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FUNCTIONALLY COMPACT SPACES

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The purpose of this note is to introduce a property which is weaker than compactness but stronger than minimality which closely relates to some filter properties and some mapping properties of compact spaces. We also indicate some procedures for constructing absolutely closed and minimal spaces.

All of the definitions used but not given in this paper may be found in [2].

DEFINITION. An open filter base on a topological space X is a filter base consisting of open subsets of X.

A Hausdorff topological space X is called *functionally compact* if whenever $\mathscr{U}$ is an open filter base on X such that the intersection A of the elements of $\mathscr{U}$ is equal to the intersection of the closures of the elements of $\mathscr{U}$, then $\mathscr{U}$ is a base for the neighborhoods of A.

THEOREM 1. *There exists a noncompact Hausdorff space which is functionally compact.*

Proof. Let $Z^+ =$ positive integers, $I = [0, 1]$. Let $0 < a_1 < a_2 < \cdots < a_n < \cdots$ be an increasing sequence in I with $\lim a_n = a_0$. For each $i \in Z^+$ let a_i^j be a strictly increasing sequence (in (a_{i-1}, a_i) for $i \geqq 2$; in $(0, a_1)$ for $i = 1$) with $\lim_j a_i^j = a_i$. Let

$$C = \bigcup_{i>0} \{a_i\} \cup \bigcup_{\substack{i>1 \\ j>1}} \{a_i^j\} .$$

Let $\alpha_0 = a_0$ and for $i \geqq 1$ let

$$\alpha_i = \{a_1^i, a_2^{i-1}, \cdots, a_i^1, a_i\} .$$

Let $C^* = \{\alpha_i : i \geqq 0\}$ and put

$$F = (I \backslash C) \cup C^* .$$

Let $\mathscr{T}$ be the topology on F such that $\mathscr{T} \mid (I\backslash C)$ is the usual topology on $I\backslash C$ and a basic open neighborhood of $\alpha_n (n \geqq 0)$ is a set of the form $N \cup \{\alpha_n\}$, where $N = O \cap (I\backslash C)$, O open in I and $\{a_1^n, \cdots, a_n^1, a_n\} \subset O$.

It is easy to see that $(F, \mathscr{T})$ is a Hausdorff space. It is not compact, however, because C^* is an infinite discrete closed set in F.

In fact, if $O_1, O_2 \in \mathscr{T}$ and $O_1 \cap C^* \neq \emptyset, O_2 \cap C^* \neq \emptyset$ then $cl_F O_1 \cap cl_F O_2 \cap C^*$ is infinite, so that points in C^* cannot be separated by closed neighborhoods. We will now show that $(F, \mathscr{T})$ is functionally compact.

Define the set function $\psi: F \to I$ via

$$\psi(\{a\}) = \begin{cases} a \text{ if } a \in I \backslash C \\ a_0 \text{ if } a = \alpha_0 \\ \{a_1^i, \cdots, a_i^1, a_i\} \text{ if } a = \alpha_i, i \geqq 1 \end{cases}$$

and $\psi(A) = \bigcup_{a \in A} \psi(\{a\})$ for $A \subseteqq F$. ψ does not preserve openness or closedness. However, if $A \subseteqq F$ is closed then there exists a set A^c, closed in I, such that

(i) $\psi(A) \cap (I \backslash C) = A^c \cap (I \backslash C)$

and

(ii) $\psi(A) \supseteqq A^c$,

namely,

$$A^c = cl_I \psi(A) \cap (I \backslash C) .$$

Hence, if $U \subseteqq F$ is open there exists an open set $U^0 \subseteqq I$ such that

(iii) $U^0 \cap (I \backslash C) = \psi(U) \cap (I \backslash C)$

(iv) $U^0 \supseteqq \psi(U)$;

namely, the set $U^0 = ((U')^c)'$ (where $'$ denotes complementation in either F or I).

Now, suppose $\mathscr{U}$ is an open filter in F such that $\cap \{U: U \in \mathscr{U}\} = \cap \{cl_F U: U \in \mathscr{U}\} = A$ and that $V \supseteqq A$, $V \in \mathscr{T}$. We claim that there exist $U_1, U_2 \in \mathscr{T}$ such that $V \supseteqq U_1 \cap U_2$.

First of all, if $O \in \mathscr{T}$ and $O \cap C^*$ is infinite, then $cl_F O \supseteqq C^*$. Hence if $A \cap C^*$ is finite then there exists $U \in \mathscr{U}$ such that $U \cap C^* = A \cap C^*$, while if $A \cap C^*$ is infinite then $A \supseteqq C^*$. Thus, in either case there exists $U_1 \in \mathscr{U}$ such that $V \cap C^* \supseteqq U_1 \cap C^*$. Furthermore,

$$V^0 \supseteqq \psi(A) = \cap \psi(cl_F U) \supseteqq \cap (cl_F U)^c .$$

Hence, since V^0 is open in I and $(cl_F U)^c$ is closed in I the compactness of I implies that there exists $U_2 \in \mathscr{U}$ such that $V^0 \supsetneqq (cl_F U_2)^c$. But then

$$\begin{aligned} V^0 \cap (I \backslash C) = \psi(V) \cap (I \backslash C) &= V \cap (I \backslash C) \supseteqq (cl_F U_2)^c \cap (I \backslash C) \\ &= (cl_F U_2) \cap (I \backslash C) \supseteqq U_2 \cap (I \backslash C) . \end{aligned}$$

Hence

$$V = (V \cap (I \backslash C)) \cup (V \cap C^*) \supseteqq (U_2 \cap (I \backslash C)) \cup (U_1 \cap C^*) \supseteqq U_1 \cap U_2 .$$

This proves the result.

There are some obvious generalizations of this construction. The above example F of a functionally compact space also shows that the property of being functionally compact is not closed-hereditary or open-hereditary or even regular-closed-hereditary.

THEOREM 2. *A Hausdorff space X is minimal Hausdorff if and only if for every point $x \in X$ and every open filter-base $\mathscr{U}$ on X such that $x = \cap \{U: U \in \mathscr{U}\}$ and $x = \cap \{cl_x U: U \in \mathscr{U}\}$, $\mathscr{U}$ is a base for the neighborhoods of x.*

Proof of the necessity. Suppose that X is minimal Hausdorff and that $\mathscr{U}$ is an open filter-base on X and $x \in X$ such that

$$x = \cap \{U: U \in \mathscr{U}\} \quad \text{and} \quad x = \cap \{cl_x U: U \in \mathscr{U}\}$$

and let R be any open set containing x. We note that x is the unique cluster point of $\mathscr{U}$. Thus by Theorem 1.3 of [1], $\mathscr{U}$ converges to x and so there exists $U \in \mathscr{U}$ such that $\mathscr{U} \subset R$. Of course this implies that $\mathscr{U}$ is a base for the neighborhoods of x.

Proof of the sufficiency. We need to show that every open filter-base with a unique cluster point converges to that point. To this end let $\mathscr{V}$ be an open filter-base on X with a unique cluster point x and let R be any open subset of X containing x. Let $\mathscr{W}$ be the collection of all open subsets of X containing x and let

$$\mathscr{U} = \{V \cap W: V \in \mathscr{V} \text{ and } W \in \mathscr{W}\} .$$

Then $\mathscr{U}$ is an open filter-base on X,

$$x = \cap \{U: U \in \mathscr{U}\} \text{ and } x = \cap \{cl_x U: U \in \mathscr{U}\} .$$

Thus by our hypothesis there exists $U \in \mathscr{U}$ such that $U \subset R$. Of course this implies that $\mathscr{V}$ converges to x and this completes the proof.

COROLLARY 2.1. *Every functionally compact Hausdorff space is minimal Hausdorff.*

Proof. This result is an immediate consequence of the definition of functionally compact and Theorem 2.

The example due to Urysohn in Remark 1.5 of [1] is a minimal Hausdorff space which is not functionally compact. In the next section we indicate a method of constructing many such spaces.

COROLLARY 2.2. *A functionally compact Hausdorff space is com-*

pact if and only if it is regular.

Proof. The proof follows immediately from the fact that a minimal Hausdorff is compact if and only if it is regular.

DEFINITIONS. By a *mapping* we will always mean a continuous function. A mapping from a space X to a space Y is *closed* provided for every closed set C in $X, f(C)$ is a closed set in Y.

THEOREM 3. *A Hausdorff space X is functionally compact if and only if every mapping of X into any Hausdorff space is closed.*

Proof of the necessity. Suppose that X is functionally compact and let f be a mapping of X into a Hausdorff space Y. Let C be a closed set in X and suppose there exists a point y in $cl_Y f(C)\backslash f(C)$. Let $\mathscr{V}$ be the collection of all open subsets of Y containing y and let $\mathscr{U} = \{f^{-1}(V): V \in \mathscr{V}\}$. By Theorem 2, X is minimal Hausdorff and hence absolutely closed. Since continuous images of absolutely closed spaces are absolutely closed, $f(X) = cl_Y f(X)$ and so $y \in f(X)$. Thus $\mathscr{U}$ is a collection of nonempty open subsets of X and therefore is an open filter-base on X. Furthermore since Y is a Hausdorff space, $f^{-1}(y) = \cap \{U: U \in \mathscr{U}\}$ and $f^{-1}(y) = \cap \{cl_X U: U \in \mathscr{U}\}$. Thus by our hypothesis $\mathscr{U}$ is a base for the neighborhoods of $f^{-1}(y)$ and so there exists $U \in \mathscr{U}$ such that $U \subseteqq X\backslash C$. But then $f(U)$ is an open subset of $f(X)$ that contains y and misses $f(C)$ (since $U = f^{-1}(f(U))$ if $U \in \mathscr{U}$) and this is a contradiction. Thus $f(C)$ is closed and this completes the proof of the necessity.

Proof of the sufficiency. Suppose that every mapping of X into a Hausdorff space is closed and let $\mathscr{U}$ be an open filter-base on X such that the intersection A of the elements of $\mathscr{U}$ equals the intersection of the closures of the elements of $\mathscr{U}$. Suppose further that there exists an open set R of X containing A such that for every $U \in \mathscr{U}, (X\backslash R) \cap U \neq \Phi$. Let Y be the decomposition of X whose only nondegenerate element is A and let f be the natural transformation of X onto Y defined by $x \in f(x)$. We topologize Y by defining a base $\mathscr{B}$ for a topology as follows:

$B \in \mathscr{B}$ if and only if (i) $f^{-1}(B)$ is an open subset of $X\backslash A$;

or

$$\text{(ii)} \quad f^{-1}(B) \in \mathscr{U} .$$

Then Y with this topology is a Hausdorff space and f is a mapping of X onto Y. By our hypothesis f must be a closed map; however

$f(X\backslash R)$ is not closed since $f(A)$ is a limit point of $f(X\backslash R)$ and $f(A) \notin f(X\backslash R)$. This is a contradiction and this completes the proof.

REMARK. The space F of Theorem 1 and Theorem 3 show that the property of being functionally compact is not productive. In fact, even if $\mathscr{S}$ is compact $F \times \mathscr{S}$ will not usually be functionally compact since the projection π: $F \times \mathscr{S} \rightarrow \mathscr{S}$ is not a closed map unless $\mathscr{S}$ is finite.

COROLLARY 3.1. *Let X be a Hausdorff space, Z a functionally compact Hausdorff space and h a mapping of Z onto X. Then X is functionally compact.*

Proof. Let f be a mapping of X into a Hausdorff space Y and let C be a closed subset of X. Since Z is functionally compact, the mapping $f \circ h$ is a closed map of Z into Y and so $f(C) = (f \circ h)(f^{-1}(C))$ is a closed subset of Y. Thus f is closed and X is functionally compact.

COROLLARY 3.2. *If a Hausdorff space X is the union of finitely many functionally compact spaces $X_1, X_2, \cdots, X_n$, X is functionally compact.*

Proof. Let f be a mapping of X into a Hausdorff space Y and let C be a closed subset of X. Then each of the restricted mappings $f \mid X_i$, $i = 1, 2, \cdots, n$, is closed and so $f(C) = \cup \{(f \mid X_i)(C \cap X_i)\colon i = 1, 2, \cdots, n\}$ is closed in Y. Thus f is closed and X is functionally compact.

DEFINITION. A closed subset C of a space X is said to be r-closed if whenever B is closed in C, $x \notin B$ there exist disjoint open sets in X containing x and B, respectively.

THEOREM 4. *An r-closed subset C of a functionally compact space X is functionally compact.*

Proof. Let $\mathscr{U}$ be an open filter-base on C such that

$$\cap \{U\colon U \in \mathscr{U}\} = \{cl_c U\colon U \in \mathscr{U}\} = A\ .$$

Let $\mathscr{V}$ be the open filter-base on X consisting of all open sets V of X such that $V \cap C \in \mathscr{U}$.

Then since C is an r-closed subset of X, $\cap \{V\colon V \in \mathscr{V}\} = A = \{cl_x V\colon V \in \mathscr{V}\}$ and since X is functionally compact, $\mathscr{V}$ is a base for the neighborhoods of A. Of course this implies that $\mathscr{U}$ is a base for the neighborhoods of A relative to C. Hence C is functionally compact.

It is also easy to see that if C is an open and closed subset of a functionally compact space X then C is functionally compact.

2. **Some related examples.** Urysohn has given an example of an nonminimal absolutely closed space (Example 1.4 of [3]) and an example of a noncompact minimal space (Remark 1.5 of [1]). We will give here two general methods for constructing such spaces. In fact, our minimal spaces will be nonfunctionally compact, as is Urysohn's.

EXAMPLE 1. Let $(X, \mathscr{K})$ be a compact Hausdorff space such that $Y \subseteqq X$ is an infinite closed subset, $\operatorname{int}_{\mathscr{K}} Y = \emptyset$. Let $\mathscr{T}$ be any topology on Y strictly stronger than $\mathscr{K} \mid Y$, e.g., the discrete topology, and let $\mathscr{U}$ be the following topology on X: $\mathscr{U} \mid (X \backslash Y) = \mathscr{K} \mid (X \backslash Y)$ and a basic open set intersecting Y is of the form $(\mathscr{O} \cap (X \backslash Y)) \cup T$, where $T \subseteqq \mathscr{O} \cap Y$, $\mathscr{O} \in \mathscr{K}$, $T \in \mathscr{T}$. Then $(X, \mathscr{U})$ is a nonminimal Hausdorff space (since $\mathscr{U}$ is stronger than $\mathscr{K}$) and $\operatorname{int}_{\mathscr{U}} Y = \emptyset$. However, we claim that $(X, \mathscr{U})$ is absolutely closed.

Let $\mathscr{C} = \{C_\alpha : \alpha \in \mathscr{A}\}$ be a $\mathscr{U}$-open covering of X. Notice that for each $U \in \mathscr{U}$ there exists $K \in \mathscr{K}$ such that $K \supseteqq U$ and $K \cap (X \backslash Y) = U \cap (X \backslash Y)$. Thus, if K_α is such a set for C_α then $\{K_\alpha : \alpha \in \mathscr{A}\}$ is a $\mathscr{K}$-open covering of X. Hence there exist $\alpha_1 \cdots, \alpha_n$ such that

$$K_{\alpha_1} \cup \cdots \cup K_{\alpha_n} \supseteqq X$$

so that

$$C_{\alpha_1} \cup \cdots \cup C_{\alpha_n} \supseteqq X \backslash Y$$

and hence

$$cl_{\mathscr{U}} C_{\alpha_1} \cup \cdots \cup cl_{\mathscr{U}} c_{\alpha_n} = X .$$

This is a generalization of one of Urysohn's examples (Example 1.4 of [3]): Let $a_i^j \rightarrow a_i$ be disjoint convergent sequences and let $X = \bigcup_{i,j} \{a_i^j\} \cup \bigcup_i^j \{a_i\} \cup \{\infty\}$ be the 1-point compactification of the union of these sequences and their limit points. Let $Y = \{a_1, a_2, \cdots\} \cup \{\infty\}$ and let $\mathscr{T}$ be the discrete topology on Y. Then $(X, \mathscr{U})$ is Urysohn's example.

EXAMPLE 2. Let $(X, \mathscr{K})$ be a compact Hausdorff space, $Y \subseteqq X$ and φ a function from Y into the 2-element subsets of Y such that

(i) Y is closed;

(ii) $\operatorname{int}_{\mathscr{K}} Y = \varnothing$

(iii) there exist $x_1, x_2 \in Y, x_1 \neq x_2$ such that $x_i \in cl_{\mathscr{K}}(Y\backslash\{x_1, x_2\})$ $(i = 1, 2)$;

(α) $\varphi(z_1) \cap \varphi(z_2) = \varnothing$ if $z_1 \neq z_2$;

(β) $\bigcup_{y \in Y} \varphi(y) = Y$;

(γ) there exist $y_1 \neq y_2$ such that $x_i \in \varphi(y_i)$, $i = 1, 2$;

(δ) for each $\mathscr{O}_1, \mathscr{O}_2 \in \mathscr{K}$ such that $x_i \in \mathscr{O}_i$, $i = 1, 2$ there exists $y \in Y$ such that $\varphi(y) \cap \mathscr{O}_i \neq \varnothing$, $i = 1, 2$.

[For instance, we could take $X = [0, 1]$, $Y = \{a_i^j: i = 1, \cdots, 4, j \geqq 1\} \cup \{a_1, a_2, a_3, a_4\}$, where the a_i are distinct and the (a_i^j) are disjoint sequences of distinct points with $\lim_j a_i^j = a_i$. We can let φ be a function which "identifies" a_1^j and a_2^j, a_3^j and a_4^j, a_1 and a_3, a_2 and a_4.]

Let $Y^* = \{\varphi(y): y \in Y\}$ and denote $\varphi(y)$ by y^*. Let $X^* = (X\backslash Y) \cup Y^*$. Let $\mathscr{T}$ be the topology on X^* defined as follows: $U \in \mathscr{T} \Leftrightarrow$ there exists $\mathscr{O} \in \mathscr{K}$ such that

(1) $U \cap (X\backslash Y) = \mathscr{O} \cap (X\backslash Y)$

(2) for each $y^* \in U \cap Y^*$ there exists $V \in \mathscr{K}, \varphi(y) \subseteqq V$, $V \subseteqq \mathscr{O}$, such that if $\varphi(z) \subseteqq V$ then $z^* \in U$.

It is to check that $\mathscr{T}$ is actually a topology and that $\mathscr{T}$ is Hausdorff. However, if $\mathscr{O}_1, \mathscr{O}_2 \in \mathscr{T}, y_1^* \in \mathscr{O}_1, y_2^* \in \mathscr{O}_2$ then (δ) implies that $cl_{\mathscr{T}}\mathscr{O}_1 \cap cl_{\mathscr{T}}\mathscr{O}_2 \neq \varnothing$. In particular, $\mathscr{T}$ is not compact.

Notice that if $C \subseteqq X^*$ is closed then there exists a closed set $C^c \subseteqq X$ such that

$$C^c \cap (X\backslash Y) = c \cap (X^*\backslash Y^*)$$

and

$$C^c \subseteqq (C \cap (X^*\backslash Y^*)) \cup (\textstyle\bigcup_{y \in C} \varphi(y)) ,$$

namely,

$$C^c = cl_{\mathscr{K}}(C \cap (X^*\backslash Y^*)) .$$

PROPOSITION 1. *$(X^*, \mathscr{T})$ is minimal.*

Proof. Let $\mathscr{U}$ be an open filter such that

$$\cap \{U: U \in \mathscr{U}\} = \cap \{cl_{\mathscr{T}} U: U \in \mathscr{U}\} = \{x\}$$

and let $V \in \mathscr{T}, x \in V$. We claim that there exists $U \in \mathscr{U}$ such that $V \supseteqq U$.

Case 1. Suppose $x = p^* \in Y^*$. Then there exists $\mathscr{W} \in \mathscr{K}$ with $\varphi(p) \subseteqq \mathscr{W}$ such that $\mathscr{W} \cap (X\backslash Y) \subseteqq V \cap (X\backslash Y)$ and such that if $y^* \in Y^*, \varphi(y) \subseteqq \mathscr{W}$ then $y^* \in V$. Then

$$\mathscr{W} \supseteqq \cap (cl_{\mathscr{T}} U)^c = \varphi(p)$$

so there exists $U \in \mathscr{U}$ such that $\mathscr{W} \supseteqq (cl_{\mathscr{T}} U)^c$. In particular,

$$V \supseteqq \mathscr{W} \cap (X \backslash Y) \supseteqq U \cap (X^* \backslash Y^*) .$$

However, if $z^* \in (U \cap Y^*) \backslash V$ then $\varphi(z) \nsubseteq \mathscr{W}$ but $\varphi(z) \subseteqq (cl_{\mathscr{T}} U)^c$ since $U \in \mathscr{T}$. Thus if $z^* \in (U \cap Y^*) \backslash V$ then there exists $y \in (cl_{\mathscr{T}} U)^c \backslash \mathscr{W}$, a contradiction. Thus $V \supseteqq (U \cap (X^* \backslash Y^*)) \cup (U \cap Y^*) = U$.

Case 2. Suppose $x \in X^* \backslash Y^*$. We may suppose that $V \subseteqq X^* \backslash Y^*$ and in fact that $cl_{\mathscr{K}} V \subseteqq X^* \backslash Y^*$ since Y is $\mathscr{K}$-closed in X. Then, again, there exists $U \in \mathscr{U}$ such that $V \cap (X^* \backslash Y^*) \supseteqq U \cap (X^* \backslash Y^*)$. Since $U \in \mathscr{T}$ and $cl_{\mathscr{K}} V \subseteqq X^* \backslash Y^*$ we must have that $U \subseteqq X^* \backslash Y^*$ so that again $V \supseteqq U$.

PROPOSITION 2. *$(X^*, \mathscr{T})$ is not functionally compact.*

Proof. Let $A = \{y_1^*, y_2^*\}$. Suppose $\varphi(y_1) = \{x_1, x_1'\}$, $\varphi(y_2) = \{x_2, x_2'\}$. If $\mathscr{O}_1, \mathscr{O}_1', \mathscr{O}_2, \mathscr{O}_2' \in \mathscr{K}$ are sets containing x_1, x_1', x_2, x_2', respectively then there are $U \in \mathscr{T}$ containing all $y^* \in Y^*$ with

$$\varphi(y) \subseteqq \mathscr{O}_1 \cup \mathscr{O}_1' \cup \mathscr{O}_2 \cup \mathscr{O}_2' ,$$

and if $\mathscr{U}$ is the filter base of all such U then $\cap U = \cap cl_{\mathscr{T}} U = A$. However, there exist $\mathscr{O} \in \mathscr{T}$, $A \subseteqq \mathscr{O}$ such that $\mathscr{O}$ does not contain any points $y^* \in Y^*$ unless $\varphi(y)$ is contained in a set of the form $\mathscr{O}_1 \cup \mathscr{O}_1'$ or one of the form $\mathscr{O}_2 \cup \mathscr{O}_2'$. Thus the filter generated by $\mathscr{U}$ does not contain all neighborhoods of A.

In [4] G. Viglino studies a property similar to functional compactness. X is C-compact if for each closed set $A \subseteqq X$ and each open cover $\mathscr{U} = \{U_\alpha\}$ of A there exist $\alpha_1, \cdots, \alpha_n$: $cl_X(U_{\alpha_1} \cup \cdots \cup U_{\alpha_n}) \supseteqq A$. It is easy to see that if X is C-compact then X is functionally compact. We do not know whether the two properties are equivalent.

Urysohn's two examples to which we have referred can both be embedded as (nondense) subsets of functionally compact spaces. The one referred to in Example 1 can be embedded in F. If we let Q be the quotient space obtained from two disjoint copies of F by identifying the points α_i in the first with α_i in the second, $i = 1, 2, \cdots$, we get a functionally compact space in which Urysohn's minimal, noncompact space can be embedded, again as a nondense subset. This is the best that can be expected, since an absolutely closed space C embedded in a Hausdorff space H would have to be a closed subset of H, and hence not dense in H is H is functionally compact unless $C = H$. Also, since open and closed subsets of functionally compact spaces are functionally compact a nonfunctionally compact absolutely closed space can not be embedded as an open subset of a functionally compact set. We do not know whether for each Hausdorff H there exists a functionally compact X with $H \subsetneqq X$.

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