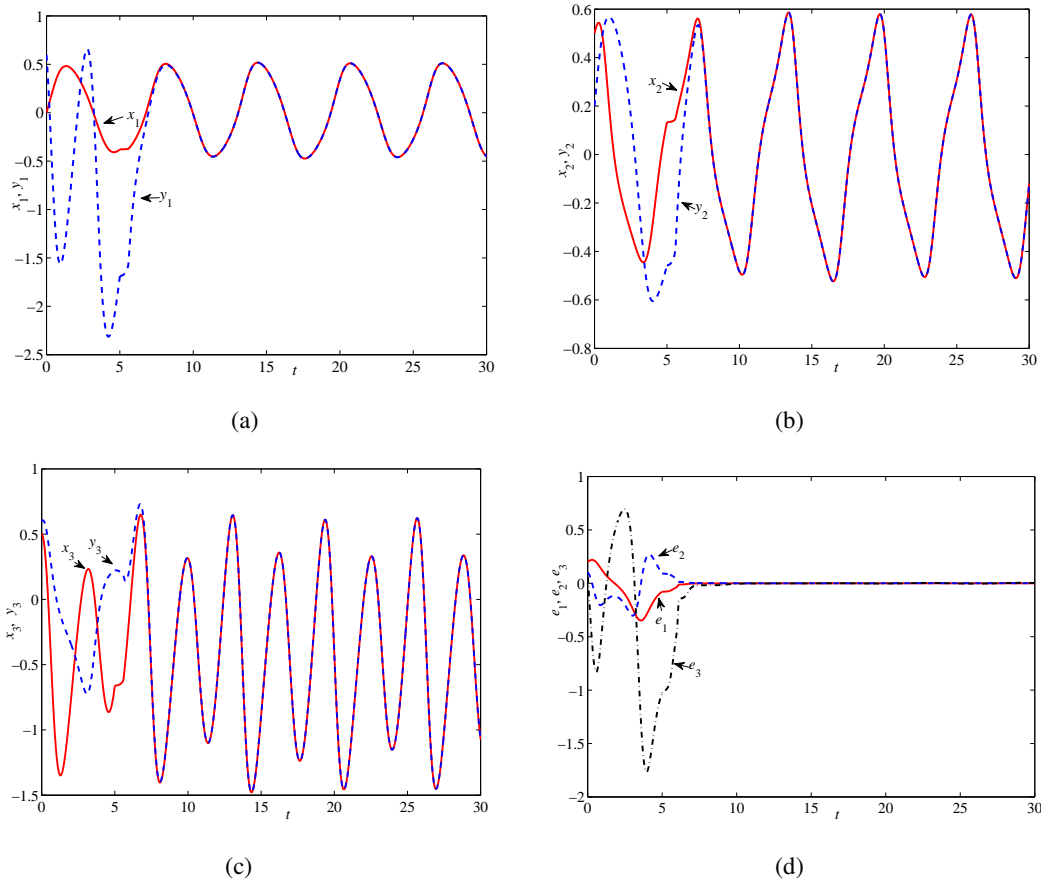


FIGURE 2. State trajectories of master and slave in the synchronization with active control law with initial values (0.1, 0.5, 0.5). (a) state trajectories of  $x_1, y_1$ , (b) state trajectories of  $x_2, y_2$ , (c) state trajectories of  $x_3, y_3$ , (d) error states in synchronization.

**Remark.** the control law designed in (9) is not unique. The general idea is to design three single nonlinear controllers  $u_1, u_2, u_3$  such that the error system (8) can be rewritten as the following form

$$(12) \quad \begin{pmatrix} {}^C_0 D_t^\alpha e_1 \\ {}^C_0 D_t^\alpha e_2 \\ {}^C_0 D_t^\alpha e_3 \end{pmatrix} = \begin{pmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & 0 \\ 0 & 0 & -p_3 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

where  $p_1, p_2, p_3$  are positive constants. In this case, it is easy to construct the positive definite matrix  $P = \text{diag}(1/p_1, 1/p_2, 1/p_3)$ , such that  $J = \mathbf{e}^T P_0^C D_t^\alpha \mathbf{e} = -e_1^2 - e_2^2 - e_3^2 \leq 0$  satisfied, hence the system is stable. However, by adopting the control strategy (9), with less controller and the error system (11) has a more general form, we can choose the corresponding control parameters flexibly according to the stability theory of fractional-order system in theorem 2.1.

## 5. Synchronization of fractional-order Jafari-Sprott chaotic system with sliding mode control

Although the active controller can synchronize the slave with the master. However, no robustness is guaranteed in the presence of uncertainties. In practice, models in real world may be imprecise. Given that the slave system is a fractional-order Jafari-Sprott system perturbed by uncertainties, in order to develop a robust synchronization, we will adopt the sliding mode control scheme which has been proved as an effective tool to control the uncertain systems robustly. Let us consider the system (5) as the master system, and the slave system rewritten as

$$(13) \quad \begin{cases} {}^C_0 D_t^\alpha y_1 = y_2 + u_1 \\ {}^C_0 D_t^\alpha y_2 = -y_1 + y_2 y_3 + u_2 \\ {}^C_0 D_t^\alpha y_3 = -y_1 - a y_1 y_2 - b y_1 y_3 + \Delta f(y_1, y_2, y_3) + u_3 \end{cases}$$

where  $\Delta f(y_1, y_2, y_3)$  presents system uncertainties. Similar to the processing method mentioned above, the state errors which yields the following dynamics:

$$(14) \quad \begin{cases} {}^C_0 D_t^\alpha e_1 = e_2 + u_1 \\ {}^C_0 D_t^\alpha e_2 = -e_1 + y_3 e_2 + x_2 e_3 + u_2 \\ {}^C_0 D_t^\alpha e_3 = (-1 - a y_2 - b y_3) e_1 - a x_1 e_2 - b x_1 e_3 + \Delta f(y_1, y_2, y_3) + u_3 \end{cases}$$

The first sliding surface for fractional-order system which is used integer derivation in sliding mode fractional controller design. Hence, we will define the fractional-order sliding surface as follows:

$$(15) \quad s(t) = D_t^{\alpha-1} \mathbf{e} + \lambda \int_0^t \mathbf{e}(\tau) d\tau$$



1 where  $\mathbf{s} = [s_1, s_2, s_3]^T$ ,  $\lambda = [\lambda_1, \lambda_2, \lambda_3]$ ,  $\mathbf{e} = [e_1, e_2, e_3]^T$

2

3

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5 (16) 
$$\begin{cases} s_1(t) = D_t^{\alpha-1} e_1 + \lambda_1 \int_0^t e_1(\tau) d\tau \\ s_2(t) = D_t^{\alpha-1} e_2 + \lambda_2 \int_0^t e_2(\tau) d\tau \\ s_3(t) = D_t^{\alpha-1} e_3 + \lambda_3 \int_0^t e_3(\tau) d\tau \end{cases}$$

6

7

8

9 In order for the error system (14) to reach the sliding surface and stay on the sliding surface, the

10 following conditions must be met:

11

12 (17) 
$$\mathbf{s}(t) = D_t^{\alpha-1} \mathbf{e}(t) + \lambda \int_0^t \mathbf{e}(\tau) d\tau = 0$$

13

14 and

15

16 (18) 
$$\dot{\mathbf{s}}(t) = D_t^\alpha \mathbf{e}(t) + \lambda \mathbf{e}(t) = 0$$

17

18 The candidate Lyapunov function can be chosen by  $V(t) = \frac{1}{2} \mathbf{s}^T \mathbf{s}$ , by taking the derivative of  $V(t)$ ,

19 the result can be rewritten as

20

21 (19) 
$$\begin{aligned} \dot{V}(t) &= \mathbf{s}^T \dot{\mathbf{s}} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + s_3 \dot{s}_3 \\ &= s_1 (D_t^\alpha e_1 + \lambda_1 e_1) + s_2 (D_t^\alpha e_2 + \lambda_2 e_2) + s_3 (D_t^\alpha e_3 + \lambda_3 e_3) \end{aligned}$$

22

23

24 from the state errors system (14), we can design the following control law

25

26

27

28 (20) 
$$\begin{cases} u_1(t) = -e_2 - \lambda_1 e_1 - k_1 \text{sign}(s_1) \\ u_2(t) = e_1 - y_3 e_2 - x_2 e_3 - \lambda_2 e_2 - k_2 \text{sign}(s_2) \\ u_3(t) = (1 + ay_2 + by_3) e_1 + ax_1 e_2 + bx_1 e_3 - \Delta f(y_1, y_2, y_3) \\ \quad - \lambda_3 e_3 - k_3 \text{sign}(s_3) \end{cases}$$

29

30

31

32 Substituting  $u_1, u_2, u_3$  into (14), and choosing suitable controller gain  $k_1, k_2, k_3$ , the following

33 sliding condition holds [27]

34

35 (21) 
$$\dot{V}(t) = -k_1 s_1 \text{sign}(s_1) - k_2 s_2 \text{sign}(s_2) - k_3 s_3 \text{sign}(s_3) \leq 0$$

36

37 It follows from (17) and (18) and combine the sliding condition (21), the trajectory is guaranteed to

38 move toward the sliding surface and the controlled system is stable. In the numerical simulations

39 we will fix the parameters  $a = 15$ ,  $b = 1$  and  $\alpha = 0.98$  to ensure the chaotic trajectory. To verify

40 the robustness of the controller, system uncertainties are modeled by  $\Delta f(y_1, y_2, y_3) = -0.3 \cos(y_1)$ ,

41 setting synovial surface parameters  $\lambda_1 = \lambda_2 = \lambda_3 = 3$  and the gain of control law  $k_1 = k_2 = k_3 = 2$ .

42

Numerical simulation results are shown in Fig. 3. Note that at the first time  $t < 5$  the control input is chattering free as the controller is turned off, it is obvious that the state trajectories are disordered and error states grow with time chaotically. Fig. 3 (a)-(c) shows the state trajectories of master and slave in the presence of uncertainties. One can see that the systems are synchronized after a transient state, when the controller is switched at time  $t = 5$ , the sliding surface is lead towards zero and synchronization is achieved. The error states are also illustrated in Fig. 3 (d), one can see the error states will asymptotically convergence to zero.

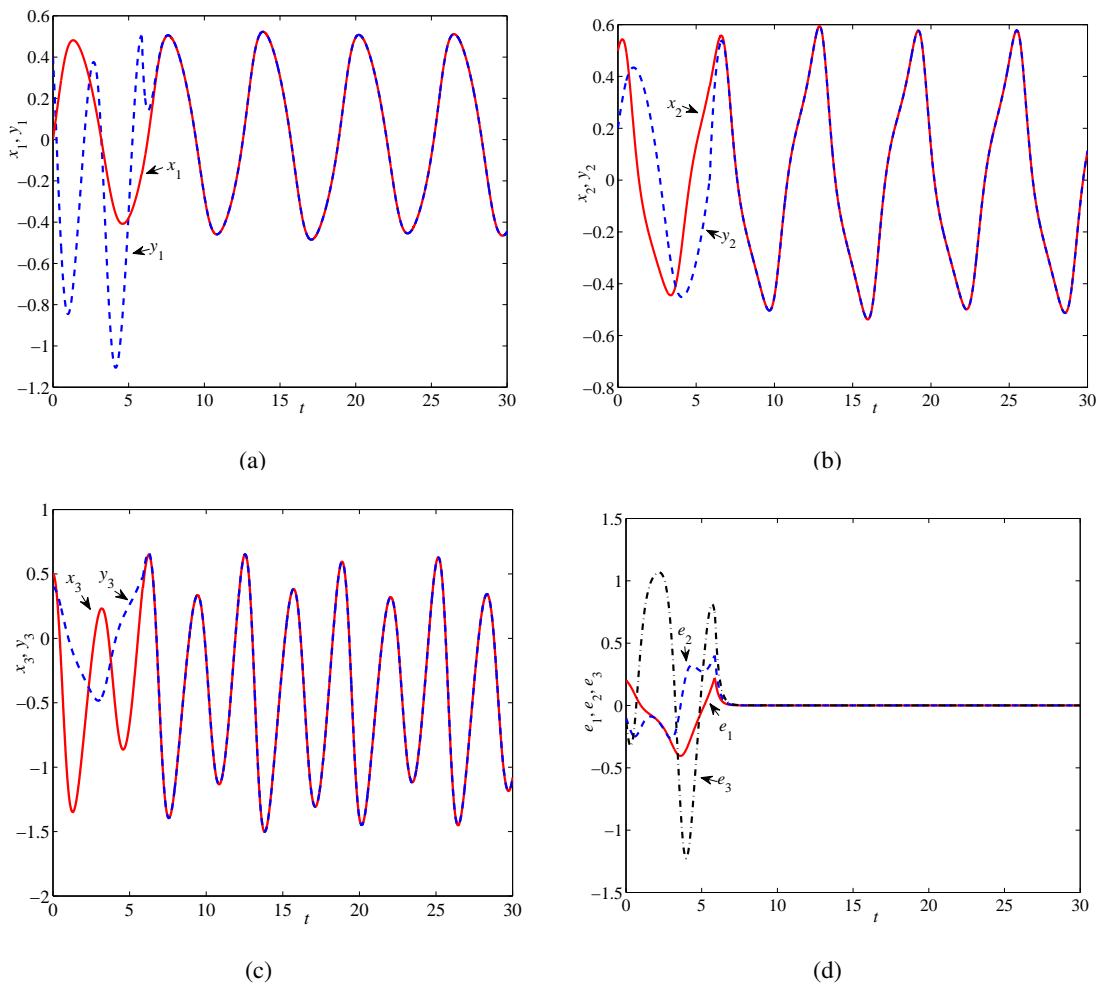


FIGURE 3. State trajectories of master and slave in the synchronization with sliding mode control with initial values (0.1, 0.5, 0.5). (a) state trajectories of  $x_1, y_1$ , (b) state trajectories of  $x_2, y_2$ , (c) state trajectories of  $x_3, y_3$ , (d) error states in synchronization.

## 6. Conclusions

This paper focus on the synchronization control law design for the fractional-order Jafari-Sprott chaotic system. Considering the system model without uncertainties, we propose the active control law to achieve the chaotic synchronization of fractional-order Jafari-Sprott system. By constructing the Lyapunov function and combining the stability of fractional-order systems, the corresponding controller parameters selection conditions are obtained. Furthermore, to realize the robust synchronization of the system model with uncertainties, the slide mode control is proposed and the corresponding controller parameters design are based on fractional-order sliding mode theory and Lyapunov stability theory. Numerical examples show that the proposed active control law and slide mode control strategy are simple, feasible to adjusted and could work efficiently and accurately. In the future work, we will consider to extend the current work to the more complex or super chaotic fractional-order systems, even to the fractional variable order systems.

**Data Availability.** Data are available on request to the corresponding author Min Shi.

**Conflicts of Interest.** The authors declare that they have no conflicts of interest.

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SYNCHRONIZATION CONTROL DESIGN OF FRACTIONAL-ORDER JAFARI-SPROTT CHAOTIC SYSTEM 14

1 THE COLLEGE OF AUTOMATION, NANJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS, NANJING  
2 210023, CHINA

3 *E-mail address:* 15996301586@163.com  
4

5 SCHOOL OF ECONOMICS AND MANAGEMENT, JIANGSU MARITIME INSTITUTE, NANJING 210023, CHINA

6 *E-mail address:* 182514823@qq.com  
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