32. *On Functions of Class U*

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We begin with the

Definition. *Let $f(z)$ $(z=re^{i\theta})$ be regular and bounded: $|f(z)|<1$ in $|z|<1$. If $|f(e^{i\theta})|=1$ almost everywhere on the arc $A(e^{i\theta}: \theta_1<\theta<\theta_2)$, then we call $f(z)$ a function of class $U(\theta_1, \theta_2)$. Instead of $U(0, 2\pi)$, we write simply U.*

The theory of class U has been studied and extended by many authors. For its full account, we refer to K. Noshiro ([1] pp. 32–48). In this note, we shall characterize the set of singularities of $f(z)\in U$ or $U(\theta_1, \theta_2)$ by its cluster sets.

Our main theorems read as follows:

Theorem 1. *For the Blaschke-product to be singular at $z=e^{i\theta}$, it is necessary and sufficient that $z=e^{i\theta}$ is a limiting point of its zeros.*

Theorem 2. *Suppose that $f(z)\in U$ and that it has at least one singular point on $|z|=1$. Then the set of singularities of $f(z)$ lying on $|z|=1$ coincides with the set of limiting points of the α-points ($|\alpha|<1$) of $f(z)$, except for a set of exceptional values α of capacity zero.*

Theorem 3. *Let $f(z)$ $(z=re^{i\theta})$ belong to class $U(\theta_1, \theta_2)$. If $f(z)$ is not regular on the arc $A(e^{i\theta}: \theta_1<\theta<\theta_2)$, then following propositions hold:*

(1) *the set S of singularities of $f(z)$ on A is the closure of the union $M_1\cup M_2$, where $M_1=A\cap E(e^{i\theta}: \alpha\in R(f, e^{i\theta}))$,*) $M_2=A\cap E(e^{i\theta}: \alpha=f(e^{i\theta}))$, and α is any fixed point of modulus less than 1.*

(2) *if at least two values in $|w|<1$ are omitted by $w=f(z)$ in the neighborhood of A, then S is a perfect set, whose capacity is positive.*

We shall give their full proof in another journal in the near future. The author wishes to express his hearty thanks to prof. K. Noshiro for his valuable criticism on this work.

Reference

[1] K. Noshiro: Cluster Sets, Berlin (1960).

*) $R(f, e^{i\theta})$ is the range of values at $e^{i\theta}$.