Notre Dame Journal of Formal Logic
Volume 34, Number 4, Fall 1993

Argument Deletion Without Events

PAUL R. GRAVES

> I am inclined to agree with Kenny that we cannot view verbs of action as usually containing a large number of standby positions, but I do not have what I consider a knock-down argument. (A knock-down argument would consist in a method for increasing the number of places indefinitely.)
>
> Donald Davidson, 1967

Abstract In this paper I describe a formal language which is adequate to represent many important features of variable polyadicity without explicit appeal to events. The formalism adequately represents inferences from simple sentences with many noun phrases to sentences with some of these noun phrases deleted, while correctly rendering the converse arguments invalid. The formalism also deals with some apparent counterexamples to the general pattern of inference. Because this is possible, the question of whether natural languages make implicit use of events cannot be settled by purely logical considerations. The formalism demonstrated to enjoy the logical virtues of extensionality, soundness, and argument completeness.

0 Introduction Several authors, beginning with Anthony Kenny,[1] have observed a feature of English and other natural languages that has no straightforward representation in standard first-order logic. In standard first-order logic, each general term has a discrete number of arguments, but in natural languages otherwise similar sentences differ in the number of noun phrases they contain. Kenny calls this feature variable polyadicity. This is of logical interest because variable polyadicity allows a pattern of inference I shall call argument deletion. A simple sentence, a sentence with only one verb, may have several noun phrases. Such a sentence often[2] implies a similar sentence with one or more of the noun phrases removed, provided that the deleted noun phrase does not contain a sentence adverb.[3] Most theories proposed in the literature to deal with this problem

Received June 22, 1992; revised June 21, 1993

involve an essential quantification over events. In this paper I propose an alternate formalism that adequately represents these inferences without quantifying over events.

Although the formalism presented here has interesting logical features, the topic is not of logical interest alone, but has broader philosophical ramifications. If these inferences cannot be represented satisfactorily without appeal to events, then no formal language lacking the apparatus to refer to events could adequately represent natural language. Thus it would follow that events are at the very least a fundamental feature of human cognition, and perhaps that events are part of the fundamental ontology of the world. I believe that variable polyadicity does not warrant such metaphysical conclusions. I here demonstrate that argument deletion can be adequately represented without quantification over events. Since this is the case, the existence of argument deletion inferences in English and other natural languages neither entails nor gives significant support to the thesis that natural languages employ essential quantifications over events.

In this paper I present one solution to the logical problem of representing argument deletion without events. I begin with a brief presentation of the problem and some of the attempts to deal with it. Next I present a formalism that I argue is adequate to represent these argument deletion inferences without recourse to explicit quantification over events. Then I sketch proofs for extensionality, soundness and argument completeness for the formalism, and I prove several theorems which are relevant to the formalism's adequacy for representing variable polyadicity.

1 Argument deletion Terence Parsons has proposed[4] several tests that an adequate account of argument deletion must pass. These tests can best be explained through an example.[5] Consider the following sentences:

(1) Brutus stabbed Caesar in the back with the knife.
(2) Brutus stabbed Caesar in the back.
(3) Brutus stabbed Caesar with the knife.
(4) Brutus stabbed Caesar.

Parsons observes that (1) implies the conjunction of (2) and (3), and that (2) and (3) each imply (4). However, the reverse argument augmentation implications fail. (4) implies neither (2) nor (3). For example, (4) might be true in virtue of the fact that Brutus stabbed Caesar only in the leg with a sword. In that case both (2) and (3) would be false. The argument conjugation from (2) and (3) does not imply (1). If Brutus stabbed Caesar in the back with the sword and in the leg with the knife, then both (2) and (3) would be true, but (1) would be false. So in any adequate representation of variable polyadicity, argument deletion must generally be valid, but argument augmentation and argument conjugation should be invalid. Finally, Parsons notes that the negation of (4) implies the negation of all of the others. Any world where Brutus did not stab Caesar is a world where Brutus stabbed Caesar neither in the back nor with the knife. I call this pattern of inference negative argument augmentation.

The most obvious representation of (1)–(4) in standard first-order logic would be to treat (1) as a four-place general term followed by 4 singular terms, to treat (2) and (3) as three-place general terms followed by three singular terms, and to treat (4) as a binary general term followed by two singular terms. Thus, on the simplifying assumption that 'the knife' and 'the back' can be treated as simple singular terms, these sentences might be represented, respectively, as[6]:

(5) STAB[BRUTUS, CAESAR, KNIFE, BACK]
(6) STAB[BRUTUS, CAESAR, KNIFE]
(7) STAB[BRUTUS, CAESAR, BACK]
(8) STAB[BRUTUS, CAESAR].

For the purpose of representing argument deletion this is unsatisfactory. Since the formulas differ in the number of arguments present, a different general term appears in each formula. In standard first-order logic there is no systematic relationship among general terms of different degree.[7] Hence any inference among these formulas will be invalid. Although these inferences could be validated by a series of meaning postulates, the pattern of inference is so pervasive that a more general theory seems desirable.

Confronted with this and other sorts of examples, Davidson proposed that a simple action sentence in English typically expresses a relation among objects and an event.[8] In this example, the analysis proposes that there is an event, characterized by the general term 'STAB' which relates the objects named by the other singular terms. Parsons proposes a Davidsonian analysis to deal with this problem through a finer analysis of the general term 'STAB'. By appealing to the λ-calculus, Parsons proposes to analyze sentences like (1)–(4) as relations captured by complex predicates.

(9) λxyzw[∃e(EVENT(e) & STAB(e) & AGENT(e,x) & PATIENT(e,y) & INSTRUMENT(e,z) & LOCATION(e,w))] (BRUTUS, CAESAR, KNIFE, BACK)

(10) λxyw[∃e(EVENT(e) & STAB(e) & AGENT(e,x) & PATIENT(e,y) & LOCATION(e,w))] (BRUTUS, CAESAR, BACK)

(11) λxyz[∃e(EVENT(e) & STAB(e) & AGENT(e,x) & PATIENT(e,y) & INSTRUMENT(e,z))] (BRUTUS, CAESAR, KNIFE,)

(12) λxy[∃e(EVENT(e) & STAB(e) & AGENT(e,x) & PATIENT(e,y))] (BRUTUS, CAESAR).

On this analysis, the downward argument deletion inferences are valid. Argument deletion is accomplished by λ-elimination, ∃-elimination, &-elimination, ∃-introduction, and λ-introduction. Yet, the existential quantification over events prohibits the reverse argument augmentation and argument conjugation inferences at the point of ∃-introduction for the intuitive reason that there is no guarantee that the sentences are true in virtue of the same events. Such cases are described in the counterexamples provided above.

The general terms 'AGENT', 'PATIENT', 'INSTRUMENT', and 'LOCATION' are intended to represent the relationship between the event in question and the object playing each of these thematic roles. A thorough discussion of the theory of thematic roles is beyond the scope of this paper.[9] The leading idea of the theory is that natural languages recognize a categorization of noun phrases according to special features of the referents of those noun phrases or special roles those objects may play. For example, 'Brutus' is identified as the AGENT-NP because 'Brutus' refers to the object which does the stabbing. 'Caesar' is identified as the PATIENT-NP because 'Caesar' refers to the object which is stabbed. 'The knife' is identified as an INSTRUMENT-NP because 'the knife' refers to the object used to perform the stabbing. 'The back' is identified as a LOCATION-NP because 'the back' refers to the place where Caesar was stabbed. Although the exact number and precise characterizations of these thematic roles are controversial, there is a rough consensus on a dozen or so such roles having special significance for natural languages. These roles are held to be universal features of natural language, and natural languages are thought to use various grammatical devices to identify the thematic roles of noun phrases. Thus the full analysis of (1) states that:

(13) There is an event of the stabbing variety which has Brutus as its agent, Caesar as its recipient, employed the knife as its instrument and was located at Caesar's back.

If we grant that there are events, this sentence does seem to be true whenever (1) is true.

Parsons' analysis passes the tests proposed for argument deletion admirably. But it does so at the cost of considerably complicating the representation of apparently simple sentences, and it makes essential use of events. In the following section I describe a formalism which adequately represents these inferences without appeal to events and preserves the representation of apparently simple English sentences as atomic formulas employing simple general terms.

2 The Logic of Argument Deletion (LAD) The central problem to be solved for any adequate treatment of argument deletion inferences is that whereas a verb of a natural language may apparently be accompanied by almost any natural number of denoting adverbial noun phrases, a general term of standard first-order logic is accompanied by a fixed number of singular terms. Accordingly, the strategy of this paper will be to relax the characterization of the formal idiom's general terms to accommodate variable polyadicity and argument deletion inferences.[10] A general term will be allowed to have any positive natural number of arguments, and the same general term will be allowed to appear with different numbers of arguments in different formulas.

Allowing general terms to have different numbers of arguments in different formulas presents several technical problems. First, as demonstrated in the inferences from (1) to (2) and (3), the adverbial phrases of English may be deleted in any order. Therefore the problem cannot be solved simply by deleting the last argument position or concatenating a new argument position onto the end of the formula. In the most general case, it will have to be possible to delete any argument from any atomic formula. Deleting arbitrary arguments presents a further problem. In the standard first-order logic, the order of the arguments has semantic significance. Consider some examples:

(14) Pat kissed Chris.
(15) Chris kissed Pat.
(16) Pat kissed.
(17) Pat was kissed.

The second pair of sentences are the result of deleting the noun phrase 'Chris' from the first pair of sentences and performing changes required to generate grammatical English sentences. Notice that (14) implies (16) but not (17) and that (15) implies (17) but not (16). If the requirement that all occurrences of a general term must be accompanied by a fixed number of singular terms, these sentences could be represented respectively as:

(18) KISS(PAT, CHRIS)
(19) KISS(CHRIS, PAT)
(20) KISS(PAT)
(21) KISS(PAT).

There is certainly a difference in meaning between (14) and (15), and standard first-order logic represents this difference with the difference in the order of arguments as in (18) and (19). Thus (18) and (19) may differ in truth value in some models just as (14) and (15) may differ in truth value. But as it stands, it is not possible for (20) to differ in truth value from (21), even though (16) may differ in truth value from (17). In standard first-order logic, such a difference could only be represented by a difference in the general term. But the theory of variably polyadic general terms requires that the same general term appears in all of these sentences. The problem arises because the order of the arguments in standard first-order logic carries information, in this case the information of who kissed whom. The identity of form in (20) and (21) demonstrates that if arbitrary arguments may be deleted, the order of the arguments cannot fulfill this role.

At the metalogical level there are further complications. In standard first-order logic, the extension of an n-place general term consists of ordered n-tuples. A formula like (18) would be true just in case the pair ⟨PAT, CHRIS⟩ appears in the interpretation of the 'KISS' general term. Variably polyadic general terms have no fixed number of arguments so their interpretations cannot be quite so neat as to include only n-tuples of some fixed size. Rather, if the general terms are to be variably polyadic, their extensions must contain elements of various sizes. Furthermore, the order of objects within an element of the extension of a general term cannot carry the information of the thematic role played by each object. Again, the singleton ⟨Pat⟩ appearing in the extension of the variably polyadic general term KISS cannot distinguish between Pat as kisser and Pat as kissee. More information is needed in the model theory.

Richard Grandy has shown that algebraic logic can accommodate the deletion of arbitrary arguments and arbitrary rearrangements in the order of arguments and indeed argues in favor of adopting algebraic logic on this basis.[11] However, it is not clear how one should deal with thematic roles within Grandy's anadic logic. He also concedes that the standard first-order theory can probably be modified to deal with these inferences and in what follows I show that with minor modifications the standard first-order logic can accommodate these patterns of inference as well. The formalism to be presented here differs from Grandy's in that rather than allowing the permutation of the arguments, LAD assigns no order whatever to the arguments.

In addition to the usual symbols of first-order logic, LAD includes arbitrarily many[12] thematic role markers which are represented by upper case letters (optionally followed by subscripts) superscripted to singular terms, i.e.: $^{A, B, C, \ldots, X, Y, Z, A1, \ldots, Zn}$. The formation rule for atomic WFFs of LAD is as follows:

Definition If Π is a general term, $\alpha_1, \ldots, \alpha_n$ are n singular terms in any order and $\theta_1, \ldots, \theta_n$ are n distinct[13] thematic role markers in any order, then

$$\Pi(\alpha_1^{\theta_1} \ldots \alpha_n^{\theta_n})$$

is an atomic well-formed formula of LAD.

The formation rules of LAD for ~, &, ∨, ⊃, ≡, ∀, and ∃ are as usual.[14]

The order of arguments in an atomic WFF or LAD is irrelevant. $L(a^A b^P)$ is a typographical variant of $L(b^P a^A)$. In LAD they are two ways of writing the same formula. The optional subscripts serve only to distinguish singular terms from each other and the optional numbers on the thematic role markers serve only to distinguish thermatic role markers from each other. No order is implied by the numbers in either case. The general terms of LAD are variably polyadic; i.e., the same general term may appear in different formulas with different numbers of arguments. For example, the same general term appears in $L(b^P a^A)$, $L(a^A)$, and $L(b^P)$.

Thematic role markers in the form of superscripts on singular terms are introduced to free the syntax of the requirement to observe a strict order of arguments. Since the thematic role of each singular term is specified by its superscript, the order of the singular terms is irrelevant. Sentences (14)–(17) are now represented, respectively, as:

(22) $\text{KISS}(\text{PAT}^{\text{AGENT}}\ \text{CHRIS}^{\text{PATIENT}})$
(23) $\text{KISS}(\text{PAT}^{\text{PATIENT}}\ \text{CHRIS}^{\text{AGENT}})$
(24) $\text{KISS}(\text{PAT}^{\text{AGENT}})$
(25) $\text{KISS}(\text{PAT}^{\text{PATIENT}})$.

With the addition of the superscripts, it is possible to distinguish between (16) and (17). With appropriate truth conditions, the inferences from (22) to (24) and from (23) to (25) can be validated. But the invalid inferences from (22) to (25) and from (23) to (24) are not instances of simple argument deletion because they involve deleting two labeled arguments and introducing a novel labeled argument. These inferences are invalid both in English and in the formal idiom.

The model theory required to support these syntactical manipulations is only slightly more complex than that of standard first-order logic. As remarked above, the extension of a general term might have some elements with as few as one object while at the same time having elements with as many as a dozen or more objects. Furthermore, the order of the members will not suffice to represent the sorts of inference and failure of inference noted above. The domain of discourse will still be regarded as a set of objects as usual. Singular terms will still be interpreted as referring to objects from the domain. However, the extension of a general term will not consist of n-tuples of any particular degree. Instead, the extension of a general term will be identified with a set whose elements are sets of labeled objects. A labeled object is just a pair whose first element is an object and whose second element is a label. For example, ⟨Pat, AGENT⟩ is a labeled object.

In general, a model of LAD consists of an ordered triple: $\langle f, \mathbf{D}, \Theta \rangle$. $\mathbf{D}$ is the domain of objects, θ is the set of thematic role markers, and f is an interpretation function from formulas to truth values; from singular terms to objects from the domain; from labeled singular terms to labeled objects, i.e., members of $\mathbf{D} \times \theta$, such that if α^{θ} is a labeled singular term then $f(\alpha^{\theta}) = \langle f(\alpha), \theta \rangle$ and from general terms to extensions, i.e., sets of sets of labeled objects such that each non-empty proper subset of each element of the extension is also an element of the extension. If ϕ is of the form:

$$\Pi(\alpha_1^{\theta_1} \ldots \alpha_n^{\theta_n})$$

then $f(\phi) = \mathrm{T}$ just in case:

$$\{f(\alpha_1^{\theta_1}), \ldots, f(\alpha_n^{\theta_n})\} \in f(\Pi).$$

Otherwise, $f(\phi) = \mathrm{F}$. The remainder of the truth definition for LAD is exactly as usual for standard first-order logic.[15] Thus, (22) will be true just in case:

$$\{f(\mathrm{PAT}^{\mathrm{AGENT}}), f(\mathrm{CHRIS}^{\mathrm{PATIENT}})\} \in f(\mathrm{KISS}).$$

Notice that the order of the arguments now makes no difference to the truth value of the formula. The information implicitly carried by the order of the arguments in standard first-order logic is now carried by the superscripts.

Theorem 1 (LAD is extensional with respect to singular terms) *For all formulas ϕ and singular terms α and δ, in all models such that $f(\alpha) = f(\delta)$ it will be the case that $f(\phi) = f(\phi(\delta/\alpha))$.*

Proof: As usual by induction on the length of the formulas of LAD. The length of a formula of LAD is the number of connectives and quantifiers in the formula.

Base case: Suppose ϕ is an atomic formula with the singular term α appearing perhaps m times with thematic role makers $\theta_{1i}, \ldots, \theta_{mi}$; i.e., ϕ is of the form:

$$\Pi(\alpha_1^{\theta_1} \ldots \alpha^{\theta_{1i}} \ldots \alpha^{\theta_{mi}} \ldots \alpha_n^{\theta_n}).$$

In that case $f(\phi) = \mathrm{T}$ iff

$$\{f(\alpha_f^{\theta_1}), \ldots, f(\alpha^{\theta_{1i}}), \ldots, f(\alpha^{\theta_{mi}}), \ldots, f(\alpha_n^{\theta_n})\} \in f(\Pi);$$

i.e., iff

$$\{\langle f(\alpha_1), \theta_1 \rangle, \ldots, \langle f(\alpha), \theta_{1i} \rangle, \ldots, \langle f(\alpha), \theta_{mi} \rangle, \ldots, \langle f(\alpha_n), \theta_n \rangle\} \in f(\Pi).$$

Suppose furthermore that $f(\alpha) = f(\delta)$. The result of substituting δ for α, $\phi(\delta/\alpha)$, would be:

$$\Pi(\alpha_1^{\theta_1} \ldots \delta^{\theta_{1i}} \ldots \delta^{\theta_{mi}} \ldots \alpha_n^{\theta_n}).$$

This will be true just in case:

$$\{\langle f(\alpha_1), \theta_1 \rangle, \ldots, \langle f(\alpha), \theta_{1i} \rangle, \ldots, \langle f(\alpha), \theta_{mi} \rangle, \ldots, \langle f(\alpha_n), \theta_n \rangle\} \in f(\Pi).$$

Thus where $f(\alpha) = f(\delta)$, $f(\phi)$ and $f(\phi(\delta/\alpha))$ name the same set. Hence if $f(\alpha) = f(\delta)$, then $f(\phi) = \mathrm{T}$ iff $f(\phi(\delta/\alpha)) = \mathrm{T}$.

The inductive step has 7 cases, one for each of the quantifiers and connectives. The arguments for the inductive step proceed exactly as usual.[16]

Derivation Definition As usual, a derivation is defined as a list of formulas each of which is either a premise, an instance of an axiom schema, or is justified in virtue of one of the intelim rules. Except for the addition of argument deletion, the intelim rules are defined exactly as usual.

Argument Deletion When an accessible line of the form

$$\Pi(\alpha_1^{\theta_1} \dots \alpha_i^{\theta_i} \dots \alpha_n^{\theta_n})$$

appears in a derivation, a new line of the form:

$$\Pi(\alpha_1^{\theta_1} \dots \alpha_n^{\theta_n})$$

may be added to the derivation.

Theorem 2 *LAD is a sound derivation system.*

Proof (Sketch): The demonstration of Theorem 2 proceeds as usual by induction on the derivations of LAD. The only novelty of the demonstration is the introduction of a new case in the inductive step to justify the soundness of argument deletion.[18]

Theorem 3 *Argument Deletion is a sound rule of inference.*

Proof: By induction on the line numbers of a derivation. Suppose all of the lines of a derivation up to line n proceed according to the usual truth preserving rules, and line n + 1 proceeds from some line by argument deletion. In that case, line n + 1 is of the form:

$$\Pi(\alpha_1^{\theta_1} \dots \alpha_n^{\theta_n})$$

and there is an accessible line *m*, where m ≤ n, of the form:

$$\Pi(\alpha_1^{\theta_1} \dots \alpha_i^{\theta_i} \dots \alpha_n^{\theta_n}).$$

By inductive hypothesis line *m* is implied by the set of assumptions accessible from line *m*, hence verified by any model which verifies all of the assumptions accessible at *m*. In any such model:

$$\{f(\alpha_1^{\theta_1}), \dots, f(\alpha_i^{\theta_i}), \dots, f(\alpha_n^{\theta_n})\} \in f(\Pi).$$

But since every non-empty proper subset of every element of $f(\Pi)$ is also included in $f(\Pi)$, this implies:

$$\{f(\alpha_1^{\theta_1}), \dots, f(\alpha_n^{\theta_n})\} \in f(\Pi).$$

Therefore, any model which verifies all of the accessible assumptions at line *m* must also verify line n + 1. Hence, the move from line n to line n + 1 by argument deletion preserves truth. Hence, LAD is a sound system of inference.

The requirement that the extension of every general term must include every non-empty proper subset of each element of the extension validates argument deletion. Thus in every model where {⟨Pat, AGENT⟩,⟨Chris, PATIENT⟩} is in the extension of the general term KISS, the extension of the general term KISS will also include {⟨Pat, AGENT⟩} and {⟨Chris, PATIENT⟩}. Thus (22) implies (24) as well as

(26) KISS(CHRIS$^{\text{PATIENT}}$).

Either sentence may be obtained from (22) by deleting one argument. This additional requirement for models of LAD is sufficient to make argument deletion a valid rule of inference.[19]

By appropriate iterations of argument deletion it is possible to delete any combination of arguments from any atomic formula. Thus the formalism can represent any argument deletion inference captured by Parsons' analysis without making essential use of events.

Theorem 4 *LAD is a complete system of inference.*

Proof (Sketch): LAD can be proved complete by a minor modification of Henkin's argument.[20] Recall that the Henkin argument proceeds by constructing a maximally consistent set of formulas by putting the formulas of the language into order and then going through the list of formulas to add those to the set which are consistent with the basis set. This implies an ordering of the WFFs of the language, and in Henkin's argument this ordering is in turn based on the order of the terms within the WFFs. In LAD, the terms of the WFFs have no order, so a further ordering convention is needed. I employ the following convention: For each atomic WFF, alphabetize the singular terms appearing in the formula. If some terms appear multiple times with multiple superscripts, alphabetize the recurring terms by their superscripts. With a standard order imposed on their arguments, the WFFs can now be ordered as usual. All of the other details of the completeness proof are exactly as usual. In particular, the definition of the model guarantees that whenever one set of labeled objects is added to the extension of a general term, every non-empty proper subset of that set of labeled objects must also be added to the extension of the general term. Therefore in generating the maximally consistent set of WFFs, all of the WFFs which should be implied by argument deletion will be included in the maximally consistent set. This in turn will guarantee that the appropriate argument deletion inferences can always be constructed.

Theorem 5 *Argument augmentation is invalid, i.e.:*

$$\Pi(\alpha_1^{\theta_1}\ldots\alpha_n^{\theta_n}) \nvDash \Pi(\alpha_1^{\theta_1}\ldots\alpha_i^{\theta_i}\ldots\alpha_n^{\theta_n}).$$

Proof: The truth of the premise requires only that $f(\Pi)$ have sets of labeled objects of cardinality n and smaller. The conclusion would require the $f(\Pi)$ to include elements of cardinality n + 1. There is nothing in the model theory to require this, hence there will be models which interpret the premise as true and the conclusion as false.

Theorem 6 *Argument conjugation is invalid, i.e.:*

$$(\Pi(\alpha_1^{\theta_1}\ldots\alpha_i^{\theta_i}\ldots\alpha_n^{\theta_n})\,\&\,\Pi(\alpha_1^{\theta_1}\ldots\alpha_j^{\theta_j}\ldots\alpha_n^{\theta_n})) \nvDash \Pi(\alpha_1^{\theta_1}\ldots\alpha_i^{\theta_i}\ldots\alpha_j^{\theta_j}\ldots\alpha_n^{\theta_n}).$$

Proof: The truth of the premise requires only that $f(\Pi)$ have sets of labeled objects of cardinality n and smaller. The conclusion would require the $f(\Pi)$ to include elements of cardinality n + 1. There is nothing in the model theory to require this; hence, there will be models which interpret the premise as true and the conclusion as false; i.e., argument conjugation is not generally valid.

Theorem 7 *Negative argument augmentation is valid: i.e., where* $\alpha_1, \dots, \alpha_n$ *are* n *singular terms, and* $\alpha_{j1}, \dots, \alpha_{jm}$ *are* m *additional singular terms*:

$$\sim\Pi(\alpha_1^{\theta_1} \dots \alpha_n^{\theta_n}) \models \sim\Pi(\alpha_1^{\theta_1} \dots \alpha_{j1}^{\theta_{j1}} \dots \alpha_{jm}^{\theta_{jm}} \dots \alpha_n^{\theta_n}).$$

Proof: Suppose the premise of the inference is true. In that case

$$\{f(\alpha_1^{\theta_1}) \dots f(\alpha_n^{\theta_n})\} \notin f(\Pi).$$

In that case, no superset of $\{f(\alpha_1^{\theta_1}) \dots f(\alpha_n^{\theta_n})\}$ will be in the extension of Π. In particular,

$$\{f(\alpha_1^{\theta_1}) \dots f(\alpha_{j1}^{\theta_{j1}}) \dots f(\alpha_{jm}^{\theta_{jm}}) \dots f(\alpha_n^{\theta_n})\} \notin f(\Pi).$$

Hence, the conclusion of the inference is true whenever the premise is true.

Theorem 8 *Negative argument augmentation is provable; i.e., where* $\alpha_1, \dots, \alpha_n$ *are* n *singular terms and* $\alpha_{j1}, \dots, \alpha_{jm}$ *are* m *additional singular terms*:

$$\sim\Pi(\alpha_1^{\theta_1} \dots \alpha_n^{\theta_n}) \vdash \sim\Pi(\alpha_1^{\theta_1} \dots \alpha_{j1}^{\theta_{j1}} \dots \alpha_{jm}^{\theta_{jm}} \dots \alpha_n^{\theta_n}).$$

Proof: Theorem 8 follows trivially from iterations of argument deletion and reductio ad absurdum.

The formalism also has the resources to block many apparent counterexamples to the argument deletion inference pattern. Consider the following invalid inference:

(27) Pat sank the Bismarck.
(28) Pat sank.

It appears at first glance that this is an instance of argument deletion. Deleting the noun phrase 'the Bismarck' from (27) yields (28). Yet the inference is clearly invalid.

To see why this inference is invalid, we need only consider the thematic roles of 'Pat' and 'the Bismarck'. In (27) 'Pat' denotes an object which sinks another object. 'Pat' denotes an agent, an active performer of an action. 'The Bismarck' denotes an object which is the recipient of the action, or undergoes the action. 'Pat' is therefore an AGENT-NP and 'The Bismarck' is a PATIENT-NP or a THEME-NP. Therefore the proper representation of (27) is:

(29) SANK(PAT$^{\text{AGENT}}$ BISMARCK$^{\text{THEME}}$).

In (28), 'Pat' plays a very different role. Here the sentence tells us that Pat moved from a higher to a lower position, and it says nothing about Pat's agency. Hence the proper representation of (28) is:

(30) SANK(PAT$^{\text{THEME}}$).

These sentences present no counterexample to argument deletion because they are not an instance of argument deletion.

3 Concluding remarks In this essay I have described a formalism which is adequate to represent argument deletion inferences. Because this is possible without explicit recourse to events, the phenomenon of argument deletion in natural languages does not entail that sentences of natural language make essential use of implicit quantification over events. The formalism enjoys the logical virtues of extensionality, soundness, and argument completeness.

Some logical work remains. This formalism shares two troublesome features with Parsons' analysis. First, it appears that some argument positions cannot be deleted from sentences of natural language. Both the formalism described here and Parsons' analysis seem to imply the truth of propositions with no clear English analogues. Consider:

(31) SANK(PAT^{AGENT}).

This is a clear consequence of (29) and argument deletion, but it has no clear English analog. One can, of course, insist that the English sentence does imply propositions which cannot be expressed in idiomatic English. While I find this a plausible line of response, some may find it capricious.

On the other hand, (27) does seem to imply the English sentence:

(32) Pat sank something.

This suggests that the THEME-NP is mandatory for English sentences where the main verb is 'sink'. It can be quantified but it cannot be deleted. The AGENT-NP, 'Pat', on the other hand, does seem to be optional. I propose that the existential quantification test suggested by (32) may be useful for identifying mandatory noun phrases. A system of markers to distinguish optional arguments from mandatory arguments will suffice to solve this problem.

The second weakness of most of these analyses is that they do depend essentially on the theory of thematic roles. In particular, the thematic role labels in the formalism presented here are largely uninterpreted, functioning here as nothing more than place holders.[21] Finally, inasmuch as the theory of thematic roles is still controversial in its particulars, the formalism incorporates a significant promissory note. The first two problems will be the subject for forthcoming papers. The last is a problem which ultimately belongs to the linguists.

NOTES

1. See Kenny [8].

2. One type of case where the inference pattern fails is where the noun phrase to be deleted introduces an intensional context. For example, 'Chris jumped over the moon in a dream' does not imply 'Chris jumped over the moon'. The former seems to be elliptical for 'Chris dreamed that she jumped over the moon'. If this is right, the first sentence is a disguised complex sentence, and the apparent counterexample illustrates only the well-known difficulties with intensional logic.

3. Sentence adverbs, e.g., 'quickly' in 'Chris buttered the toast quickly', appear to have different logical properties from adverbial noun-phrases. Whereas adverbial noun-phrases denote objects together with their relation to the predication of the main verb, sentence adverbs appear to be selection functions on sentences; i.e., they appear to select a subclass from the extension of the main verb of the sentence. For example, 'quickly' selects from all of the instances of buttering, those which were

accomplished quickly. Hence, from the standpoint of the present account, they seem to be second order. One of the admitted strengths of the Davidsonian account vis-a-vis the present account is the ease with which sentence adverbs are accommodated as first-order predicates of events. On the other hand, since these sentence adverbs do not appear to be denoting phrases, they do not correspond in any straightforward way to arguments. Therefore, the present account, inasmuch as it is intended to deal with argument deletion, is not descriptively incomplete for its failure to account for sentence adverbs.

4. Cf. Parsons [10].

5. This example appears in Davidson [3].

6. Here, and throughout the informal exposition, I adopt the device of using suggestive English words written in all capital letters in place of the general terms, singular terms and thematic role markers of the formal language.

7. Davidson discusses this point on p. 136 in [3].

8. See Davidson [3]. Parsons [9] traces this move to Ramsey [10].

9. For a thorough discussion of thematic roles, see Dowty [4].

10. Richard Grandy discusses relaxing this restriction in the context of algebraic logic in Grandy [6].

11. Grandy [5].

12. The precise number of thematic roles is a matter of some controversy. David Dowty countenances as many thematic roles as there are argument positions in atomic sentences. From this multitude he advocates selecting a small set which is useful for doing linguistics. A dozen or so seem to have significant linguistic motivation. The formalism here is deliberately vague. Note that all of the metalogical results are proved for this stronger system with arbitrarily many thematic role markers. If natural language requires few thematic roles, it can be treated as a finite special case.

13. It is generally held that a given thematic role can appear at most once in any simple sentence. It should be noted that none of the theorems proved in this article depend on this restriction.

14. For a complete characterization of the syntax of standard first-order logic, see Bergmann et al. [2] or Bencivenga et al. [1].

15. Full statements of the truth conditions for standard first-order logic can be found in [1] and [2]. These accounts are derived from Tarski's work in the early 1930's. Cf. Tarski. [11].

16. For a full statement of the argument, see [1].

17. For a complete statement of the intelim rules, see [2].

18. For a detailed exposition of these arguments, see [2].

19. In an axiomatic approach, this could be achieved by introducing the **Argument Deletion Axiom Schema (ADAS):**

    If Π is a general term, $\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n$ are n singular terms in any order and $\theta_1, \ldots, i, \ldots, n$ are n case markers in any order, then any formula of the form:

    $$(\Pi(\alpha_1^{\theta_1} \ldots \alpha_i^{\theta_i} \ldots \alpha_n^{\theta_n}) \supset \Pi(\alpha_1^{\theta_1} \ldots \alpha_n^{\theta_n}))$$

    is an axiom of LAD.

Theorem *Each instance of ADAS is logically true.*

Proof: Any instance of ADAS will be a conditional formula of the form:

$$(\Pi(\alpha_1^{\theta_1}\ldots\alpha_i^{\theta_i}\ldots\alpha_n^{\theta_n}) \supset \Pi(\alpha_1^{\theta_1}\ldots\alpha_n^{\theta_n})).$$

Such formula could only be false if its antecedent were true and its consequent false. But any model which verifies the antecedent must also verify the consequent, for the antecedent is true just in case:

$$\{f(\alpha_1^{\theta_1}),\ldots,f(\alpha_i^{\theta_i}),\ldots,f(\alpha_n^{\theta_n})\} \in f(\Pi).$$

But since every non-empty proper subset of every element of $f(\Pi)$ is also included in $f(\Pi)$,

$$\{f(\alpha_1^{\theta_1}),\ldots,f(\alpha_n^{\theta_n})\} \in f(\Pi).$$

Therefore, any model which verifies the antecedent must verify the consequent as well. Hence, any instance of ADAS is logically true.

20. See Henkin [7].

21. The outline of a solution to this problem in an event theory can be found in [5]. Dowty insists that in order for a theory to count as a thematic role theory, the theory must contain more than a collection of uninterpreted thematic role labels; some inference(s) must follow from the fact that a given term bears a given thematic role label. Dowty proposes that from the set of all relations among all argument positions in the language, a small subset is relevant to linguistics. This set he identifies with what I have called the thematic roles. With each thematic role he associates a list of characteristic properties for objects participating in that role. He then advocates adding to the semantical representation a meaning postulate reflecting these characteristic properties for each of the thematic roles. If θ is one of the thematic roles, and $P\theta_1,\ldots,P\theta_m$ represent the possibly logically complex characteristic properties for θ, then such a meaning postulate has the form:

$$\forall x\forall e[(\text{EVENT}(e)\ \&\ \theta(e,x)) \supset (P_{\theta_1}(x)\ \&\ldots\&\ P_{\theta_m}(x))].$$

An axiomatic development of LAD could include a corresponding thematic role axiom:

$$\Pi(\alpha_1^{\theta_1}\ldots\alpha^{\theta}\ldots\alpha_n^{\theta_n}) \supset (P_{\theta_1}(\alpha)\ \&\ldots\&\ P_{\theta_m}(\alpha)).$$

In the intelim format of the present account, a thematic role implication inference rule may be introduced:

Whenever a line of the form:

$$\Pi(\alpha_1^{\theta_1}\ldots\alpha^{\theta}\ldots\alpha_n^{\theta_n})$$

appears in a derivation, a new line of the form:

$$P_{\theta_1}(\alpha)\ \&\ldots\&\ P_{\theta_m}(\alpha)$$

may be added to the derivation.

Each of these representations of thematic role implication involves significant difficulties. Each would involve a considerable complication of the model theory to guarantee the legitimacy of these inferences/meaning postulates; i.e., it would have to be stipulated that whenever a singular term appears with a given thematic role marker, it would also have to appear in the extension of some subset the general terms appearing in the $P\theta_i$ associated with that thematic role sufficient to guarantee the truth of the meaning postulates/validity of the inferences. This may succeed in representing more English inferences, but only ad hoc.

REFERENCES

[1] Bencivenga, E., Lambert, K., and van Fraassen, B., *Logic, Bivalence and Denotation: An Essay in Philosophical Logic*, Ridgeview Press, Atascadero, 1987.

[2] Bergmann, M., Moor, J., and Nelson, R., *The Logic Book*, McGraw-Hill, New York, 1990.

[3] Davison, D., "The Logical Form of Action Sentences," pp. 105–121, in *Essays on Actions and Events*, Clarendon Press, Oxford, 1980.

[4] Dowty, D., "On the Semantic Content of the Notion of 'Thematic Role'," pp. 69–130 in *Properties, Types, and Meaning*, vol. 2, edited by G. Chierchia, Partee, B., and Turner, R., Kluwer, Dordrecht, 1989.

[5] Grandy, R., "Anadic Logic and English," *Synthese*, vol. 32 (1976), pp. 393–402.

[6] Grandy, R., *Advanced Logic for Applications*, Reidel, Dordrecht, Holland, 1977.

[7] Henkin, L., "The Completeness of the First-Order Functional Calculus," *Journal of Symbolic Logic*, vol. 14 (1949), pp. 159–166.

[8] Kenny, A., *Action, Emotion, and Will*, Humanities Press, New York, 1963.

[9] Parsons, T., *Events in the Analysis of English: A Study of Subatomic Semantics*, MIT Press, Cambridge, 1990.

[10] Ramsey, F., "Facts and Propositions," pp. 138–155 in *The Foundations of Mathematics*, K. Paul, Trench, Trubner & Co., Ltd., London, 19831.

[11] Tarski, A., "Projecie prawdy w jezykach nauk dedukcyjnych," *Travaux de la Societe des Sciences et des Lettres de Varsovie*, Classe III, no. 34, translated into English as "The Concept of Truth in Formalized Languages," pp. 152–278, in his *Logic, Semantics, Metamathematics*, Oxford University Press, Oxford, 1956.

Department of Philosophy
Oakland University
Rochester, Michigan 48309