NEW FORMS OF CONTRA-CONTINUITY IN IDEAL TOPOLOGY SPACES

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Abstract. In this paper, we apply the notion of e-$\mathcal{I}$-open sets [1] in ideal topological spaces to present and study new classes of functions called contra e-$\mathcal{I}$-continuous functions, almost-e-$\mathcal{I}$-continuous, almost contra-e-$\mathcal{I}$-continuous, and almost weakly-e-$\mathcal{I}$-continuous along with their several properties, characterizations and mutual relationships. Relationships between their new classes and other classes of functions are established and some characterizations of their new classes of functions are studied. Further, we introduce new types of graphs, called e-$\mathcal{I}$-closed, contra-e-$\mathcal{I}$-closed, and strongly contra-e-$\mathcal{I}$-closed graphs via e-$\mathcal{I}$-open sets. Several characterizations and properties of such notions are investigated.

1. Introduction and Preliminaries

In 1996, Dontchev [4] introduced a new class of functions called contra-continuous functions. He defined a function $f\colon X \to Y$ to be contra-continuous if the preimage of every open set of Y is closed in X. A new weaker form of this class of functions, called contra-e-continuous functions, contra e-continuous functions, and contra a-continuous functions were introduced and investigated by Ekici [6]. In this direction, we will introduce the concept of contra e-$\mathcal{I}$-continuous functions. In Section 2 we introduce and study fundamental properties of contra-e-$\mathcal{I}$-continuous functions, almost contra-e-$\mathcal{I}$-continuous, almost-e-$\mathcal{I}$-continuous etc.; and using such functions we characterize e-$\mathcal{I}$-connectedness. Section 3 is devoted to the investigation of almost contra-e-continuous functions. Section 4 deals with notions of e-$\mathcal{I}$-closed, contra-e-$\mathcal{I}$-closed, and strongly contra e-$\mathcal{I}$-closed graphs.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following conditions:

$$A \in \mathcal{I} \text{ and } B \subset A \text{ implies } B \in \mathcal{I};\quad A \in \mathcal{I} \text{ and } B \in \mathcal{I} \text{ implies } A \cup B \in \mathcal{I}.$$

Applications to various fields were further investigated by Jankovic and Hamlett [9], Dontchev et al. [4], Mukherjee et al. [12], Arenas et al. [2], Nasef and Mahmoud [13], etc. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^* : \wp(X) \to \wp(X)$, called a local function [20, 9] of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A \subseteq X$,

$$A^*(\mathcal{I}, \tau) = \{x \in X : U \cap A \notin \mathcal{I}, \text{ for every } U \in \tau(x)\}$$

where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator is defined as $Cl^*(x) = A \cup A^*$ for τ^*. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I}, \tau)$. X^* is often a proper subset of X.

A subset A of a space $(X, \tau, \mathcal{I})$ is said to be regular open (resp. regular closed) [19] if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$). A is called δ-open [21] if for each $x \in A$, there exist a regular open set G such that $x \in G \subset A$. The complement of δ-open set is called δ-closed. A point $x \in X$ is called a δ-cluster point of A if $Int(Cl(U)) \cap A \neq \emptyset$ for each open set U containing x. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $Cl_\delta(A)$ [21]. The set δ-interior of A [21] is the union of all regular open sets of X contained in A and is denoted by $Int_\delta(A)$. A is δ-open if $Int_\delta(A) = A$. δ-open sets forms a topology τ^δ. The collection of all δ-open sets in X is denoted by $\delta O(X)$.

A subset A of an ideal topological space (X, τ) is said to be R-I-open (resp. R-I-closed) [22] if $A = Int(Cl^*(A))$ (resp. $A = Cl^*(Int(A))$). A point $x \in X$ is called a δ-I-cluster point of A if $Int(Cl^*(U)) \cap A \neq \emptyset$ for each open set U containing x. The family of all δ-$\mathcal{I}$-cluster points of A is called the δ-$\mathcal{I}$-closure of A and is denoted by $\delta Cl_I(A)$. The set δ-$\mathcal{I}$-interior of A is the union of all R-I-open sets of X contained in A and is denoted by $\delta Int_I(A)$. A is said to be δ-$\mathcal{I}$-closed if $\delta Cl_I(A) = A$ [22]. The class of e-open sets contains all δ-preopen [15] sets and δ-semiopen [14] sets.

Definition 1.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1) *e-$\mathcal{I}$-open if* [1] *$A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$ and e-$\mathcal{I}$-closed* [1] *if $Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A)) \subset A$.*

The class of all e-$\mathcal{I}$-open sets of an ideal topological space $(X, \tau, \mathcal{I})$ is denoted by $EIO(X)$.

Theorem 1.2. [1]

(1) *The union of any family of e-$\mathcal{I}$-open sets is an e-$\mathcal{I}$-open set;*
(2) *The intersection of even two e-$\mathcal{I}$-open open sets need not be e-$\mathcal{I}$-open.*

Definition 1.3. [6] *A function $f\colon (X, \tau) \longrightarrow (Y, \sigma)$ is called contra e-continuous if $f^{-1}(B)$ is e-closed in X for every open set B of Y.*

2. Contra e-$\mathcal{I}$-Continuous Functions

Definition 2.1. *A function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is called contra e-$\mathcal{I}$-continuous functions if $f^{-1}(V)$ is e-$\mathcal{I}$-closed in $(X,\tau,\mathcal{I})$ for every open set V in (Y,σ).*

Example 2.2. *Let $X = \{a,b,c,d\}$ with a topology $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b,c\}, \{a,b\}, \{a,b,c\}, \{c,b,d\}\}$ and an ideal $\mathcal{I} = \{\emptyset, \{c\}\}$. Let $f\colon (X,\tau,\mathcal{I}) \longrightarrow (X,\tau)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = d$, and $f(d) = c$. Observe that f is contra-e-continuous. But f is not contra-e-$\mathcal{I}$-continuous, since $\{a,b\}$ is open and $f^{-1}\{a,b\} = \{a,b\}$ is not e-$\mathcal{I}$-closed.*

Definition 2.3.

1. *Let A be a subset of an ideal topological space $(X,\tau,\mathcal{I})$. Then the set $\cap\{U \in \tau : A \subset U\}$ is called the kernel of A and denoted by $Ker(A)$* [11].
2. *The intersection of all e-$\mathcal{I}$-closed containing A is called the e-I-closure of A and its denoted by $Cl^*_e(A)$* [1].
3. *The e-$\mathcal{I}$-interior of A, denoted by $Int^*_e(A)$, is defined by the union of all e-$\mathcal{I}$-open sets contained in A* [1].

Lemma 2.4. [8] *Let A be a subset of an ideal topological space $(X,\tau,\mathcal{I})$.*

1. *$x \in Ker(A)$ if and only if $A \cap F \neq \emptyset$ for any closed subset of X with $x \in F$,*
2. *$A \subset Ker(A)$ and $A = Ker(A)$ if A is open in X,*
3. *if $A \subset B$, then $Ker(A) \subset Ker(B)$.*

Lemma 2.5. *The following properties hold for a subset A of an topological ideal space $(X,\tau,\mathcal{I})$.*

1. *$Int^*_e(A) = X - Cl^*_e(X - A)$.*
2. *$x \in Cl^*_e(A)$ iff $A \cap U \neq \emptyset$ for each $U \in EIO(X,x)$.*
3. *A is e-$\mathcal{I}$-closed if and only if $Cl^*_e(A) = A$* [1].
4. *B is e-$\mathcal{I}$-open if and only if $Int^*_e(B) = B$* [1].

Theorem 2.6. *The following statements are equivalent for a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$.*

1. *f is contra e-$\mathcal{I}$-continuous,*
2. *for each $x \in X$ and each closed set F in Y with $f(x) \in F$, $f^{-1}(F)$ is e-$\mathcal{I}$-open in X,*
3. *for each $x \in X$ and each closed set F in Y with $f(x) \in F$, there exist e-$\mathcal{I}$-open set U containing x such that $f(U) \subset F$,*
4. *for every subset A of X, $f(Cl^*_e(A)) \subset Ker(f(A))$,*
5. *for every subset B of Y, $Cl^*_e(f^{-1}(B)) \subset f^{-1}(Ker(B))$.*

Proof.
(1)⇒(3). Let $x \in X$ and let F be any closed set in Y such that $f(x) \in F$. Since f is contra e-$\mathcal{I}$-continuous, we have $f^{-1}(Y-F) = X - f^{-1}(F)$ is e-$\mathcal{I}$-closed in X and so $f^{-1}(F)$ is e-$\mathcal{I}$-open. By butting $U = f^{-1}(F)$ containing x, we have $f(U) \subset F$.
(3)⇒(2). Let F be an closed set in Y and $x \in f(x)$. Then $f(x) \in F$ and there exists e-$\mathcal{I}$-open subset U_x containing x such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$ which is e-$\mathcal{I}$-open in X.
(2)⇒(1). Let U be any open set of Y. Then since $(Y-U)$ is closed in Y, by (2), $f^{-1}(Y-U) = X - f^{-1}(U)$ is e-$\mathcal{I}$-open in X. Therefore, $f^{-1}(U)$ is e-$\mathcal{I}$-closed in X.
(2)⇒(4). Let A be any subset of X. Suppose that $y \notin Ker(A)$. Then, by Lemma 2.4, there exists a closed set F of Y such that $y \in F$ and $f(A) \cap F = \emptyset$. This implies that $A \cap f^{-1}(F) = \emptyset$ and $Cl_e^*(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(Cl_e^*(A)) \cap (F) = \emptyset$ and $y \notin f(Cl_e^*(A))$. This implies that $f(Cl_e^*(A)) \subset Ker(f(A))$.
(4)⇒(5). Let B be any subset of Y. By (4) and Lemma 2.4, we have $f(Cl_e^*(f^{-1}(B))) \subset Ker(f(f^{-1}(B))) \subset Ker(B)$ and
$Cl_e^*(f^{-1}(B)) \subset f^{-1}(Ker(B))$.
(5)⇒(1). Let V be any subset of Y. By (5) and Lemma 2.4, we have $Cl_e^*(f^{-1}(V)) \subset f^{-1}(Ker(V))$=$f^{-1}(V)$ and $Cl_e^*(f^{-1}(V))$=$f^{-1}(V)$. This shows that $f^{-1}(V)$ is e-$\mathcal{I}$-closed in X. □

Lemma 2.7. [1] *The following statements are equivalent for a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$.*

(1) *f is e-$\mathcal{I}$-continuous,*
(2) *for each $x \in X$ and each open set V in Y with $f(x) \in V$, there exist e-$\mathcal{I}$-open set U containing x such that $f(U) \subset V$.*

Definition 2.8. *If a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is e-$\mathcal{I}$-continuous* [1] *if $f^{-1}(B)$ is e-$\mathcal{I}$-open in X for every open set B of Y.*

Theorem 2.9. *If a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is contra e-$\mathcal{I}$-continuous functions and Y is regular, then f is an e-$\mathcal{I}$-continuous functions.*

Proof. Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $Cl(W) \subset V$. Since f is contra e-$\mathcal{I}$-continuous, by Theorem 2.6 there exists $U \in EIO(X)$ containing x such that $f(U) \subset Cl(W)$. Then $f(U) \subset Cl(W) \subset V$. Hence, f is e-$\mathcal{I}$-continuous. □

Definition 2.10. *An ideal topological space $(X,\tau,\mathcal{I})$ is said to be e-$\mathcal{I}$-connected if X is not the union of two disjoint non-empty e-$\mathcal{I}$-open subsets of X.*

Theorem 2.11. *If $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is a contra e-$\mathcal{I}$-continuous function from an e-$\mathcal{I}$-connected space X onto any space Y, then Y is not a discrete space.*

Proof. Suppose that Y is discrete. Let A be a proper non-empty clopen set in Y. Then $f^{-1}(A)$ is a proper non-empty e-$\mathcal{I}$-clopen subset of X, which contradicts the fact that X is e-$\mathcal{I}$-connected. □

Theorem 2.12. *If $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is a contra e-$\mathcal{I}$-continuous surjection function and X is e-$\mathcal{I}$-connected, then Y is connected.*

Proof. Let $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ be a contra e-$\mathcal{I}$-continuous function from an e-$\mathcal{I}$-connected space X onto a space Y. If possible assume that Y is disconnected. Then $Y = A \cup B$, where A and B are non-empty clopen sets in Y with $A \cap B = \emptyset$. Since f is contra e-$\mathcal{I}$-continuous, we have that $f^{-1}(A)$ and $f^{-1}(B)$ are e-$\mathcal{I}$-open non-empty sets in X with $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A\cup B) = f^{-1}(Y) = X$ and $f^{-1}(A)\cap f^{-1}(B) = f^{-1}(A\cap B) = f^{-1}(\emptyset) = \emptyset$. This means that X is not e-$\mathcal{I}$-connected, which is a contradiction. Then Y is connected. □

Definition 2.13. *A function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\mathcal{J})$ is called almost-e-$\mathcal{I}$-continuous if, for each $x \in X$ and for each open set V of Y containing $f(x)$, there exist $U \in EIO(X,x)$ such that $f(U) \subset Int^*_e(Cl(V))$.*

Definition 2.14. *A function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\mathcal{J})$ is called Pre-e-$\mathcal{I}$-open if image of each e-$\mathcal{I}$-open set of X is an e-$\mathcal{J}$-open set in Y.*

Definition 2.15. *The e-$\mathcal{I}$-frontier of a subset A of a space X, denoted by e-$\mathcal{I}$-$Fr(A)$, is denoted as e-$\mathcal{I}$-$Fr(A) = Cl^*_e(A) \cap Cl^*_e(X-A) = Cl^*_e(A) \cap Int^*_e(A)$.*

Theorem 2.16. *The set of all points x of X at which $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is not contra-e-$\mathcal{I}$-continuous is identical with the union of e-$\mathcal{I}$-frontier of the inverse images of closed sets of Y containing $f(x)$.*

Proof.
Necessity. Let f be not contra-e-$\mathcal{I}$-continuous at a point x of X. Then by Theorem 2.6, there exists a closed set F of Y containing $f(x)$ such that $f(U) \cap (Y-F) \neq \emptyset$, for every $U \in EIO(X,x)$, which implies $U \cap f^{-1}(Y-F) \neq \emptyset$. Therefore, $x \in Cl^*_e(f^{-1}(Y-F)) = Cl^*_e(X - f^{-1}(F))$. Again, since $x \in f^{-1}(F)$, we get $x \in Cl^*_e(f^{-1}(F))$ and so $x \in e\text{-}\mathcal{I} - Fr(f^{-1}(F))$.
Sufficiency. Suppose that $x \in e\text{-}\mathcal{I} - Fr(f^{-1}(F))$ for some closed set F of Y containing $f(x)$ and f is contra-e-$\mathcal{I}$-continuous at x. Then there exists $U \in EIO(X,x)$ such that $f(U) \subset F$. Therefore, $x \in U \subset f^{-1}(F)$ and hence, $x \in Int^*_e(f^{-1}(F)) \subset X - e\text{-}\mathcal{I}\text{-}Fr(f^{-1}(F))$, which is a contradiction. So f is not contra-e-$\mathcal{I}$-continuous at x. □

Theorem 2.17. *If $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\mathcal{J})$ is pre-e-$\mathcal{I}$-open, contra-e-$\mathcal{I}$-continuous then its almost-e-$\mathcal{I}$-continuous.*

Proof. Let $x \in X$ and V be an open set containing $f(x)$. Since f is contra-e-$\mathcal{I}$-continuous, then by Theorem 2.6, there exists $U \in EIO(X,x)$ such that $f(U) \subset Cl(V)$. Again, since f is *pre*-e-$\mathcal{I}$-open, $f(U)$ is e-$\mathcal{J}$-open in Y. Therefore, $f(U) = Int_e^*(f(U))$ and hence, $f(U) \subset Int_e^*(Cl(f(V))) \subset Int_e^*(Cl(V))$. So f is almost-e-$\mathcal{I}$continuous. □

Definition 2.18. *A function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is called almost weakly-e-$\mathcal{I}$-continuous if, for each $x \in X$ and for each open set V of Y containing $f(x)$, there exist $U \in EIO(X,x)$ such that $f(U) \subset Cl(V)$.*

Theorem 2.19. *If function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is contra e-$\mathcal{I}$-continuous then f is almost weakly-e-$\mathcal{I}$-continuous.*

Proof. For any open set V of Y, $Cl(V)$ is closed in Y. Since f is contra e-$\mathcal{I}$-continuous, $f^{-1}(Cl(V))$ is e-$\mathcal{I}$-open set in X. We take $U = f^{-1}(Cl(V))$, then $f(U) \subset Cl(V)$. Hence, f is almost weakly-e-$\mathcal{I}$-continuous. □

Example 2.20. *Let $X = \{a,b,c\}$ with a topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and an ideal $\mathcal{I} = \{\emptyset, \{a\}\}$. Let $f\colon (X,\tau,\mathcal{I}) \longrightarrow (X,\tau)$ be the identity function. Observe that f is almost weakly-e-$\mathcal{I}$-continuous. But f is not contra-e-$\mathcal{I}$-continuous.*

Definition 2.21. *An ideal topological space $(X,\tau,\mathcal{I})$ is said to be e-$\mathcal{I}$-T_1 if for each pair of distinct points x and y in X, there exist e-$\mathcal{I}$-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.*

Definition 2.22. *An ideal topological space $(X,\tau,\mathcal{I})$ is said to be e-$\mathcal{I}$-T_2 if for each pair of distinct points x and y in X, there exist e-$\mathcal{I}$-open sets U and V containing x and y, respectively, such that $U \cap V = \emptyset$.*

Theorem 2.23. *Let $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ be a contra e-$\mathcal{I}$-continuous injection. If Y is a Urysohn space, then X is e-$\mathcal{I}$-T_2.*

Proof. Let x and y be a pair of distinct points in X. Then $f(x) \neq f(y)$. Since Y is a Urysohn space, there exist open sets U and V of Y such that $f(x) \in U$, $f(y) \in V$ and $Cl(U) \cap Cl(V) = \emptyset$. Since f is contra e-$\mathcal{I}$-continuous at x and y, there exist e-$\mathcal{I}$-open sets A and B in X such that $x \in A$, $y \in B$ and $f(A) \subset Cl(U)$, $f(B) \subset Cl(V)$. Then, $f(A) \cap f(B) = \emptyset$, so $A \cap B = \emptyset$. Hence, X is e-$\mathcal{I}$-T_2. □

Proposition 2.24. *Let $(X,\tau,\mathcal{I})$ be an e-$\mathcal{I}$-connected space and (Y,σ) a T_1-space. If $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is a contra e-$\mathcal{I}$-continuous function, then f is constant.*

Proof. Let X be e-$\mathcal{I}$-connected. Now, since Y is a T_1 space, $\mho = \{f^{-1}(y) : y \in Y\}$ is disjoint e-$\mathcal{I}$-open partition of X. If $|\mho| \geq 2$ (where $|\mho|$ denotes the cardinality of $\mho$), then X is the union of two nonempty disjoint e-$\mathcal{I}$-open sets. Since X is e-$\mathcal{I}$-connected, we get $|\mho| = 1$. Hence, f is constant. □

Definition 2.25. *A function $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be e-$\mathcal{I}$-irresolute if $f^{-1}(B) \in EIO(X)$ for each $B \in EIO(Y)$.*

Theorem 2.26. *For the function $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ and $g\colon (Y, \sigma, \mathcal{J}) \longrightarrow (Z, \eta)$ the following hold.*

(1) *If f is contra-e-$\mathcal{I}$-continuous function and g is a continuous function, then $g \circ f$ is contra-e-$\mathcal{I}$-continuous.*
(2) *If f is e-$\mathcal{I}$-irresolute function and g is a contra-e-$\mathcal{I}$-continuous, then $g \circ f$ is contra-e-$\mathcal{I}$-continuous.*

Proof.
(1) For $x \in X$, let W be any closed set of Z containing $(g \circ f)(x)$. Since g is continuous, $V = g^{-1}(W)$ is closed in Y. Also, since f is contra-e-$\mathcal{I}$-continuous, there exists $U \in EIO(X, x)$ such that $f(U) \subset V$. Therefore, $(g \circ f(U)) \subset W$. Hence, $g \circ f$ is contra-e-$\mathcal{I}$-continuous.
(2) For $x \in X$, let W be any closed set of Z containing $g \circ f(x)$. Since g is contra-e-$\mathcal{I}$-continuous, there exist $V \in EJO(Y, f(x))$ such that $g(V) \subset W$. Again, since f is e-$\mathcal{I}$-irresolute there exist $U \in EIO(X, x)$ such that $f(U) \subset V$. This shows that $(g \circ f(U)) \subset W$. Hence, $g \circ f$ is contra-e-$\mathcal{I}$-continuous. □

Definition 2.27. *A space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-normal if each pair of nonempty disjoint closed sets can be separated by disjoint e-$\mathcal{I}$-open sets.*

Definition 2.28. [18] *A space (X, τ) is said to be ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.*

Theorem 2.29. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ is a contra e-$\mathcal{I}$-continuous, closed injection and Y is ultra normal, then X is e-$\mathcal{I}$-normal.*

Proof. Let C_1 and C_2 be disjoint closed subsets of X. Since f is closed and injective, $f(C_1)$ and $f(C_2)$ are disjoint closed subsets of Y. But Y is ultra normal, so $f(C_1)$ and $f(C_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Since f is contra e-$\mathcal{I}$-continuous, $f^{-1}(V_1)$, and $f^{-1}(V_2)$ are e-$\mathcal{I}$-open, with $C_1 \subset f^{-1}(V_1)$, $C_2 \subset f^{-1}(V_2)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Hence, X is e-$\mathcal{I}$-normal. □

Definition 2.30. *A topological space (X, τ) is ultra Hausdorff* [18] *if for each pair of distinct points x and y of X there exist closed sets U and V such that $x \in U$, $y \in V$, and $U \cap V = \emptyset$. A topological space (X, τ) is said* *to be weakly Hausdorff* [17] *each element of* X *is the intersection of regular closed sets of* X*.*

Theorem 2.31. *If* $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma,\mathcal{J})$ *is a contra e-*$\mathcal{I}$*-continuous injection and* Y *is ultra Hausdorff, then* X *is e-*$\mathcal{I}$*-*T_2*.*

Proof. Let $x, y \in X$ where $x \neq y$. Then, since f is an injection and Y is ultra Hausdorff, $f(x) \neq f(y)$ and there exist disjoint closed sets U and V containing $f(x)$ and $f(y)$, respectively. Again, since f is contra-e-$\mathcal{I}$-continuous, $f^{-1}(U) \in EIO(X,x)$ and $f^{-1}(V)EIO(X,y)$ with $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is e-$\mathcal{I}$-T_2. □

Definition 2.32. *An ideal topological space* $(X,\tau,\mathcal{I})$ *is said to be e-*$\mathcal{I}$*-compact* [1] *if every e-*$\mathcal{I}$*-open cover of* X *has a finite subcover.*

3. Almost Contra e-$\mathcal{I}$-Continuous

Definition 3.1. *A function* $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ *is called almost contra e-*$\mathcal{I}$*-continuous if* $f^{-1}(V)$ *is e-*$\mathcal{I}$*-closed in* $(X,\tau,\mathcal{I})$ *for every regular open set* V *in* (Y,σ)*.*

Theorem 3.2. *The following statements are equivalent for a function* $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$*:*

1. (1) *f is almost contra e-$\mathcal{I}$-continuous,*
2. (2) *$f^{-1}(F)$ is e-$\mathcal{I}$-open in X for each regular closed set F of Y,*
3. (3) *for each $x \in X$ and each regular closed set F of Y with $f(x) \in F$, there exist e-$\mathcal{I}$-open set U containing x such that $f(U) \subset F$,*
4. (4) *for each $x \in X$ and each regular open set F of Y non-containing $f(x)$, there exist an e-$\mathcal{I}$-closed set K of X non-containing x such that $f^{-1}(F) \subset K$,*

Proof.
(1)⇒(2). Let $x \in X$ and let F be any regular closed set in Y such that $f(x) \in F$. Then $(Y - F)$ is regular open. Since f is contra e-$\mathcal{I}$-continuous, we have $f^{-1}(Y - F) = X - f^{-1}(F)$ is e-$\mathcal{I}$-closed in X and so $f^{-1}(F)$ is e-$\mathcal{I}$-open. By butting $U = f^{-1}(F)$ which is containing x, we have $f(U) \subset F$.
(2)⇒(3). Let F be an regular closed set in Y such that $f(x) \in F$. Then $f^{-1}(F)$ is e-$\mathcal{I}$-open in X and $x \in f^{-1}(F)$. Taking $U = f^{-1}(F)$ we get $f(U) \subset F$.
(3)⇒(2). Let F be any regular closed set of Y and $x \in f^{-1}(F)$. Then there exist $U_x \in EIO(X,x)$ such that $f(U_x) \subset F$ and so $U_x \subset f^{-1}(F)$. Also, we have $f^{-1}(F) \subset \cup\{U_x : x \in f^{-1}(F)\}$. Hence, $f^{-1}(F) \in EIO(X)$.
(3)⇒(4). Let V be any regular open set of Y non-containing $f(x)$. Then $(Y - V)$ is a regular closed set of Y containing $f(x)$. Hence by (3), there exist $U \in EIO(X,x)$ such that $f(U) \subset (Y - V)$. Hence, $U \subset f^{-1}(Y - V) \subset$

$X - f^{-1}(V)$ and so $f^{-1}(V) \subset (X - U)$. Now, since $U \in EIO(X)$, $(X - U)$ is e-$\mathcal{I}$-closed set of X not containing x. The converse part is obvious. □

Proposition 3.3. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost contra e-$\mathcal{I}$-continuous then f is almost weakly-e-$\mathcal{I}$-continuous.*

Proof. For $x \in X$, let Q be any open set of Y containing $f(x)$. Then $Cl(Q)$ is a regular closed set of Y containing $f(x)$. Then by Theorem 3.2 (3), there exist $P \in EIO(X, x)$ such that $f(P) \subset Cl(Q)$. So f is almost weakly-e-$\mathcal{I}$continuous. □

The following Lemma can be easily verified.

Lemma 3.4. *A function $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost e-$\mathcal{I}$-continuous, if and only if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exist $U \in EIO(X, x)$ such that $f(U) \subset V$.*

We recall that a topological (X, τ) is said to be extremely disconnected if the closure of every open set of X is open in X.

Theorem 3.5. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a function, where Y is extremely disconnected. Then f is almost contra-e-$\mathcal{I}$-continuous if and only if it is almost e-$\mathcal{I}$-continuous.*

Proof. Suppose $x \in X$ and V is a regular open set of Y containing $f(x)$. Since Y is extremely disconnected, V is clopen and so it is regular closed. Then using Theorem 3.2, there exist $U \in EIO(X, x)$ such that $f(U) \subset V$. Hence by Lemma 3.4, f is almost e-$\mathcal{I}$-continuous.
Conversely, let f be almost e-$\mathcal{I}$-continuous and W be any regular closed set of Y. Since Y is extremally disconnected, W is also regular open in Y. Therefore, $f^{-1}(W)$ is e-$\mathcal{I}$-open in X. This shows that f is almost contra-e-$\mathcal{I}$-continuous. □

Theorem 3.6. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost a contra-e-$\mathcal{I}$-continuous injection and Y is weakly Hausdorff, then X is e-$\mathcal{I}$-T_1.*

Proof. Let $x, y \in X$ where $x \neq y$. Then, since f is an injection and Y is weakly Hausdorff, $f(x) \neq f(y)$ and there exist disjoint regular closed sets U and V such that $f(x) \in U$, $f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$. Assuming f is almost contra-e-$\mathcal{I}$-continuous, $f^{-1}(U) \in EIO(X, x)$ and $f^{-1}(V) \in EIO(X, y)$ such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{-1}(V)$. This shows that X is e-$\mathcal{I}$-T_1. □

Corollary 3.7. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is an almost contra-e-$\mathcal{I}$-continuous injection and Y is weakly Hausdorff, then X is e-$\mathcal{I}$-T_1.*

Theorem 3.8. *If $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost contra e-$\mathcal{I}$-continuous surjection and X is weakly Hausdorff, then Y is connected.*

Proof. If possible, suppose that Y is not connected. Then there exist disjoint non-empty open sets U and V of Y such that $Y = U \cup V$. Since U and V are clopen sets in Y, they are regular open sets of Y. Again, since f is almost contra-e-$\mathcal{I}$-continuous surjection, $f^{-1}(U)$ and $f^{-1}(V)$ are e-$\mathcal{I}$-open sets of X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not e-$\mathcal{I}$-connected, a contradiction. Hence, Y is connected. □

Definition 3.9. *An ideal topological space* $(X, \tau, \mathcal{I})$ *is said to be*

1. *e-$\mathcal{I}$-compact* [1] *if every e-$\mathcal{I}$-open cover of X has a finite subcover.*
2. *countably e-$\mathcal{I}$-compact if every countable cover of X by e-$\mathcal{I}$-open sets has a finite subcover.*
3. *e-$\mathcal{I}$-Lindelöf if every e-$\mathcal{I}$-open cover of X has a countable subcover.*

Definition 3.10. *A topological space X is called S-closed* [10] *(resp. countably S-closed* [3]*, S-Lindelöf* [7]*) if every regular closed (resp. countably regular closed, regular closed) cover of X has a finite (resp. finite, countable) subcover. A topological space X is said to be nearly compact* [16] *(resp. nearly countably compact* [16]*, nearly Lindelöf* [16]*) if every regular open (resp. countable regular open, regular open) cover of X has a finite (resp. finite, a countably) subcover.*

Theorem 3.11. *Let $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is an almost contra-e-$\mathcal{I}$-continuous surjection. Then the following statements hold:*

1. *If X e-$\mathcal{I}$-compact, then Y is S-closed.*
2. *If X e-$\mathcal{I}$-Lindelöf, then Y is S-Lindelöf.*
3. *If X countably e-$\mathcal{I}$-compact, then Y is countably S-closed.*

Proof.
(1) Let $\{V_\alpha : \alpha \in \triangle\}$ be any regular closed cover of Y. Since f is almost contra-e-$\mathcal{I}$-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in \triangle\}$ is a e-$\mathcal{I}$-open cover of X. Again, since X is e-$\mathcal{I}$-compact, there exist a finite subset I_0 of $\triangle$ such that $X = \cup\{f^{-1}(V)_\alpha : \alpha \in I_0\}$ and hence, $Y = \cup\{V_\alpha : \alpha \in I_0\}$. Therefore, Y is S-closed.
Other proofs are similar to (1) and are therefore omitted. □

4. Graphs via e-$\mathcal{I}$-Open Sets

Recall that for a function $f\colon X \to Y$, the subset $f(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 4.1. *The graph $G(f)$ of a function $f\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be e-$\mathcal{I}$-closed (resp. contra-e-$\mathcal{I}$-closed) if for each $(x, y) \in (X \times Y) - G(f)$, there exist an $U \in EIO(X, x)$ and an open (resp. a closed) set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 4.2. *The graph $G(f)$ of a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is e-$\mathcal{I}$-closed (resp. contra-e-$\mathcal{I}$-closed) in $X \times Y$ if and only if for each $(x,y) \in (X \times Y) - G(f)$ there exist $U \in EIO(X,x)$ and an open set (resp. a closed set) V in Y containing y such that $f(U) \cap V = \emptyset$.*

Proof. We shall prove that $f(U) \cap V = \emptyset$, $(U \times V) \cap G(f) = \emptyset$. Let $(U \times V) \cap G(f) \neq \emptyset$. Then there exist $(x,y) \in (U \times V)$ and $(x,y) \in G(f)$. This implies that $x \in U$, $y \in V$ and $y = f(x) \in V$. Therefore, $f(U) \cap V \neq \emptyset$. Hence, the result follows. □

Theorem 4.3. *If a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is contra-e-$\mathcal{I}$-continuous and Y is Urysohn, then $G(f)$ is contra-e-$\mathcal{I}$-closed in $X \times Y$.*

Proof. Let $(x,y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is Urysohn, there exist open sets S, T in Y such that $f(x) \in S$, $y \in T$ and $Cl(S) \cap Cl(T) = \emptyset$. Now, since f is contra-e-$\mathcal{I}$-continuous, there exist $U \in EIO(X,x)$ such that $f(U) \subset Cl(S)$ which implies that $f(U) \cap Cl(T) = \emptyset$. Hence by Lemma 4.2, $G(f)$ is contra-e-$\mathcal{I}$-closed in $X \times Y$. □

Theorem 4.4. *If $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ and $g\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ are contra-e-$\mathcal{I}$-continuous functions, where Y is Urysohn, then $D = \{x \in X : f(x) = g(x)\}$ is e-$\mathcal{I}$-closed in X.*

Proof. Let $x \in (X - D)$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exist open sets U and V such that $f(x) \in U$ and $g(x) \in V$ with $Cl(U) \cap Cl(V) = \emptyset$. Again, since f and g are contra-e-$\mathcal{I}$-continuous, then $f^{-1}(Cl(U))$ and $f^{-1}(Cl(V))$ are e-$\mathcal{I}$-open sets in X. Let $P = f^{-1}(Cl(U))$ and $Q = f^{-1}(Cl(V))$, then P and Q are e-$\mathcal{I}$-open sets of X containing x. Let $M = P \cap Q$, then M is e-$\mathcal{I}$-open in X. Hence, $f(M) \cap g(M) = f(P \cap Q) \cap g(P \cap Q) \subset f(P) \cap g(Q) = Cl(U) \cap Cl(V) = \emptyset$. Therefore, $D \cap M = \emptyset$ and hence, $x \notin Cl^{*}_{e}(D)$. Thus, D is e-$\mathcal{I}$-closed in X. □

Definition 4.5. *The graph $G(f)$ of a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is said to be strongly contra-e-$\mathcal{I}$-closed if for each $(x,y) \in X \times Y - G(f)$, there exist $U \in EIO(X,x)$ and regular closed set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 4.6. *The graph $G(f)$ of a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is strongly contra-e-$\mathcal{I}$-closed in $X \times Y$ if and only if for each $(x,y) \in (X \times Y) - G(f)$ there exist $U \in EIO(X,x)$ and regular closed set V in Y containing y such that $f(U) \cap V = \emptyset$.*

Theorem 4.7. *If a function $f\colon (X,\tau,\mathcal{I}) \longrightarrow (Y,\sigma)$ is almost weakly-e-$\mathcal{I}$-continuous and Y is Urysohn, then $G(f)$ is strongly contra-e-$\mathcal{I}$-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is Urysohn there exist open sets P, Q in Y such that $f(x) \in P$, $y \in Q$ and $Cl(P) \cap Cl(Q) = \emptyset$. Now, since f is almost weakly-e-$\mathcal{I}$-continuous, there exist $U \in EIO(X, x)$ such that $f(U) \subset Cl(P)$. This implies that $f(U) \cap Cl(Q) = f(U) \cap Cl(Int(Q)) = \emptyset$, where $Cl(Int(Q))$ is regular closed in Y. Hence by Lemma 4.6, $G(f)$ is strongly contra-e-$\mathcal{I}$-closed in $X \times Y$. □

A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is called almost e-$\mathcal{I}$-continuous if $f^{-1}(V)$ is e-$\mathcal{I}$-open in X for every regular open set V of Y.

Lemma 4.8. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost e-$\mathcal{I}$-continuous if and only if for each $x \in X$ and each regular open set Q of Y containing $f(x)$, there exists $P \in EIO(X, x)$ such that $f(P) \subset Q$.*

Theorem 4.9. *If $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is almost e-$\mathcal{I}$-continuous and Y is T_2, then $G(f)$ is strongly contra-e-$\mathcal{I}$-closed.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and since Y is T_2, there exist open sets P and Q containing y and $f(x)$, respectively, such that $P \cap Q = \emptyset$, which is equivalent to $Cl(P) \cap Int(Cl(Q)) = \emptyset$. Again, since f is almost e-$\mathcal{I}$-continuous and Q is regular open, by Lemma 4.8, there exists $S \in EIO(X, x)$ such that $f(S) \subset Q \subset Int(Cl(Q))$. This implies that $f(S) \cap Cl(P) = \emptyset$ and so by Lemma 4.6, $G(f)$ is strongly contra-e-$\mathcal{I}$-closed. □

Definition 4.10. [1] *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called e-$\mathcal{I}$-dense if $Cl^*_e(A) = X$.*

Proposition 4.11. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ and $g : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be any two functions. If Y is Urysohn, f, g are contra-e-$\mathcal{I}$-continuous functions and $f = g$ on e-$\mathcal{I}$-dense set $A \subset X$, then $f = g$ on X.*

Proof. Since f, g are contra-e-$\mathcal{I}$-continuous and Y is Urysohn, using Theorem 4.4, $D = \{x \in X : f(x) = g(x)\}$ is e-$\mathcal{I}$-closed in X. Also, we have $f = g$ on e-$\mathcal{I}$-dense set $A \subset X$. Now, since $A \subset D$ and A is e-$\mathcal{I}$-dense in X, we have $X = Cl^*_e(A) \subset Cl^*_e(D) = D$. Hence, $f = g$ on X. □

Definition 4.12. *A filter base $\mathcal{F}$ on a topological space $(X, \tau, \mathcal{I})$ is said to e-$\mathcal{I}$-converge to a point $x \in X$ if for each $V \in EIO(X, x)$, there exists $F \in \mathcal{F}$ such that $F \subset V$.*

Theorem 4.13. *Every function $\psi : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$, where Y is compact with e-$\mathcal{I}$-closed graph is e-$\mathcal{I}$-continuous.*

Proof. Let ψ be not e-$\mathcal{I}$-continuous at $x \in X$. Then there exists an open set S in Y containing $\psi(x)$ such that $\psi(T) \not\subset S$ for every $T \in EIO(X, x)$. It is obvious to verify that $\vartheta = \{T \subset X : T \in EIO(X, x)\}$ is a filter base on

X that e-$\mathcal{I}$-converges to x. Now we shall show that $Y_\vartheta = \{\psi(T) \cap (Y - S) : T \in EIO(X, x)\}$ is a filter base on Y. Here for every $T \in EIO(X, x)$, $\psi(T) \not\subset S$, i.e. $\psi(T) \cap (Y - S) \neq \emptyset$. So $\emptyset \notin Y_\vartheta$. Let G, $H \in Y_\vartheta$. Then there are T_1, $T_2 \in \vartheta$ such that $G = \psi(T_1) \cap (Y - S)$ and $H = \psi(T_2) \cap (Y - S)$. Since ϑ is a filter base, there exists a $T_3 \in \vartheta$ such that $T_3 \subset T_1 \cap T_2$ and so $W = \psi(T_3) \cap (Y - S) \in Y_\vartheta$ with $W \subset G \cap H$. It is clear that $G \in Y_\vartheta$ and $G \subset H$ imply $H \in Y_\vartheta$. Hence, Y_ϑ is a filter base on Y. Since $Y - S$ is closed in compact space Y, S is itself compact. At some point, Y_ϑ must adhere to $y \in Y - S$. Here $y \neq \psi(x)$ ensures that $(x, y) \notin G(\psi)$. Thus Lemma 4.2 gives us an $U \in EIO(X, x)$ and a open set V in Y containing y such that $\psi(U) \cap V = \emptyset$, i.e. $(\psi(U) \cap (Y - S)) \cap V = \emptyset$. This is a contradiction. □

Theorem 4.14. *If a surjection $\psi\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ possesses an e-$\mathcal{I}$-closed graph, then Y is T_1.*

Proof. Let p_1, $p_2 \in Y$ with $p_1 \neq p_2$. Since ψ is a surjection, there exists an $x_1 \in X$ such that $\psi(x_1) = p_1$ and $\psi(x_1) \neq p_2$. Therefore, $(x_1, p_2) \notin G(\psi)$ and so by Lemma 4.2, there exist $U_1 \in EIO(X, x_1)$ and open set V_1 in Y containing p_2 such that $\psi(U_1) \cap V_1 = \emptyset$. Then $p_1 \in \psi(U_1)$ but $p_1 \notin V_1$. Similarly, there exists an $x_2 \in X$ such that $\psi(x_2) = p_2$ and $\psi(x_2) \neq p_1$. Therefore, $(x_2, p_1) \notin G(\psi)$ and so by Lemma 4.2, there exists $U_2 \in EIO(X, x_2)$ and open set V_2 in Y containing p_1 such that $\psi(U_2) \cap V_2 = \emptyset$. Then $p_2 \in \psi(U_2)$ but $p_2 \notin V_2$. Hence, V_1 and V_2 are two open sets containing p_1 and p_2, respectively but $p_1 \notin V_1$ and $p_2 \notin V_2$. So Y is T_1. □

Theorem 4.15. *If an open surjection $\psi\colon (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ possesses an e-$\mathcal{I}$-closed graph, then Y is T_2.*

Proof. Let p_1, $p_2 \in Y$ with $p_1 \neq p_2$. Since ψ is a surjection, there exists $x_1 \in X$ such that $\psi(x_1) = p_1$ and $\psi(x_1) \neq p_2$. Therefore, $(x_1, p_2) \notin G(\psi)$ and so by Lemma 4.2, there exist $U_1 \in EIO(X, x_1)$ and open set V in Y containing p_2 such that $\psi(U) \cap V = \emptyset$. Since ψ is e-$\mathcal{I}$-open, $\psi(U)$ and V are disjoint e-$\mathcal{I}$-open sets containing p_1 and p_2. So Y is e-$\mathcal{I}$-T_2. □

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