31. *Congruences between Siegel Modular Forms of Degree Two. II*

By Nobushige KUROKAWA
Department of Mathematics, Tokyo Institute of Technology

(Communicated by Kunihiko KODAIRA, M. J. A., Feb. 12, 1981)

Introduction. We supplement the previous note [6] by describing liftings of congruences. In particular, the congruences in Theorems 2 and 3 of [6] are considered to be congruences lifted from degree 1 to degree 2. The author would like to thank Prof. H. Maass for communicating that Prof. D. Zagier ([16]) proved completely the Conjectures 1 and 2 of [5] by using recent results of Prof. W. Kohnen after Maass [10] [11] [12] and Andrianov [2] (cf. § 1 below).

§ 1. **Liftings.** We denote by $M_k(\Gamma_n)$ (resp. $S_k(\Gamma_n)$) the vector space over the complex number field $\boldsymbol{C}$ consisting of all Siegel modular (resp. cusp) forms of degree n and weight k for integers $n \geqq 0$ and $k \geqq 0$. The space of Eisenstein series is denoted by $E_k(\Gamma_n)$ which is the orthogonal complement of $S_k(\Gamma_n)$ in $M_k(\Gamma_n)$ with respect to the Petersson inner product. We say that a modular form f in $M_k(\Gamma_n)$ is eigen if f is a non-zero eigenfunction of all Hecke operators on $M_k(\Gamma_n)$. Let f be an eigen modular form in $M_k(\Gamma_n)$ for $n=1, 2$. We define the (standard) Hecke polynomial at a prime p by $H_p(T, f)=1-\lambda(p, f)T+p^{k-1}T^2$ if $n=1$, and $H_p(T, f)=1-\lambda(p, f)T+(\lambda(p)^2-\lambda(p^2)-p^{2k-4})T^2-p^{2k-3}\lambda(p)T^3+p^{4k-6}T^4$ if $n=2$, where T is an indeterminate and $\lambda(m, f)$ is the eigenvalue of the Hecke operator $T(m)$ for $f: T(m)f=\lambda(m, f)f$. We define the (standard) L-function by $L(s, f)=\prod_p H_p(p^{-s}, f)^{-1}$ where p runs over all prime numbers. We denote by $\boldsymbol{Q}(f)$ the field generated by $\{\lambda(m, f) \mid m \geqq 1\}$ over the rational number field $\boldsymbol{Q}$, and we put $\boldsymbol{Z}(f)=\boldsymbol{Q}(f) \cap \bar{\boldsymbol{Z}}$, where $\boldsymbol{Z}$ is the rational integer ring, and $\bar{\boldsymbol{Z}}$ is the ring of all algebraic integers in $\boldsymbol{C}$. Then $\boldsymbol{Q}(f)$ is a totally real finite extension of $\boldsymbol{Q}$, and $\boldsymbol{Z}(f)$ is the integer ring of $\boldsymbol{Q}(f)$. See [7] which contains the case of general degree.

We consider the following two liftings from degree 1 to degree 2 for each even integer $k \geqq 4$.

(A) The first lifting is the $\boldsymbol{C}$-linear injection $[\ ]: M_k(\Gamma_1) \to M_k(\Gamma_2)$ defined in [8] (cf. [6] [9]), which is given by the (generalized) Eisenstein series. For each eigen modular form f in $M_k(\Gamma_1)$ we have that: $[f]$ is an eigen modular form satisfying $H_p(T, [f])=H_p(T, f)H_p(p^{k-2}T, f)$ for all p and $L(s, [f])=L(s, f)L(s-k+2, f)$.

(B) The second lifting is the $\boldsymbol{C}$-linear injection $\sigma_k: M_{2k-2}(\Gamma_1)$

$\to M_k(\Gamma_2)$ constructed by Maass [10] [11] [12], Andrianov [2] and Zagier [16], which was conjectured in [5]. For each eigen modular form f in $M_{2k-2}(\Gamma_1)$ we have that: $\sigma_k(f)$ is an eigen modular form satisfying $H_p(T, \sigma_k(f))=(1-p^{k-2}T)(1-p^{k-1}T)H_p(T, f)$ for all p and $L(s, \sigma_k(f)) =\zeta(s-k+2)\zeta(s-k+1)L(s, f)$. Here we define $\sigma_k(E_{2k-2})=\varphi_k$ for the Eisenstein series; see [5, 2.2(5)].

These liftings give the following decompositions of $M_k(\Gamma_2): M_k(\Gamma_2) = E_k(\Gamma_2)\oplus S_k(\Gamma_2) = M_k^{\mathrm{I}}(\Gamma_2)\oplus M_k^{\mathrm{II}}(\Gamma_2) = E_k^{\mathrm{I}}(\Gamma_2)\oplus E_k^{\mathrm{II}}(\Gamma_2)\oplus S_k^{\mathrm{I}}(\Gamma_2)\oplus S_k^{\mathrm{II}}(\Gamma_2)$. The notation is as follows. We put $E_k^{\mathrm{I}}(\Gamma_2)=[E_k(\Gamma_1)]=\boldsymbol{C}\cdot\varphi_k$ and $E_k^{\mathrm{II}}(\Gamma_2) =[S_k(\Gamma_1)]$, then we have $E_k(\Gamma_2)=[M_k(\Gamma_1)]=E_k^{\mathrm{I}}(\Gamma_2)\oplus E_k^{\mathrm{II}}(\Gamma_2)$. We put

$$M_k^{\mathrm{I}}(\Gamma_2)=\left\{f \in M_k(\Gamma_2) \mid a(T, f)=\sum_{d|e(T)} d^{k-1}a\left(\left\langle\frac{1}{d}T\right\rangle, f\right) \text{ for all } T\geqq 0,\ T\neq 0\right\}$$

and $S_k^{\mathrm{I}}(\Gamma_2)=M_k^{\mathrm{I}}(\Gamma_2)\cap S_k(\Gamma_2)$ with the notation of [5, § 4]. Then we have $M_k^{\mathrm{I}}(\Gamma_2) = \sigma_k(M_{2k-2}(\Gamma_1)) = E_k^{\mathrm{I}}(\Gamma_2)\oplus S_k^{\mathrm{I}}(\Gamma_2)$. Here it holds that $E_k^{\mathrm{I}}(\Gamma_2) =M_k^{\mathrm{I}}(\Gamma_2)\cap E_k(\Gamma_2)=\boldsymbol{C}\cdot\varphi_k$. We denote by $S_k^{\mathrm{II}}(\Gamma_2)$ the orthogonal complement of $S_k^{\mathrm{I}}(\Gamma_2)$ in $S_k(\Gamma_2)$ with respect to the Petersson inner product ([6, Remark 3]), and we put $M_k^{\mathrm{II}}(\Gamma_2)=E_k^{\mathrm{II}}(\Gamma_2)\oplus S_k^{\mathrm{II}}(\Gamma_2)$.

We note on ℓ-adic representations. We fix a prime number ℓ. Let f be an eigen modular form in $M_k(\Gamma_1)$ for even $k\geqq 4$. Let $\mathfrak{l}$ be a prime ideal of $\boldsymbol{Q}(f)$ dividing ℓ. We denote by $\boldsymbol{Q}(f)_{\mathfrak{l}}$ the $\mathfrak{l}$-adic completion of $\boldsymbol{Q}(f)$ and by $\boldsymbol{Z}(f)_{\mathfrak{l}}$ the integer ring of $\boldsymbol{Q}(f)_{\mathfrak{l}}$. Then, Deligne ([3] and [4, Th. 6.1]) constructed a continuous $\mathfrak{l}$-adic representation $\rho_{\mathfrak{l}}(f)$: $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})\to GL(2, \boldsymbol{Z}(f)_{\mathfrak{l}})$ ($\bar{\boldsymbol{Q}}$ being the algebraic closure of $\boldsymbol{Q}$ in $\boldsymbol{C}$) attached to f such that $\rho_{\mathfrak{l}}(f)$ is unramified outside of ℓ and satisfies $\det(1-\rho_{\mathfrak{l}}(f)(\mathrm{Frob}(p))T)=H_p(T, f)$ for all prime numbers $p\neq\ell$, where $\mathrm{Frob}(p)$ denotes the Frobenius conjugacy class at p. We denote by χ_ℓ: $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})\to GL(1, \boldsymbol{Z}_\ell)$ the cyclotomic ℓ-adic representation, where $\boldsymbol{Z}_\ell$ is the ring of ℓ-adic integers. Next, let F be an eigen modular form in $M_k(\Gamma_2)$ for (even) $k\geqq 4$. Let $\mathfrak{l}$ be a prime ideal of $\boldsymbol{Q}(F)$ dividing ℓ. We denote by $\boldsymbol{Q}(F)_{\mathfrak{l}}$ the $\mathfrak{l}$-adic completion of $\boldsymbol{Q}(F)$ and by $\boldsymbol{Z}(F)_{\mathfrak{l}}$ the integer ring of $\boldsymbol{Q}(F)_{\mathfrak{l}}$. Then, it is conjectured that there exists a continuous $\mathfrak{l}$-adic representation $\rho_{\mathfrak{l}}(F)$: $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})\to GL(4, \boldsymbol{Z}(F)_{\mathfrak{l}})$ such that $\rho_{\mathfrak{l}}(F)$ is unramified outside of ℓ and satisfies $\det(1-\rho_{\mathfrak{l}}(F)(\mathrm{Frob}(p))T) =H_p(T, F)$ for all prime numbers $p\neq\ell$. For liftings (A)(B) such an $\mathfrak{l}$-adic representation is defined as follows. (A) If $F=[f]$ with an eigen $f\in M_k(\Gamma_1)$, then we put $\rho_{\mathfrak{l}}(F)=\rho_{\mathfrak{l}}(f)\oplus\chi_\ell^{k-2}\otimes\rho_{\mathfrak{l}}(f)$. (B) If $F=\sigma_k(f)$ with an eigen $f\in M_{2k-2}(\Gamma_1)$, then we put $\rho_{\mathfrak{l}}(F)=\chi_\ell^{k-2}\oplus\chi_\ell^{k-1}\oplus\rho_{\mathfrak{l}}(f)$. Note that $\boldsymbol{Z}(F)=\boldsymbol{Z}(f)$ in both cases. It might be natural to consider as $\rho_{\mathfrak{l}}(F)$: $\mathrm{Gal}(\bar{\boldsymbol{Q}}/\boldsymbol{Q})\to CSp(4, \boldsymbol{Z}(F)_{\mathfrak{l}})$ by a slight modification.

§ 2. **Congruences.** We recall the definition of Hecke operators following Andrianov [1, § 1.3] (cf. [7]). For integers $n\geqq 1$ and $m\geqq 1$ we put $S^{(n)}=\{M\in M(2n, \boldsymbol{Z}) \mid {}^tMJ_nM=\nu(M)J_n$ with an integer $\nu(M)\geqq 1\}$

and $S_m^{(n)}=\{M\in S^{(n)}|\nu(M)=m\}$, where ${}^tM$ denotes the transposed matrix of M and $J_n=\begin{pmatrix}0 & E_n\\ -E_n & 0\end{pmatrix}$ with the identity matrix E_n of size n. For each subring R of $\boldsymbol{C}$ we denote by $L_R^{(n)}$ (resp. $L_{R,m}^{(n)}$) the R-module generated by the double cosets $\Gamma_n M\Gamma_n$ for all $M\in S^{(n)}$ (resp. $S_m^{(n)}$). Under the usual multiplication, $L_R^{(n)}$ is an R-algebra (the abstract Hecke algebra of degree n over R). We put $H_R^{(n)}=\bigcup_{m\geq 1}L_{R,m}^{(n)}$ (the set of "homogeneous" elements of $L_R^{(n)}$), and we define a map $\nu: H_R^{(n)}\to\boldsymbol{Z}$ by $\nu(X)=m$ if $X\in L_{R,m}^{(n)}$, $X\neq 0$, and $\nu(0)=0$. Then ν is a homomorphism between (multiplicative) semi-groups. We denote by $\tau=\tau_k^{(n)}: L_C^{(n)}\to\mathrm{End}_C(M_k(\Gamma_n))$ the representation of the Hecke algebra $L_C^{(n)}$ on $M_k(\Gamma_n)$ defined in Andrianov [1, (1.3.3)].

Let $f\in M_k(\Gamma_n)$ and $g\in M_{k-r}(\Gamma_n)$ be eigen modular forms for an integer $n\geqq 1$ and even integers $k\geqq r\geqq 0$. In [6], we defined the eigencharacter $\lambda(f)$ (resp. $\lambda(g)$) and a totally real finite number field $\boldsymbol{Q}(f)$ (resp. $\boldsymbol{Q}(g)$) attached to f (resp. g). We denote by $\boldsymbol{Q}(f,g)=\boldsymbol{Q}(f)\boldsymbol{Q}(g)$ the composite field and by $\boldsymbol{Z}(f,g)$ the integer ring of $\boldsymbol{Q}(f,g)$. For an ideal $\mathfrak{c}$ of $\boldsymbol{Z}(f,g)$ we write $\lambda(f)\equiv\nu^{nr/2}\lambda(g) \bmod \mathfrak{c}$ if $\lambda(f)(\tau_k^{(n)}(X))-\nu(X)^{nr/2}\cdot\lambda(g)(\tau_{k-r}^{(n)}(X))$ belongs to $\tilde{\mathfrak{c}}$ for all $X\in H_Z^{(n)}$, where $\tilde{\mathfrak{c}}=\{\alpha/\beta|\alpha\in\mathfrak{c},\beta\in\boldsymbol{Z}(f,g),((\beta),\mathfrak{c})=\boldsymbol{Z}(f,g)\}$. (The case $r=0$ coincides with the definition in [7, §4].) For $n=1$ and 2, this condition is equivalent to the following: $\lambda(m,f)\equiv m^{nr/2}\lambda(m,g) \bmod \mathfrak{c}$ for all integers $m\geqq 1$. Moreover we can restrict to $m=p$ (resp. $m=p,p^2$) for $n=1$ (resp. $n=2$) where p runs over all prime numbers, and this is equivalent to the following congruence between Hecke polynomials: $H_p(T,f)\equiv H_p(p^{nr/2}T,g) \bmod \mathfrak{c}$ for all prime numbers p. In fact, $\sum_{\delta\geq 0}(\lambda(p^\delta,f)-p^{nr\delta/2}\lambda(p^\delta,g))T^\delta$ $=(H_p(T,f)^{-1}-H_p(p^{nr/2}T,g)^{-1})\times\begin{cases}1 & \text{if } n=1,\\ (1-p^{2k-4}T^2) & \text{if } n=2.\end{cases}$

Eigenvalues of Hecke operators in [5] suggest, for example, the following congruences: $\lambda(\chi_{20}^{(3)})\equiv\nu^2\lambda([\Delta_{18}]) \bmod 7^2$, $\lambda(\chi_{20}^{(3)})\equiv\nu^4\lambda([\Delta_{16}]) \bmod 11$, $\lambda(\chi_{20}^{(3)})\equiv\nu^8\lambda([\Delta_{12}]) \bmod 7\cdot 29$. These congruences supplement the following congruence proved in Theorem 1 of [6]: $\lambda(\chi_{20}^{(3)})\equiv\lambda([\Delta_{20}]) \bmod 71^2$ which is equivalent to $H_p(T,\chi_{20}^{(3)})\equiv H_p(T,[\Delta_{20}]) \bmod 71^2$ for all p. They seem to suggest to use a derivation $\partial=\oplus_{k\geq 0}\partial_k$ of $M(\Gamma_n)=\oplus_{k\geq 0}M_k(\Gamma_n)$ (a graded $\boldsymbol{C}$-algebra) such that $\partial_k(M_k(\Gamma_n))\subset M_{k+2}(\Gamma_n)$ and $\partial(M(\Gamma_n)_Z)\subset M(\Gamma_n)_Z$ where $M(\Gamma_n)_Z$ denotes the graded $\boldsymbol{Z}$-algebra $\oplus_{k\geq 0}M_k(\Gamma_n)_Z$ consisting of Siegel modular forms of degree n with Fourier coefficients in $\boldsymbol{Z}$. See Ramanujan [13], Serre [14] and Swinnerton-Dyer [15] for the case $n=1$. We remark that similar congruences such as $\lambda(\chi_{10})\equiv\nu^2\lambda(\varphi_8) \bmod 5$ are proved by reducing to the elliptic modular case; see the next section (type (B)).

§3. Liftings of congruences. We note three types of congruences lifted from degree 1 to degree 2.

Theorem. *Let $k \geqq 4$ be an even integer. Then the following hold.*

(A) *Let f and g be eigen modular forms in $M_k(\Gamma_1)$ satisfying $\lambda(f) \equiv \lambda(g) \bmod \mathfrak{c}$ with an ideal $\mathfrak{c}$ of $\boldsymbol{Z}(f, g)$. Then we have $\lambda([f]) \equiv \lambda([g]) \bmod \mathfrak{c}$.*

(B) *Let $f \in M_{2k-2}(\Gamma_1)$ and $g \in M_{2k-2r-2}(\Gamma_1)$ be eigen modular forms for an even integer r in $0 \leqq r \leqq k-4$. Assume that $\lambda(f) \equiv \nu^r \lambda(g) \bmod \mathfrak{c}$ for an ideal $\mathfrak{c}$ of $\boldsymbol{Z}(f, g)$. Then we have $\lambda(\sigma_k(f)) \equiv \nu^r \lambda(\sigma_{k-r}(g)) \bmod \mathfrak{c}$.*

(C) *(Mixed type) Let $f \in M_k(\Gamma_1)$ and $g \in M_{2k-2}(\Gamma_1)$ be eigen modular forms. Let $r=0$ or 1. Assume that $\lambda(f) \equiv \nu^r \lambda(E_{k-2r}) \bmod \mathfrak{c}$ and $\lambda(g) \equiv \nu^r \lambda(E_{2k-2r-2}) \bmod \mathfrak{c}$ for an ideal $\mathfrak{c}$ of $\boldsymbol{Z}(f, g)$. Then we have $\lambda([f]) \equiv \lambda(\sigma_k(g)) \bmod \mathfrak{c}$.*

Proof. It is sufficient to show the congruences for Hecke polynomials. Let p be a prime number and T an indeterminate.

(A) $H_p(T, [f]) \equiv H_p(T, [g]) \bmod \mathfrak{c}$ follows from $H_p(T, f) \equiv H_p(T, g) \bmod \mathfrak{c}$.

(B) $H_p(T, \sigma_k(f)) \equiv H_p(p^r T, \sigma_{k-r}(g)) \bmod \mathfrak{c}$ follows from $H_p(T, f) \equiv H_p(p^r T, g) \bmod \mathfrak{c}$.

(C) We have $H_p(T, f) \equiv (1-p^r T)(1-p^{k-r-1}T) \bmod \mathfrak{c}$ from $\lambda(f) \equiv \nu^r \lambda(E_{k-2r}) \bmod \mathfrak{c}$. Hence $H_p(T, [f]) \equiv (1-p_r T)(1-p^{k-r-1}T)(1-p^{k+r-2}T)(1-p^{2k-r-3}T) \bmod \mathfrak{c}$. We have $H_p(T, g) \equiv (1-p^r T)(1-p^{2k-r-3}T) \bmod \mathfrak{c}$ from $\lambda(g) \equiv \nu^r \lambda(E_{2k-2r-2}) \bmod \mathfrak{c}$. Hence $H_p(T, \sigma_k(g)) \equiv (1-p^r T)(1-p^{k-2}T)(1-p^{k-1}T)(1-p^{2k-r-3}T) \bmod \mathfrak{c}$. Since $r=0$ or 1, we have $H_p(T, [f]) \equiv H_p(T, \sigma_k(g)) \bmod \mathfrak{c}$.

Alternatively we can use the equality of the following type (here we note on (B) as an example): $\sum_{\delta \geqq 0} (\lambda(p^\delta, \sigma_k(f)) - p^{r\delta} \lambda(p^\delta, \sigma_{k-r}(g)))T^\delta = (1-p^{2k-4}T^2)(1-p^{k-2}T)^{-1}(1-p^{k-1}T)^{-1} \sum_{\delta \geqq 0} (\lambda(p^\delta, f) - p^{r\delta} \lambda(p^\delta, g))T^\delta$.

Q.E.D.

Examples. From some congruences in the elliptic modular case (see Ramanujan [13], Serre [14], and Swinnerton-Dyer [15]) we have the following congruences. We use the notation of [5] for modular forms.

(A) We note a typical example. From the Ramanujan's congruence $\lambda(\Delta_{12}) \equiv \lambda(E_{12}) \bmod 691$, we have $\lambda([\Delta_{12}]) \equiv \lambda(\varphi_{12}) \bmod 691$. This is proved also as in [6].

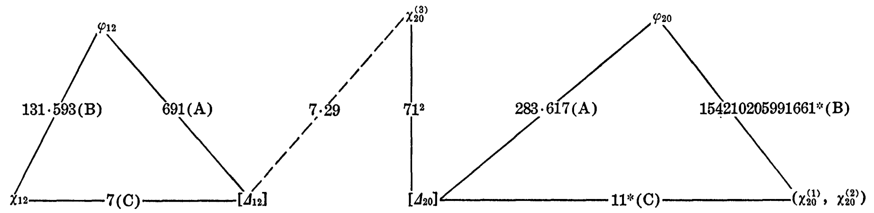
(B) $\lambda(\Delta_{18}) \equiv \lambda(E_{18}) \bmod 43867 \Rightarrow \lambda(\chi_{10}) \equiv \lambda(\varphi_{10}) \bmod 43867$.
$\lambda(\Delta_{22}) \equiv \lambda(E_{22}) \bmod 131 \cdot 593 \Rightarrow \lambda(\chi_{12}) \equiv \lambda(\varphi_{12}) \bmod 131 \cdot 593$.
$\lambda(\Delta_{26}) \equiv \lambda(E_{26}) \bmod 657931 \Rightarrow \lambda(\chi_{14}) \equiv \lambda(\varphi_{14}) \bmod 657931$.

The above three congruences coincide with Theorem 2 of [6].

$\lambda(\Delta_{18}) \equiv \nu^2 \lambda(E_{14}) \bmod 5 \Rightarrow \lambda(\chi_{10}) \equiv \nu^2 \lambda(\varphi_8) \bmod 5$.
$\lambda(\Delta_{22}) \equiv \nu^2 \lambda(E_{18}) \bmod 5 \Rightarrow \lambda(\chi_{12}) \equiv \nu^2 \lambda(\varphi_{10}) \bmod 5$.
$\lambda(\Delta_{26}) \equiv \nu^2 \lambda(E_{22}) \bmod 5 \cdot 7 \Rightarrow \lambda(\chi_{14}) \equiv \nu^2 \lambda(\varphi_{12}) \bmod 5 \cdot 7$.

(C) From $\lambda(\Delta_{12})\equiv\nu\lambda(E_{10})$ mod 7 and $\lambda(\Delta_{22})\equiv\nu\lambda(E_{20})$ mod 7 we have $\lambda(\chi_{12})\equiv\lambda([\Delta_{12}])$ mod 7. This congruence coincides with Theorem 3 of [6]. We may consider $7\,|\,L_2^*(22, \Delta_{12})$ as an interpretation for $\lambda(\Delta_{12})\equiv\nu\lambda(E_{10})$ mod 7.

We may list some congruences according to the decomposition $M_k(\Gamma_2)=E_k^{\mathrm{I}}(\Gamma_2)\oplus E_k^{\mathrm{II}}(\Gamma_2)\oplus S_k^{\mathrm{I}}(\Gamma_2)\oplus S_k^{\mathrm{II}}(\Gamma_2)$ for weight $k=12$ and 20 as follows.



We remark that $\boldsymbol{Q}(\chi_{20}^{(1)})=\boldsymbol{Q}(\chi_{20}^{(2)})=\boldsymbol{Q}(\sqrt{63737521})$, and the two congruences related to $\chi_{20}^{(i)}$ for $i=1$ and 2 indicate that: $N(\lambda(m, \chi_{20}^{(i)})-\lambda(m, \varphi_{20}))\equiv 0$ mod 154210205991661 and $N(\lambda(m, \chi_{20}^{(i)})-\lambda(m, [\Delta_{20}]))\equiv 0$ mod 11, for all $m\geqq 1$, where N: $\boldsymbol{Q}(\sqrt{63737521})\to\boldsymbol{Q}$ denotes the norm map. These congruences are proved as in [6]. On the other hand, they are also reduced to the elliptic modular case by (B) with $r=0$ and (C) with $r=1$ respectively.

We note a congruence for Fourier coefficients. From [6] we see that the Fourier coefficients $7a(T, [\Delta_{12}])$ are integers for all $T\geqq 0$, and some numerical values (cf. [9, Table I]) suggest that $7a(T, [\Delta_{12}])\equiv 0$ mod 23 for all $T>0$. We remark that $\ell=23$ is an exceptional prime for Δ_{12} of type (ii) in the sense of Serre [14] and Swinnerton-Dyer [15] and $23=2k-1$ with $k=12$. Similar possible examples are $\ell=31$ (resp. 47) for $k=16$ (resp. 24).

References

[1] A. N. Andrianov: Euler products corresponding to Siegel modular forms of genus 2. Russian Math. Surveys, **29**, 45–116 (1974) (English translation).

[2] ——: Modular descent and the Saito-Kurokawa conjecture. Invent. Math., **53**, 267–280 (1979).

[3] P. Deligne: Formes modulaires et représentations ℓ-adiques. Séminaire Bourbaki, exp. 355 (February 1969), Lect. Notes in Math., vol. 179, Springer-Verlag, pp. 139–186 (1971).

[4] P. Deligne and J.-P. Serre: Formes modulaires de poids 1. Ann. sci. E.N.S. 4e ser., **7**, 507–530 (1974).

[5] N. Kurokawa: Examples of eigenvalues of Hecke operators on Siegel cusp forms of degree two. Invent. Math., **49**, 149–165 (1978).

[6] ——: Congruences between Siegel modular forms of degree two. Proc.

Japan Acad., **55**A, 417–422 (1979).

[7] N. Kurokawa: On Siegel eigenforms. Proc. Japan Acad., **57**A, 47–50 (1981).

[8] ——: On Eisenstein series for Siegel modular groups. Proc. Japan Acad., **57**A, 51–55 (1981); II (preprint).

[9] N. Kurokawa and S. Mizumoto: On Eisenstein series of degree two. Proc. Japan Acad., **57**A, 134–139 (1981).

[10] H. Maass: Über eine Spezialschar von Modulformen zweiten Grades. Invent. Math., **52**, 95–104 (1979).

[11] ——: Über eine Spezialschar von Modulformen zweiten Grades (II). Invent. Math., **53**, 249–253 (1979).

[12] ——: Über eine Spezialschar von Modulformen zweiten Grades (III). Invent. Math., **53**, 255–265 (1979).

[13] S. Ramanujan: On certain arithmetical functions. Trans. Cambridge Philos. Soc., **22**, 159–184 (1916).

[14] J.-P. Serre: Congruences et formes modulaires. Séminaire Bourbaki, exp. 416 (June 1972), Lect. Notes in Math., vol. 317, Springer-Verlag, pp. 319–338 (1973).

[15] H. P. F. Swinnerton-Dyer: On ℓ-adic representations and congruences for coefficients of modular forms. Lect. Notes in Math., vol. 350, Springer-Verlag, pp. 1–55 (1973).

[16] D. Zagier: A talk at the symposium "Modular Functions of Several Variables". Oberwolfach, 1980.