INTERPOLATION BY FUNCTIONS IN BERGMAN SPACES

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I. Introduction. We begin by describing a special case of our main result. Let U be the upper half plane and let A be the Bergman space consisting of functions f which are holomorphic in U and for which

$$(1.1)\qquad \|f\| = \iint_U |f(x+iy)|\,dx\,dy$$

is finite. Let $S=\{\zeta_n\}$ be a sequence of points in U; $\zeta_n = x_n + iy_n$. We are interested in the relation between the geometry of S and the values which functions in A can take on S. Using the area mean value theorem for a disk centered at $\zeta = x+iy$ in U and having radius y we find that for any f in A

$$(1.2)\qquad |f(\zeta)| \leqslant c\|f\| y^{-2}.$$

Using a similar estimate for derivatives we find that

$$(1.3)\qquad |f'(\zeta)| \leqslant c\|f\| y^{-3}.$$

On the basis of (1.2) we see that the sequence $Tf = \{y_n^2 f(\zeta_n)\}$ is in $l^\infty(S)$. On the basis of the form of (1.1) and the analogy with the known results for the Hardy spaces we then ask for the relation between $l^1(S)$ and $\{Tf; f\in A\}$. (1.3) suggests one constraint. In order for there to be functions f_n in A which satisfy

$$(1.4)\qquad f_n(\zeta_m) = \delta_{n,m}$$

and which have $\|f_n\|$ uniformly bounded it is necessary that

$$(1.5)\qquad \inf_{\substack{n,m\\ n\neq m}} d(\zeta_n, \zeta_m) = K > 0.$$

Here $d(\cdot,\cdot)$ denotes the invariant distance (i.e., the hyperbolic distance) on U.

A particular case of our main result is that this condition is very close to being sufficient.

THEOREM. *There is a K_0 so that if S satisfies* (1.5) *with $K>K_0$ then $\{Tf; f\in A\} = l^1(S)$. In fact for all f in A*

$$\text{(a)}\qquad \sum_n y_n^2 |f(\zeta_n)| \leqslant c\|f\|$$

and for any $\{\lambda_n\}\in l^1(S)$, there is an f in A with $\|f\| \leqslant c\|\{\lambda_i\}\|$ and

$$\text{(b)}\qquad y_n^2 f(\zeta_n) = \lambda_n \quad n = 1, 2 \ldots$$

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The proof of (a) is based on the fact that the sum in (a) is essentially a Riemann sum for the integral in (1.1). The proof of (b) uses the theory of molecular decomposition of functions in Bergman spaces which was developed by R. R. Coifman and the author in [2]. That theory gives specific functions f_n in A which almost satisfy (1.4) and which have uniformly bounded norms. Thus the function $g_1 = \Sigma \lambda_n f_n$ is the first approximation to the function f which satisfies (b). The estimates in [2] are sufficient to insure that iteration of this approximation scheme will converge to the required function f.

A virtue of this approach is that, using the results of [2], the proof is seen to work with little extra effort in various other contexts. For example, if p and r are given; $0<p<\infty$, $-1<2r$, then we obtain analogous results for the space of functions f holomorphic in U which satisfy $\iint_U |f|^p y^{-2r} < \infty$. We also obtain similar results for analogously defined spaces of functions defined on domains in $\mathbf{C}^n$ which are biholomorphically equivalent to bounded symmetric domains. In particular we obtain results for the Bergman spaces on the ball and polydisk.

In Section 2 we give definitions, background and a precise statement of our results. The proof is in Section 3. Additional comments are in the final section.

These problems have a rich history which we will not summarize here. The interested reader can consult the books by Shapiro [9], Duren [3], Koosis [7], and Garnett [5].

Our results extend recent work by E. Amar [1]. Discussion of the relation between our results and methods and his is given in the final section.

II. Statement of the theorem. Let D be a symmetric homogenous domain in $\mathbf{C}^n$. For example, D could be the unit ball, unit polydisk or the product of half planes. Associated with such a D is a Bergman kernel function $B = B_D(\cdot,\cdot)$ and an invariant distance $d = d_0(\cdot,\cdot)$. (For background on these objects see [6] or [10].) Although we will never use estimates based on explicit formulas we note that for D the unit ball in $\mathbf{C}^n$, $z = (z_1, \ldots, z_n)$ and $\zeta = (\zeta_1, \ldots, \zeta_n)$ in D, $B_D(z,\zeta) = c_n(1 - \Sigma z_k \bar{\zeta}_k)^{-n-1}$. For the unit polydisk $B_D(z,\zeta) = c_n \prod (1 - z_k \bar{\zeta}_k)^{-2}$. Associated with any such D is a number ϵ_D which is defined to be the constant of Theorem 2 of [2] for the unbounded realization of D. ϵ_D is between 0 and 1, $\epsilon_D = 1/2$ for a product of half planes, $\epsilon_D = 1/(n+1)$ for the ball in $\mathbf{C}^n$.

For p, r with $0<p<\infty$, $-\epsilon_D < r$; we let $A^{p,r}(D) = A^{p,r}$ be the generalized Bergman space consisting of holomorphic functions f defined on D for which

$$\|f\|_{p,r}^p = \int_D |f(z)|^p B(z,z)^{-r}\, dV(z) < \infty.$$

Here $dV(z)$ denotes the volume element of the ambient $\mathbf{C}^n$. The choice $p=1$, $r=0$, $D=U$ produces the space A considered in the introduction.

In order to make full use of the results of [2] we assume for now that

(∗) D has a transitive group of affine automorphisms.

We will motivate the formulation of the theorem and prove the theorem using (∗). We will then observe that the formulation is invariant under changes of variable and hence (∗) is irrelevant. Suppose (∗) holds.

Let $S=\{\zeta_i\}_{i=1}^{\infty}$ be a set of points in D. We regard D, S, r and p as fixed and wish to study the relation between the geometry of S and the values taken on S by functions in $A^{p,r}(D)$.

Let $\alpha=(1+r)/p$.

The first observation is an analogue of (1.2). (Proofs of the lemmas are in the next section.)

LEMMA 2.1. *Suppose* (*) *holds. There is a constant* $c=c(D,p,r)$ *so that for all* ζ *in* D *and all* f *in* $A^{p,r}(D)$,

$$|f(\zeta)| \leqslant cB(\zeta,\zeta)^{\alpha}\|f\|_{p,r}. \tag{2.1}$$

On the basis of (2.1) we define the map T from $A^{p,r}$ to $l^{\infty}(S)$ by $(Tf)(\zeta_k)=B(\zeta_k,\zeta_k)^{-\alpha}f(\zeta_k)$. The next lemma (which is based on a derivative estimate analogous to (1.3)) shows the relation between the separation of points of S and variability of Tf.

LEMMA 2.2. *Suppose* (*) *holds. There is a constant* $c=c(D,p,r)$ *so that for all* z_1,z_2 *in* D *with* $d(z_1,z_2)<1$

$$|B(z_1,z_1)^{-\alpha}f(z_1)-B(z_2,z_2)^{-\alpha}f(z_2)| \leqslant cd(z_1,z_2)\|f\|_{p,r}. \tag{2.2}$$

Thus, in order for the characteristic function of every singleton of S to be realized as Tf with f in $A^{p,r}$ and $\|f\|_{p,r}$ dominated by a uniform constant, it is necessary that

$$\inf_{i\neq j} d(\zeta_i,\zeta_j)=K>0. \tag{2.3}$$

Our main result is that if K is large then this is a sufficient condition to insure that T maps $A^{p,r}$ onto $l^p(S)$.

THEOREM. *Suppose* D *is biholomorphically equivalent to a bounded symmetric domain. Suppose* p, r, S *are given and* S *satisfies* (2.3). *Then* T *is a continuous map of* $A^{p,r}$ *into* $l^p(S)$. *That is, there is a constant* c *so that for all* f *in* $A^{p,r}$

$$\|Tf\|_{l^p}^p=\sum|B(\zeta_k,\zeta_k)^{-\alpha}f(\zeta_k)|^p \leqslant c\|f\|_{p,r}^p. \tag{2.4}$$

There is a K_0 *so that if* K *of* (2.3) *is larger than* K_0 *then* T *maps onto* $l^p(S)$. *In fact, there is a continuous linear map* R *of* $l^p(S)$ *into* $A^{p,r}(D)$ *so that* $TR=I_{l^p(S)}$.

III. Proofs. Many of the details of the proof are direct consequences of calculations and estimates in [2]. We will make free use of those results and often only indicate how those calculations can be modified to our current purposes. We first give a proof using (*). We begin with the lemmas. Pick a base point e of D. If h is holomorphic in D and B is a small Euclidean ball in D centered at e then, using the subharmonicity of $|h|$, we have

$$|h(e)| \leqslant c_B \int_B |h(z)|\,dV(z).$$

We now replace the integral by an integral over a slightly larger invariant ball centered at e. Thus, $|h(e)|\leqslant c\int_{\tilde{B}}|h|\,dV$ where $\tilde{B}$ is an invariant ball and c is a constant which can be chosen to depend only on the domain D. Let ζ be a different point in D

and let g be an affine automorphism of D which takes e to ζ. We apply the previous estimate to $h(z)=f(g(z))$ and obtain $|f(\zeta)|\leqslant c\int_{\bar{B}}|f(g(z))|\,dV(z)$. Let $w=g(z)$ be a new integration variable. The Jacobian factor obtained when we change variables, $|\det g'|^{-2}$ (which is constant because g is affine) equals $cB(\zeta,\zeta)$ (equation (2.2) of [2]). Thus $|f(\zeta)|\leqslant c(\int_{\bar{B}}|f(w)|\,dV(w))B(\zeta,\zeta)$. The desired conclusion is now obtained by using Lemma 2.5 of [2] to estimate the integral. □

To prove the second lemma we estimate two terms; $B(z_1,z_1)^{-\alpha}|f(z_1)-f(z_2)|$ and $|(B(z_2,z_2)/B(z_1,z_1))^{\alpha}-1|B(z_2,z_2)^{-\alpha}|f(z_2)|$. The required estimate on the first term is a consequence of a volume integral estimate of the gradient of f. (This is done in detail in the proof of Lemma 2.6 of [2]). The second term is estimated using the previous Lemma and Lemma 2.3 of [2]). □

We now prove the theorem. We start with the data D, p, r, e (the base point of D), S, and K (the constant of (2.3)). Without loss of generality we can assume that S is maximal with respect to the condition (2.3). For ζ_i, ζ_j in S we write $B_{ij}=|B(\zeta_i,\zeta_j)|$.

We now verify (2.4). Rereading the beginning of the proof of Lemma 2.1 and using the fact that $|f(z)|^p$ is subharmonic for any positive p we obtain the estimate $|f(\zeta)|^p\leqslant cB(\zeta,\zeta)\int_{\bar{B}}|f(w)|^p\,dV(w)$. Here $\bar{B}$ is the invariant ball of fixed radius and centered at ζ. By adjusting c we may assume that this radius is $K/2$. Thus, denoting the ball centered at ζ_i with radius $K/2$ by $B(i)$ we have

$$\sum B_{ii}^{-p\alpha}|f(\zeta_i)|^p\leqslant c\sum B_{ii}^{-r}\int_{B(i)}|f(w)|^p\,dV(w). \tag{3.1}$$

By Lemma 2.3 of [2], the ratio $B_{ii}^{-r}/B(w,w)^{-r}$ with w in $B(i)$ is bounded above and below by constants which depend only on K, r, and D. Thus the sum in (3.1) is dominated by $c\sum\int_{B(i)}|f(w)|^pB(w,w)^{-r}\,dV(w)$. Because the $B(i)$ are disjoint, this last integral is dominated by $\|f\|_{p,r}^p$ and (2.4) is established.

We now prove the second half of the theorem. We start with the case $p\leqslant 1$. In that range it suffices to exhibit functions $f_1,\ldots$ which are uniformly bounded, i.e., $\|f_i\|_{p,r}\leqslant c$ and for which

$$\sum_j|B_{jj}^{-\alpha}f_i(\zeta_j)-\delta_{ij}|^p\leqslant\frac{1}{2}\quad i=1,2\ldots \tag{3.2}$$

Once we have such an estimate we can then construct R iteratively as follows. Let e_i be the function in $l^p(S)$ given by $e_i(\zeta_j)=\delta_{ij}$. Define $R_1e_i=f_i$ and extend by linearity. For p with $p\leqslant 1$ we have the estimate $\|R_1(\sum\lambda_ie_i)\|_{p,r}^p\leqslant\sum|\lambda_i|^p\|f_i\|_{p,r}^p$ and hence R_i is continuous. Using (3.2) we find that

$$\|(TR_1-I)(\sum\lambda_ie_i)\|_p^p\leqslant\sum|\lambda_i|^p\|TR_1e_i-e_i\|_p^p\leqslant\tfrac{1}{2}\sum|\lambda_i|^p.$$

Thus TR_1 is invertible. The required operator R is $R_1(TR_1)^{-1}$.

We now describe the required f_i. In fact we give a family of such with a free parameter ϵ. Pick and fix $\epsilon>0$. Let $f_i(\zeta)=B_{ii}^{-\epsilon}B(\zeta,\zeta_i)^{\alpha+\epsilon}$. The uniform estimate on the $A^{p,r}$ norm of f_i is Lemma 2.2 of [2]. To obtain (3.2) we first note that $B_{ii}^{-\alpha}(f_i(\zeta_i))=1$. Thus we must estimate the sum over all j distinct from i of terms $B_{jj}^{-\alpha p}B_{ii}^{-\epsilon p}B_{ij}^{\alpha p+\epsilon p}$. It is in making this estimate that crucial use is made of the assumption that K is large and of (*). □

LEMMA 3.1. *Given $\delta > 0$ there is a constant $c = c(D,p,r,K,\delta)$ so that for all i*

$$\sum_{\substack{j=1 \\ j\neq i}}^{\infty} B_{ij}^{1+r+\delta} B_{jj}^{-1-r} \leqslant c B_{ii}^{\delta}. \tag{3.3}$$

For fixed D, p, r, δ it is possible to choose c to be arbitrarily small if we allow K to be arbitrarily large.

Proof of the lemma. The basic invariance property of the Bergman kernel is that if g is a holomorphic automorphism of D then $B(gz, g\zeta)\det g'(z)\overline{\det g'(\zeta)} = B(z,\zeta)$. If g is affine then g' is constant and we have

$$B(gz, g\zeta)|\det g'|^2 = B(z,\zeta). \tag{3.4}$$

(This is (2.2) of [2].) Let g_i be an automorphism of D such that $g_i(e) = \zeta_i$. Let $\xi_j = g_i^{-1}(\zeta_j)$. Using (3.4) we find

$$B_{ij}^{1+r+\delta} B_{jj}^{-1-r} B_{ii}^{-\delta} = B(e,\xi_j)^{1+r+\delta} B(\xi_j,\xi_j)^{-1-r} B(e,e)^{-\delta}.$$

The automorphism g_i is an isometry with respect to the invariant distance hence the set $\{\xi_i\}$ satisfies (2.3) with the same constant. Thus the problem of showing (3.3) for general S and general i is reduced to establishing (3.3) for $i=1$ and an S which has $\zeta_1 = e$. We now consider that case.

The sum (3.3) is roughly Riemann sum for $c\int_D |B(e,\zeta)|^{1+r+\delta} B(\zeta,\zeta)^{-r}\, dV(z)$ which is bounded by Lemma 2.2 of [2]. More precisely, let $B(i)$ be the invariant ball of radius one centered at ζ_i. Let $|B(i)|$ be the Euclidean volume of $B(i)$. Write $B_{1j}^{1+r+\delta} B_{jj}^{-1-r}$ as $\int_{B(j)} B_{1j}^{1+r+\delta} B_{jj}^{-1-r} |B(j)|^{-1}\, dV(\zeta)$. By Lemma 2.3 of [2] we can find a constant c which depends on D, p, r, δ (but not S, j or K) so that

$$B_{jj}^{-1}|B(j)|^{-1} \leqslant c$$

and

$$\sup\{|B(e,\zeta)^{1+r+\delta} B(\zeta,\zeta)^{-r}|;\ \zeta \in B(j)\} \leqslant c B_{1j}^{1+r+\delta} B_{jj}^{-1-r}.$$

Using this and the fact that the $B(j)$ are disjoint we see that the sum in (3.3) is dominated by

$$c\sum_{j=2}^{\infty} \int_{B(j)} |B(e,\zeta)|^{1+r+\delta} B(\zeta,\zeta)^{-r}\, dV(\zeta) = c\int_{\bigcup_{j=2}^{\infty} B(j)}.$$

All points in $\bigcup_{j=2}^{\infty} B(j)$ are at invariant distance at least $K-1$ from e. Thus we have the estimate

$$\sum_{j=2}^{\infty} B_{1j}^{1+r+\epsilon} B_{jj}^{-1-r} \leqslant c\int_{d(\zeta,e)>K-1} |B(e,\zeta)|^{1+r+\epsilon} B(\zeta,\zeta)^{-r}\, dV(\zeta).$$

The integral on the right is finite by Lemma 2.2 of [2]. Hence, by selecting K large we can obtain an arbitrarily small upper bound for the sum. The lemma is proved. □

For $p > 1$ we will need to estimate the norm of a sum of terms with coefficients in l^p. One technique for doing this which is often used in this context (e.g., Lemma 2.1.2 of [1]) is the following Lemma of Schur. (For a proof, see [4].)

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LEMMA 3.2. *Suppose p, q are given $1<p,q<\infty$; $1/p+1/q=1$. Let A be an infinite matrix, $A=(a_{ij})$ with nonnegative entries. Suppose there is a constant c and a sequence h_i of nonnegative numbers so that $\sum_j a_{ij}h_j^q \leqslant ch_i^q$, $i=1,2,\ldots$ and $\sum_i a_{ij}h_i^p \leqslant ch_j^p$. Then the map A of l^p to itself which takes $f=(f_i)$ to Af defined by $(Af)_i=\sum_j a_{ij}f_j$ is bounded and has operator norm at most c.*

We now suppose $p>1$. Let q be the conjugate index; $p^{-1}+q^{-1}=1$. We will identify $l^p(S)$ with the abstract l^p space consisting of sequences $\{\lambda_i\}$ for which $\sum|\lambda_i|^p<\infty$. Pick ϵ so that

$$\epsilon > \alpha(p-1) = (1+r)/q \tag{3.5}$$

(further constraints on ϵ will be imposed later). For each i, let $f_i(\zeta)=B(\zeta,\zeta_i)^{\alpha+\epsilon}B_{ii}^{-\epsilon}$. We wish to construct a map R from l^p to $A^{p,r}$. We start with an approximation R_0 defined by $R_0(\{\lambda_i\})=\sum\lambda_i f_i$. Theorem 2 of [2] insures that R_0 is a bounded map of l^p into $A^{p,r}$. TR_0 maps the sequence $\{\lambda_i\}$ to the sequence $\{U_i\}$ given by $U_i=\sum_j(B_{ii}^{-\alpha}B_{ij}^{\alpha+\epsilon}B_{jj}^{-\epsilon})\lambda_j$. We now claim that TR_0-I is an operator of small norm. The matrix TR_0-I is (a_{ij}) with $a_{ij}=0$ if $i=j$, $a_{ij}=B_{ii}^{-\alpha}B_{ij}^{\alpha+\epsilon}B_{jj}^{-\epsilon}$ if $i\neq j$. We now apply the Lemma 3.2 with $h_j=B_{jj}^{\beta}$. Thus we must show

$$\sum_{\substack{j=1\\ j\neq i}}^{\infty} B_{ij}^{\alpha+\epsilon}B_{jj}^{-\epsilon+\beta q} \leqslant cB_{ii}^{\alpha+\beta q} \tag{3.6}$$

for some small c which is independent of i and we must similarly show that for some small c which is independent of j

$$\sum_{\substack{i=1\\ i\neq j}}^{\infty} B_{ij}^{\alpha+\epsilon}B_{ii}^{-\alpha+\beta q} \leqslant cB_{jj}^{\epsilon+\beta p}. \tag{3.7}$$

Both of these estimates follow from Lemma 3.1 as soon as we verify that the appropriate inequalities are satisfied by the exponents. The quantity r in (3.3) must satisfy $r>-\epsilon_D$; thus $-1-r<-1+\epsilon_D$. Hence to have (3.6) and (3.7) be instances of (3.3) we must have:

$$-\epsilon+\beta q < -1+\epsilon_D \tag{3.8}$$

$$\alpha+\beta q > 0 \tag{3.9}$$

$$-\alpha+\beta p < -1+\epsilon_D \tag{3.10}$$

$$\epsilon+\beta p > 0. \tag{3.11}$$

From (3.9) and (3.10) we find $-\alpha/q<\beta<(-1+\epsilon_D+\alpha)/p$. We can find a β which satisfies this if $-\alpha/q<(1+\epsilon_D+\alpha)/p$. However, this inequality simplifies as $r>-\epsilon_D$. Once β is selected we can then choose ϵ large enough to insure that (3.5), (3.8) and (3.11) are all satisfied. Thus if the separation constant K is large then $S=TR_0-I$ has small norm. $R=R_0(I+S)^{-1}$ is the required operator. The proof is now complete in the case where (*) holds. □

The general result follows upon noting that the formulation is invariant. That is, let B be any domain which is biholomorphically equivalent to a bounded symmetric domain. There is a D which satisfies (*) and a biholomorphic map g of D to B [8]. The mapping from functions $f(z)$ on B to $F(\zeta)=f(g(\zeta))(\det g'(\zeta))^{2\alpha}$ is a norm preserving map from $A^{p,r}(B)$ onto $A^{p,r}(D)$. This is an immediate consequence of the invariant property of the Bergman kernel ((2.9 of [2]).

$$B_B(g(z),g(\zeta))(\det g'(z))\overline{(\det g'(\zeta))} = B_D(z,\zeta).$$

Also, using this invariance we find that $B_D^{-\alpha}(\zeta,\zeta)F(\zeta)=B_B^{-\alpha}(g(\zeta),g(\zeta))f(g(\zeta))$. g is an isometry from (D,d_D) to (B,d_B) and hence the theorem for B follows from the result for D.

IV. Comments.

A. Some restrictions on K in (2.3) are needed. In fact, if D, p, r are given and K is sufficiently small then any S which satisfies (2.3) with that choice of K and which is maximal with respect to that condition will fail to satisfy the conclusions of the theorem. (See Proposition 4.1 of [2].) Also, the examples in [1] show that S can satisfy the conclusion of the theorem for some p and r and not for others. This suggests that K in the theorem depends in an essential way on p and r.

B. There are many unexplored questions related to these ideas. One is the question of the validity of these results for harmonic functions on domains in $\mathbf{R}^n$ (one approach would be to develop the analogues of the results of [4]). Another is the finding of the appropriate $p=\infty$ (i.e., $A=$the Bloch space) result. This might be more difficult (note, for example, that Lemma 3.2 does not apply).

C. Suppose D is the upper half plane and $p=2$. The spaces $A^{2,r}$ converge to $H^2(\mathbf{R})$ as $r\to -\frac{1}{2}$. The interpolation problem for $H^2(\mathbf{R})$ is known to have a different (and more delicate) solution. Roughly, our results are based on Lemmas 3.1 and 3.2 and hence are related to conditions on points $z_j=x_j+iy_j$ in U which insure that the matrix $\left(\dfrac{y_i^{1/2+\epsilon_1}y_j^{1/2+\epsilon_2}}{|z_i-\bar{z}_j|^{1+\epsilon_1+\epsilon_2}}\right)$ be a bounded operator on l^2 for various positive ϵ_1,ϵ_2. The numbers ϵ_1 and ϵ_2 can be taken near zero if r is close to $-\frac{1}{2}$. The H^2 interpolation theorem is related to conditions on the z_j which insure that the matrix $\left(\dfrac{y_i^{1/2}y_j^{1/2}}{|z_i-\bar{z}_j|}\right)$ is bounded on l^2 (see [7]).

D. If D is the ball or polydisk and $r=0$ then most of these results were obtained earlier by Amar [1]. His proof uses estimates which are more explicit than ours. In part this is because we lean on [2] and in part because we exploit the affine homogeneity to reduce Lemma 3.1 to the case $\zeta_i=e$. If we had tried to prove the Lemma directly for the ball or polydisk we would have been led to calculations very much in the spirit of [1]. Thus the difference in appearance between our analysis and his should not obscure the fact that our analysis can be understood as Amar's proof done "in general".

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