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ORBITS OF HERMANN ACTIONS

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1. Introduction

In this paper we consider the parallel translations of the normal bundles of the orbits of Hermann actions on compact symmetric spaces and represent such parallel translations by the group actions (Theorem 2.1). Using this we can show that their mean curvature vectors are parallel (Corollary 2.8), moreover those of hyperpolar actions are parallel (Corollary 2.13).

We first review some definitons and previous results concerning isometric group actions on compact symmetric spaces. Let (G, K_1) and (G, K_2) be compact symmetric pairs. Then K_2 acts isometrically on G/K_1, which is a compact symmetric space. This action of K_2 on G/K_1 is called a *Hermann action*.

The Hermann actions are examples of hyperpolar actions, which is defined in the following. Let G be a Lie group acting isometrically on a Riemannian manifold M. A closed submanifold Σ of M is called a *section*, if all orbits of the action of G meet Σ perpendicularly. The action of G on M is said to be *hyperpolar*, if there exists a section which is flat with respect to the induced Riemannian metric. The codimension of the orbit of highest dimension is called the *cohomogeneity*. The isometric actions on compact symmetric spaces of cohomogeneity one are another examples of hyperpolar actions. Recently Kollross [8] proved that the hyperpolar actions on compact symmetric spaces are Hermann actions or cohomogeneity one actions.

We next review previous results concerning geometry of orbits of isometric group actions on symmetric spaces. The linear isotropy representations of symmetric pairs have sections which are maximal Abelian subspaces, so they are hyperpolar actions. All of their orbits have parallel mean curvature vectors, which was proved by Kitagawa-Ohnita [6]. Ohnita [9] considered the parallel translations of the normal bundles of the orbits of the linear isotropy actions on compact symmetric spaces and represent such parallel translations by the group actions. One can prove the result of Kitagawa-Ohnita mentioned above by this. Heintze-Olmos [1] also considered such parallel translations and described the normal holonomy groups of the orbits. For compact symmetric space G/K, Hirohashi-Song-Takagi-Tasaki [4] and Hirohashi-Ikawa-Tasaki [3] considered some geometric properties of orbits of the linear isotropy action on $T_o(G/K)$ and the isotropy action on G/K.

2. Orbits of Hermann actions

Let θ_1 and θ_2 be two involutive automorphisms of a compact connected Lie group G furnished with a biinvariant Riemannian metric $\langle\ ,\ \rangle$. We denote by G_{θ_i} $(i = 1, 2)$ the closed subgroup consisting of all fixed points of θ_i in G. For a closed subgroup K_i $(i = 1, 2)$ of G which lies between G_{θ_i} and the identity component of G_{θ_i}, (G, K_1) and (G, K_2) are Riemannian symmetric pairs. We consider the Hermann action K_2 on a compact symmetric space $M_1 = G/K_1$ with the induced Riemannian metric from the biinvariant Riemannian metric $\langle\ ,\ \rangle$ on G. We denote by $\mathfrak{g}$, $\mathfrak{k}_1$ and $\mathfrak{k}_2$ the Lie algebras of G, K_1 and K_2, respectively. The involutive automorphisms θ_1 and θ_2 of G induce involutive automorphisms of $\mathfrak{g}$, also denoted by θ_1 and θ_2, respectively. Since θ_1 and θ_2 are involutive, we have

$$\mathfrak{g} = \mathfrak{k}_1 + \mathfrak{m}_1 = \mathfrak{k}_2 + \mathfrak{m}_2,$$

where we put

$$\mathfrak{m}_i = \{X \in \mathfrak{g} \mid \theta_i(X) = -X\} \qquad (i = 1, 2).$$

We can identify $\mathfrak{m}_1$ with $T_o(M_1)$ in a natural manner. For $H \in \mathfrak{m}_1$, we consider the K_2-orbit $K_2 \operatorname{Exp} H \subset M_1$, where Exp is the exponential mapping from $\mathfrak{m}_1$ into M_1. The tangent space of $K_2 \operatorname{Exp} H$ at $\operatorname{Exp} H$ is given by

$$T_{\operatorname{Exp} H}(K_2 \operatorname{Exp} H) = (\exp H)_*(\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1},$$

where $(\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}$ is the $\mathfrak{m}_1$-component of $\operatorname{Ad}(\exp(-H))\mathfrak{k}_2$. We define a closed subgroup $N^H_{K_2}[K_1]$ in K_2 by

$$N^H_{K_2}[K_1] = \{k \in K_2 \mid \exp(-H)k \exp H \in K_1\}.$$

Then we have the following diffeomorphism from $K_2 \operatorname{Exp} H$ onto $K_2/N^H_{K_2}[K_1]$:

$$K_2 \operatorname{Exp} H \to K_2/N^H_{K_2}[K_1];\, k \operatorname{Exp} H \mapsto kN^H_{K_2}[K_1].$$

We denote by $\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1]$ the Lie algebra of $N^H_{K_2}[K_1]$. Then we have

$$\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1] = \{X \in \mathfrak{k}_2 \mid \operatorname{Ad}(\exp(-H))X \in \mathfrak{k}_1\}.$$

We denote by $(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp$ the orthogonal complement of $\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1]$ in $\mathfrak{k}_2$. We can identify $(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp$ with $T_o(K_2/N^H_{K_2}(K_1))$ in a natural manner. The above diffeomorphism $K_2 \operatorname{Exp} H \cong K_2/N^H_{K_2}[K_1]$ induces a linear isomorphism from $T_{\operatorname{Exp} H}(K_2 \operatorname{Exp} H)$ onto $(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp$.

Theorem 2.1. *Let Y be in $(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp$. We define a curve $c(t)$ in $K_2 \operatorname{Exp} H$ by*

$$c(t) = \exp tY \operatorname{Exp} H.$$

Let ξ *be in* $(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}$. *We define a normal vector field* $\xi(t)$ *of* $K_2 \operatorname{Exp} H$ *along* $c(t)$ *by*

$$\xi(t) = (\exp tY)_*(\exp H)_*\xi.$$

Then $\xi(t)$ *is parallel with respect to the normal connection.*

In order to show the theorem we prove the lemmas below.

Lemma 2.2. *We denote by* $\nabla^{\perp}$ *the normal connection of* $K_2 \operatorname{Exp} H \subset M_1$. *We define a curve* $g(t)$ *in* G *by*

$$g(t) = \exp tY \exp H.$$

Then

$$\nabla^{\perp}_{\dot{c}}\xi(t) = g(t)_*[(\mathrm{Ad}(\exp(-H))Y)_{\mathfrak{k}_1}, \xi]^{\perp}.$$

Proof. Let π be the natural projection from G onto $M_1 = G/K_1$. We consider the principal fiber bundle $G(M_1, K_1, \pi)$. The canonical decomposition $\mathfrak{g} = \mathfrak{k}_1 + \mathfrak{m}_1$ induces an invariant connection on $G(M_1, K_1, \pi)$. It is known that the Levi-Civita connection of M_1 is reduced to the invariant connection. The tangent bundle $E = T(M_1)$ of M_1 is the vector bundle associated with $G(M_1, K_1, \pi)$ with standard fiber $\mathfrak{m}_1$. We denote by $A^p(E)$ the vector space of E-valued p-forms on M_1, and by $A^p_{\mathrm{Ad}}(G)$ the vector space of tensorial p-forms $\tilde{\xi}$ of type $\mathrm{Ad}(K_1)$ on G, that is, $\tilde{\xi}$ satisfies the following conditions.
(1) $R_a^*\tilde{\xi} = \mathrm{Ad}(a^{-1})\tilde{\xi} \qquad a \in K_1$
(2) $\tilde{\xi}(X_1, \dots, X_p) = 0$ when X_1 is vertical, $\quad X_1, \dots, X_p \in T_gG$
It is well known that the linear mapping given by

$$A^p(E) \to A^p_{\mathrm{Ad}}(G) \,;\xi \mapsto \tilde{\xi}$$
$$\tilde{\xi}(X_1, \dots, X_p) = g_*^{-1}(\xi(\pi_*X_1, \dots, \pi_*X_p))$$

is an isomorphism. We denote by ∇ the covariant derivative on $T(M_1)$. When X in $A^0(E) = \mathfrak{X}(M_1)$ corresponds to $\tilde{\xi}$ in $A^0_{\mathrm{Ad}}(M_1)$ by this correspondence, ∇X in $A^1(E)$ corresponds to $d\tilde{\xi} \circ h$ in $A^1_{\mathrm{Ad}}(M_1)$ (see [7] Chapter II), where we denote by $h(Y)$ the horizontal component of Y in $\mathfrak{X}(G)$. By this relationship, we get the following expression. Let X be in $\mathfrak{X}(M_1)$, v in $\mathfrak{m}_1$ and A in $\mathfrak{k}_1$. Let $\alpha(s)$ be a curve in G such that $\alpha(0) = e$ and $\dot{\alpha}(0) = v + A$. Then

$$\nabla_{g_*v}X = g_*\left(\frac{d}{ds}\bigg|_{s=0} \alpha(s)_*^{-1}g_*^{-1}X_{\pi(g\alpha(s))} + [A, g_*^{-1}X_{\pi(g)}]\right)$$

For fixed t, we define $\alpha(s)$ by $\alpha(s) = g(t)^{-1}g(t+s)$. Then $\alpha(0) = e$ and $\dot{\alpha}(0) = \mathrm{Ad}(\exp(-H))Y$. Hence the lemma follows immediately. $\square$

Hence in order to show the theorem it is sufficient to prove

$$[(\mathrm{Ad}(\exp(-H))(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp)_{\mathfrak{k}_1}, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}] \subset (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}.$$

The following lemma is trivial.

Lemma 2.3. $\mathrm{Ad}(\exp H)\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2] = \mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1]$

Lemma 2.4. $(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1} = \{X \in \mathfrak{m}_1 \mid \mathrm{Ad}(\exp H)X \in \mathfrak{m}_2\}$. *In particular* $(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}$ *is a Lie triple system in* $\mathfrak{m}_1$.

Proof.

$$\begin{aligned}(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1} &= \{X \in \mathfrak{m}_1 \mid \langle X, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}\rangle = 0\}\\ &= \{X \in \mathfrak{m}_1 \mid \langle X, \mathrm{Ad}(\exp(-H))\mathfrak{k}_2)\rangle = 0\}\\ &= \{X \in \mathfrak{m}_1 \mid \langle \mathrm{Ad}(\exp H)X, \mathfrak{k}_2\rangle = 0\}\\ &= \{X \in \mathfrak{m}_1 \mid \mathrm{Ad}(\exp H)X \in \mathfrak{m}_2\}.\end{aligned}$$

Hence the lemma is proved. □

The following lemma immediately follows from the lemma above.

Lemma 2.5. $[(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}] \subset \mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2]$.

Lemma 2.6. $(\mathrm{Ad}(\exp(-H))(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp)_{\mathfrak{k}_1} \subset (\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2])^\perp$.

Proof.

$$\begin{aligned}&\langle(\mathrm{Ad}(\exp(-H))(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp)_{\mathfrak{k}_1}, \mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2]\rangle\\ &\quad= \langle \mathrm{Ad}(\exp(-H))(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp, \mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2]\rangle \qquad (\text{by } \mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2] \subset \mathfrak{k}_1)\\ &\quad= \langle(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp, \mathrm{Ad}(\exp H)\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2]\rangle\\ &\quad= \langle(\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^\perp, \mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1]\rangle \qquad (\text{by Lemma 2.3})\\ &\quad= \{0\}.\end{aligned}$$

□

Lemma 2.7. $[(\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2])^\perp, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}] \subset (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}$.

Proof.

$$\begin{aligned}&\langle[(\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2])^\perp, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}], (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}\rangle\\ &\quad= \langle(\mathfrak{n}^{-H}_{\mathfrak{k}_1}[\mathfrak{k}_2])^\perp, [(\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)^\perp_{\mathfrak{m}_1}]\rangle\end{aligned}$$

$$\begin{aligned} &\subset \langle (\mathfrak{n}_{\mathfrak{k}_1}^{-H}[\mathfrak{k}_2])^{\perp}, \mathfrak{n}_{\mathfrak{k}_1}^{-H}[\mathfrak{k}_2]\rangle \qquad \text{(by Lemma 2.5)} \\ &= \{0\}. \end{aligned}$$

Hence the lemma is proved. □

By Lemmas 2.6 and 2.7 we have

$$[(\mathrm{Ad}(\exp(-H))(\mathfrak{n}_{\mathfrak{k}_2}^{H}[\mathfrak{k}_1])^{\perp})_{\mathfrak{k}_1}, (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}^{\perp}] \subset (\mathrm{Ad}(\exp(-H))\mathfrak{k}_2)_{\mathfrak{m}_1}.$$

This completes the proof of Thoerem 2.1.

Corollary 2.8. *The mean curvature vector of $K_2 \operatorname{Exp} H \subset M_1$ is parallel with respect to the normal connection.*

Proof. We denote by m the mean curvature vector of $K_2 \operatorname{Exp} H \subset M_1$. Since $m_{\exp tX \operatorname{Exp} H} = (\exp tX)_* m_{\operatorname{Exp} H}$ $(X \in (\mathfrak{n}_{\mathfrak{k}_2}^{H}[\mathfrak{k}_1])^{\perp})$, we have $\nabla_X^{\perp} m = 0$ by Theorem 2.1. Hence $(\nabla^{\perp} m)_{\operatorname{Exp} H} = 0$. Therefore $\nabla^{\perp} m$ vanishes everywhere by the homogeneity of $K_2 \operatorname{Exp} H$. □

The decomposition

$$\mathfrak{k}_2 = \mathfrak{n}_{\mathfrak{k}_2}^{H}[\mathfrak{k}_1] \oplus (\mathfrak{n}_{\mathfrak{k}_2}^{H}[\mathfrak{k}_1])^{\perp}$$

defines an invariant connection ∇^c of $K_2 \operatorname{Exp} H$. We denote by α the second fundamental form of $K_2 \operatorname{Exp} H \subset M_1$ We define $\nabla^c \alpha$ by

$$(\nabla_X^c \alpha)(Y, Z) = \nabla_X^{\perp}(\alpha(Y, Z)) - \alpha(\nabla_X^c Y, Z) - \alpha(Y, \nabla_X^c Z).$$

Corollary 2.9. $\nabla^c \alpha = 0$.

Proof. Let X, Y and Z be in $(\mathfrak{n}_{\mathfrak{k}_2}^{H}[\mathfrak{k}_1])^{\perp}$. The vector fields $(\exp tX)_*(\exp H)_* Y$ and $(\exp tX)_*(\exp H)_* Z$ of $K_2 \operatorname{Exp} H$ along a curve $\exp tX \operatorname{Exp} H$ are ∇^c-parallel. Thus we get

$$\begin{aligned} &(\nabla_{(\exp H)_* X}^c \alpha)(Y, Z) \\ &\quad = \nabla_{(\exp H)_* X}^{\perp}(\alpha((\exp tX)_*(\exp H)_* Y, (\exp tX)_*(\exp H)_* Z)) \\ &\quad = \nabla_{(\exp H)_* X}^{\perp}((\exp tX)_*(\alpha((\exp H)_* Y, (\exp H)_* Z)) \\ &\quad = 0 \qquad \text{(Theorem 2.1)}. \end{aligned}$$

Hence we have $(\nabla^c \alpha)_{\operatorname{Exp} H} = 0$. By homogeneity we have $\nabla^c \alpha = 0$. □

By Corollary 2.9, for any vector fields X, Y and Z of $K_2 \operatorname{Exp} H$ we have

$$(\nabla_X^{\perp}(\alpha(Y, Z)))_p \in \operatorname{span}\{\alpha(Y, Z) \mid Y, Z \in T_p(K_2 \operatorname{Exp} H)\}.$$

In another words we get:

Corollary 2.10. *The degree of* $K_2 \operatorname{Exp} H \subset M_1$ *is at most* 2.

We consider the normal holonomy representation of $K_2 \operatorname{Exp} H \subset M_1$. By Lemma 2.4, we can define an action ρ of $N^H_{K_2}[K_1]$ on $(\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}$ by

$$\rho(k)X = \operatorname{Ad}(\exp(-H)k \exp H)X$$

for $k \in N^H_{K_2}[K_1]$, $X \in (\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}$. This action is equivalent to the differential representation of $N^H_{K_2}[K_1]$ on $T^{\perp}_{\operatorname{Exp} H}(K_2 \operatorname{Exp} H)$.

Corollary 2.11. *The normal holonomy representation of* $K_2 \operatorname{Exp} H \subset M_1$ *is equivalent to the* (*effectively made*) *action of a subgroup of* $N^H_{K_2}[K_1]$ *on* $(\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}$.

Proof. Every geodesic $c(t)$ of $K_2/N^H_{K_2}[K_1]$ through the origin $\operatorname{Exp} H$ with respect to the normal homogeneous Riemannian metric is given by

$$c(t) = \exp tY \operatorname{Exp} H \quad \text{for some} \quad Y \in (\mathfrak{n}^H_{\mathfrak{k}_2}[\mathfrak{k}_1])^{\perp}.$$

By Theorem 2.1, the parallel translation along $c(t)$ with respect to the normal connection is given by $(\exp tY)_*$. Now any curve in $K_2 \operatorname{Exp} H$ can be approximated by broken geodesics with respect to the normal homogeneous Riemannian metric. It follows that the normal holonomy representation is equivalent to the action of K/L on $(\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}$ where

$$L = \{k \in N^H_{K_2}[K_1] \mid \rho(k) = 1 \quad \text{on} \quad (\operatorname{Ad}(\exp(-H))\mathfrak{k}_2)^{\perp}_{\mathfrak{m}_1}\}$$

and where K is a subgroup of $N^H_{K_1}[K_2]$ with $L \subset K$. □

We shall prove that the mean curvature vector of any orbit of any hyperpolar action is parallel with respect to the normal connection. In order to do this we review a result of Hsiang.

Let G be a compact, connected Lie group of isometries of a Riemannian manifold M. We denote by G_p the isotropy subgroup at $p \in M$. Two orbits $G(p)$ and $G(q)$ are said to be *of the same type* if there exists $g \in G$ such that $G_q = gG_pg^{-1}$. Each orbit of G has a well defined volume by the induced metric as a submanifold. The volume of an orbit $N_0 = G(p_0)$ is said to be *extremal among nearby orbits of the same type* if

$$\left.\frac{d}{dt}\right|_{t=o} \operatorname{vol}(N_t) = 0$$

for all smooth families N_t, $|t| < \varepsilon$, of G-orbits of the same type on M.

Theorem 2.12 (Wu-Yi Hsiang [5]). *Let G be a compact connected Lie group of isometries of a Riemannian manifold M. Then any orbit of G whose volume is extremal among nearby orbits of the same type is a minimal submanifold of M. In particular if there exists a neighborhood of N_0 in which there are no other orbits of the same type, then N_0 is a minimal submanifold of M.*

Corollary 2.13. *The mean curvature vector of any orbit of any hyperpolar action on a compact symmetric space is parallel with respect to the normal connection.*

Proof. Kollross [8] showed that hyperpolar actions on compact symmetric spaces are either cohomogeneity one actions or ω-equivalent to Hermann actions. Here two actions are ω-equivalent if all of their orbits coincide. We have already proved that the mean curvature vector of any orbit of any Hermann action is parallel in Corollaly 2.8, so it is sufficient to consider the case of cohomogeneity one actions.

We assume that the cohomogeneity of the action is equal to one. When a compact, connected Lie group G acts a Riemannian manifold M isometlically, an orbit $G(p)$ through $p \in M$ is said to be *of the principal type* if for any $q \in M$ there exists $g \in G$ such that $G_p \subset gG_qg^{-1}$. Since the codimensions of orbits of the principal type are equal to one, they have parallel mean curvature vectors. It is known that the set of all orbits of the principal type is open and dense in the orbit space. Since the cohomogeneity is equal to one, the set of orbits which are not of the principal type does not include any nonempty open set. Hence we have the conclusion by Theorem 2.12. □

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