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KILLING FIELDS PRESERVING TOTALLY GEODESIC, CODIMENSION-ONE FOLIATIONS

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§1. Introduction

Let M be a complete manifold, endowed with a codimension-one foliation $\mathcal{F}$. We want to study the Lie algebra $\mathcal{G}$ of Killing fields preserving the foliation (i.e., Killing fields such that the isometries of their one-parameter group send leaves of $\mathcal{F}$ onto leaves of $\mathcal{F}$).

In [5], Johnson and Whitt proved that when the foliation is totally geodesic (i.e., leaves are totally geodesic submanifolds) and all the leaves are compact, then any Killing field preserves $\mathcal{F}$. Later, Oshikiri (see [7]) proved the same result for the case when the manifold is compact and $\mathcal{F}$ is totally geodesic. Nevertheless, in the general case all Killing fields do not preserve foliations. For example, in the euclidean plane foliated by lines parallel to the 0X-axis, Killing fields associated to rotations do not preserve the foliation.

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§2. Totally geodesic foliations

First of all, let us recall that any codimension-one foliation which admits an orthogonal Killing field must be totally geodesic (see [3] for instance). For this reason, from now on we shall only consider totally geodesic foliations. The universal cover of a manifold with such a structure verifies the following

THEOREM 1. (see [2]) *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. Let $\tilde{M}$ be the universal cover of M. Then $\tilde{M}$ is trivially foliated as $\tilde{L}\times\boldsymbol{R}$, where $\tilde{L}$ is the universal cover of any leaf and the induced metric reads $ds^2_{\tilde{M}}=ds^2_{\tilde{L}}+f^2d\tilde{t}^2$, where $f:\tilde{M}\to(0, \infty)$ is a C^∞ function.*

In order to simplify calculations, it will be convenient to give a characterization of Killing fields preserving foliations. Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. With the notations of Theorem 1,

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PROPOSITION 1. *Any Killing field* $X \in i(\tilde{M})$ *is of the form* $X = X^t + \lambda \partial t$, *where*

i) X^t *is a Killing field on* $\tilde{L}$ *with respect to* ds_L^2;

ii) $X^t f = -\partial_t(\lambda f)$;

iii) $f^2 \cdot (Y\lambda) = \langle Y, [X^t, \partial t] \rangle$, $\forall Y \in T(\mathcal{F})$ *with* $[Y, \partial t] = 0$.

Moreover, X preserves the foliation if and only if it verifies also that

iv) $Y\lambda = 0$, $\forall Y \in T(\mathcal{F})$, *or, equivalently,* $[X^t, \partial t] = 0$.

Proof. See Propositions 1.1 and 1.2 of [4]. ■

Passing, if necessary, to a 2-fold cover, we may suppose foliations to be transversally oriented. Thus, from now on, we will assume this fact and call N the normal field to $\mathcal{F}$ (i.e. an unitary vector field orthogonal to $\mathcal{F}$). According to the characteristics of the function f in Theorem 1, we will consider two cases:

i) $Yf = 0$, $\forall Y \in T(\mathcal{F})$. Then, $\mathcal{F}$ is a bundle-like foliation.

ii) Otherwise, we have the general case.

There is not much to say about bundle-like, totally geodesic foliations. Thus, we begin with case ii): let us assume for the moment that f is not constant in the leaves. Our goal now is to give the best bound possible for the dimension of $\mathcal{G}$, the Lie algebra of Killing fields preserving the foliation. First of all, let us give an upper bound for the dimension of its subalgebra $\mathcal{G}^t$ of Killing fields tangent to $\mathcal{F}$. If n=dimension $\mathcal{F}$, it is clear that dimension $\mathcal{G}^t \leqq (1/2)n(n+1)$. We want to show that actually dimension $\mathcal{G}^t \leqq (1/2)n(n-1)$. Before this, we need some Lemmas.

LEMMA 1. *Let M be a complete n-dimensional manifold, endowed with a foliation* $\mathcal{F}$ *of dimension* $m<n$ *and let* $X \in i(M) \cap T(\mathcal{F})$. *If there is some leaf L in which* $X_{|L} \equiv 0$, *then* $X \equiv 0$.

Proof. Let (ϕ_t) be the one-parameter group associated to X. We shall see that, for any $p \in L$, $(\phi_t)_{*p} = Id$, $\forall t$. In a neighborhood of p, let $(\partial x^1, \cdots, \partial x^m, \partial y^1, \cdots, \partial y^{n-m})$ be a basis such that the leaves of $\mathcal{F}$ are locally of the form $\{y^1 = ctt., \cdots, y^{n-m} = ctt.\}$. As in [9], we can modify it to a new basis $(\partial x^1, \cdots, \partial x^m, \nu^1, \cdots, \nu^{n-m})$, with $\nu^j = y\partial^j + \sum b_{ji} \partial x^i$, $\forall j$ and such that $\langle \nu^j, \partial x^i \rangle = 0$, $\forall i, j$. Now:

$$(\phi_t)_{*p}(\partial x^i) = \partial x^i, \qquad (\phi_t)_{*p}(\partial y^j) = \partial y^j + \sum_{i=1}^{m} \mu_{ij} \partial x^i,$$

$$(\phi_t)_{*p}(\nu^j - \partial y^j) = \nu^j - \partial y^j \qquad (\text{as}(\nu^j - \partial y^j) \in T(\mathcal{F})),$$

because X is tangent to $\mathcal{F}$ and vanishes at the leaf L. Thus,

$$(\phi_t)_{*p}(\nu^j) = \nu^j + \sum_{i=1}^{m} \mu_{ij} \partial x^i. \tag{1}$$

$(\phi_t)_*$ is an isometry and preserves the foliation. Then $(\phi_t)_{*p}(\nu^j)$ must be orthogonal to $\mathcal{F}$. It follows from (1) and the expression of the riemannian metric in the basis $(\partial x^1, \cdots, \partial x^m, \nu^1, \cdots, \nu^{n-m})$ that $(\phi_t)_{*p}(\nu^j)=(\nu^j)$, $\forall j$. That is, $(\phi_t)_{*p} = Id_{T_pM}$. p was an arbitrary point and the manifold is complete, thus $(\phi_t)=Id$, i.e., $X\equiv 0$. ∎

LEMMA 2. *Let M be a complete manifold and $\mathcal{T}$ a subalgebra of $i(M)$. Assume that $\forall p\in M$, dimension $\mathcal{T}_p \leqq m$. Then dimension $\mathcal{T} \leqq r =: (1/2)m(m+1)$.*

Proof. Let $p\in M$ with dimension $\mathcal{T}_p=m$. We can choose m fields, $X_1, \cdots, X_m\in\mathcal{T}$, independent (as vectors) in a neighborhood U of p. In U let $\mathcal{S}$ be the distribution generated by $\{X_1, \cdots, X_m\}$. It is easy to see that $\mathcal{S}$ is involutive and then defines a foliation $\mathcal{F}_U$ of dimension m in U. If dimension $\mathcal{T}>r$, let $Y_1, \cdots, Y_{r+1}\in\mathcal{T}$ be $r+1$ independent vector fields. Their restrictions to U are Killing fields tangent to $\mathcal{F}_U$, because $\mathcal{T}_q=(\mathcal{F}_U)_q$. Let L be (an m-dimensional) leaf of $\mathcal{F}_U$. Thus there are constants $c_1, \cdots, c_{r+1}$ such that $\sum c_j(Y_j)_{|L}=0$. Let us assume, for example, $c_{r+1}\neq 0$ and let $Y=: c_{r+1}Y_{r+1}-\sum_{j=1}^{r} c_jY_j$. By Lemma 1, $Y_{|U}=0$. But U is open on M; then $Y\equiv 0$, which contradicts the assumption on $Y_1, \cdots, Y_{r+1}$ ∎

PROPOSITION 2. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, n-dimensional, totally geodesic (not bundle-like) foliation. Then, dimension $G^t \leqq (1/2)n(n-1)$.*

Proof. It is enough to show the theorem for the universal cover of $(M, \mathcal{F})$. Therefore we may assume $M=L\times\boldsymbol{R}$ and $ds_M^2=ds_L^2+f^2dt^2$. Let $Y\in\mathcal{G}^t$. Thus, by Proposition 1, $[Y, \partial t]=0$ and $\mathcal{G}^t$ has constant dimension along any $\mathcal{F}^\perp$-leaf. Therefore we may define $W=\{p\in L \mid dimension\ \mathcal{G}^t_{pxt_0}=n=dimension\ L\}$. If W were dense in L, the foliation should be bundle-like, by Proposition 1. Thus there is an open subset $U\subset L\setminus W$. If $\mathcal{G}_U:=\{X_{|U}, \forall X\in\mathcal{G}^t\}$, then $\mathcal{G}_U\subset i(U)$ and $\forall p\in U$, dimension $(\mathcal{G}_U)_p$=dimension $\mathcal{G}^t_p\leqq n-1$. By Lemma 2, dimension $\mathcal{G}_U\leqq 1/2(n-1)n$. $\mathcal{G}^t\subset i(L)$ and U is open on L, thus, independent vector fields on $\mathcal{G}^t$ give independent vector fields on $\mathcal{G}_U$ and dimension $\mathcal{G}^t\leqq$dimension $\mathcal{G}_U\leqq(1/2)n(n-1)$. ∎

Let us introduce some definitions:
Let $\mathcal{G}^n =: \{X\in T^\perp(\mathcal{F})\cap\mathcal{G}\}$.
Let $\mathcal{J} =: \{Y\in T(\mathcal{F}) \mid \exists X\in\mathcal{G}$ and $Y=X^t\}$.
$\mathcal{G}^n$ and $\mathcal{J}$ are subalgebras of $\mathcal{G}$ and $\mathcal{G}^t\subset\mathcal{J}$. Moreover

PROPOSITION 3. *Dimension $\mathcal{G}^n\leqq 1$ and dimension $\mathcal{G}^n=1$ if and only if the universal cover $(\tilde{M}, \tilde{\mathcal{F}})$ is a warped product (in the sense that $\partial_t f=0$, in Theorem 1).*

Proof. Let us assume M to be simply connected. If $(M, \mathcal{F})$ is a warped product, it is clear from Proposition 1 that ∂t is a Killing field.

Suppose now that $X=\lambda\partial t$ is a Killing field. Then $\partial_t(\lambda f)=0$ and $\lambda=\lambda(t)$. (Moreover, λ never vanishes. See [4] for instance). If we reparametrize $\boldsymbol{R}$

with $\bar{t}=\int(1/\lambda)dt$, then the metric reads as $ds^2=ds_L^2+(\lambda f)^2 d\bar{t}^2$, which is a warped product and now $X=\partial\bar{t}$. Finally let $\mu\partial\bar{t}$ be another element of $\mathcal{G}$. Then, from ii) of Proposition 1 we may see that μ=cotstant. ■

§ 3. Warped product foliations.

We shall restrict now our attention to totally geodesic foliations with a warped product structure in the universal cover, (but not bundle-like). I.e., $\partial_t f=0$ in Theorem 1, but $f\neq constant$. This is equivalent to the fact that the 1-form θ associated to the vector field $\nabla_N N$ will be closed ($\theta(X)= :\langle\nabla_N N, X\rangle$, for any vector field X):

PROPOSITION 4. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. Let $\tilde{M}$ be the universal cover of M. Then, the structure of $(\tilde{M}, \tilde{\mathcal{F}})$ stated in Theorem 1 is a warped product if and only if $d\theta=0$.*

Proof. Suppose that $(\tilde{M}, \tilde{\mathcal{F}})$ is a warped product. We may consider in M an orthonormal (local) basis $\{X_1, \cdots, X_n, N\}$ for $T(M)$, with $X_i\in T(\mathcal{F})$; $[X_i, \partial t]=0$, $N=(1/f)\partial t$; and such that $\partial_t f=0$. Then $\nabla_N N=-\sum(X_i\cdot\log f)X_i$ and $\theta(X_i)=-X_i\cdot\log f$. Thus

$$d\theta(X_i, X_j)=-X_iX_j\log f+X_jX_i\log f+<\sum_{k=1}^{n}(X_k\cdot\log f)X_k, [X_iX_j]>=0,$$

$$d\theta(X_i, \partial t)=\partial_t X_i\log f=X_i\partial_t\log f=0.$$

For the converse, let us assume M to be simply connected. With the same notations as above, we have $X_i\partial t\log f=0$, $\forall i$. Then f should be of the form $f=e^g\cdot e^h$, where $g=g(t)$ and h is defined on L, the generic leaf. With the change of parameter $\bar{t}=\int e^{g(t)}dt$ we obtain a warped product metric for M. ■

Remark. After the change of parameter, the manifold should remain of the form $M=L\times\boldsymbol{R}$. For if M were equal to $L\times(a, b)$ and $a>-\infty$, for instance, we will consider geodesics with initial tangent vector $-\partial t$. Then, since leaves are totally geodesic submanifolds and translations in the direction of the 0t-axis are isometries (where they are defined), these geodesics should cross the extreme leaf $\{t=a\}$, which will contradicts the fact that M is complete.

PROPOSITION 5. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation, whose universal cover has a warped product structure. Let $X\in\mathcal{G}$. Then $\nabla_N X^t=kN$, with k=constant.*

Proof. Let us work in the universal cover of $(M, \mathcal{F})$. As $X=(X^t+\lambda\partial t)\in\mathcal{G}$,

$$\nabla_N X^t = -[X^t, N] = -\left[X^t, \frac{1}{f}\partial t\right] = \frac{X^t \cdot f}{f^2}\partial t = -\frac{\partial_t(\lambda f)}{f^2}\partial t = -\frac{\lambda'}{f}\partial t .$$

Actually, ∂t is a Killing field. Thus, $\lambda'\partial t=[\partial t, X]\in \mathcal{G}$ and $0=\partial_t(\lambda' f)=\lambda'' f$, so $\lambda''=0$. If we put $\lambda'=-k$, then $\nabla_N X^t=(k/f)\partial t=kN$. ■

Remarks. (1) Observe that the constant k verifies: $k=(X^t \log f)$. As a consequence, Killing fields tangent to the foliation are just vector fields $Y\in T(\mathcal{F})$ such that are Killing fields with respect to the metric of the leaves and verify $\nabla_N Y=0$. Moreover, every $X\in\mathcal{G}$ is of the form $X=X^t+(h-kt)fN$, for some constant h.

(2) The converse result is true when the manifold is simply connected (see [8]).

PROPOSITION 6. *Let $(M. \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation, whose universal cover is a warped product. Then dimension $\mathcal{J}\leqq$ dimension $\mathcal{G}^t+1$ and dimension $\mathcal{G}=$ dimension $\mathcal{J}+$ dimension $\mathcal{G}^n$.*

Proof. If there is some $Y_1\in\mathcal{J}\setminus\mathcal{G}^t$, we may take $Y_2, \cdots, Y_r$ in order to form a basis $(Y_1, Y_2, \cdots, Y_r)$ of $\mathcal{J}$. Thus, $\nabla_N Y_i=k_i N$, $\forall i$; and $k_1\neq 0$. Let $Z_j:=((k_j/k_1)Y_1-Y_j)$, $j: 2, \cdots, r$. It is easy to see that $(Y_1, Z_2, \cdots, Z_r)$ is a new basis of $\mathcal{J}$ with $Z_2, \cdots, Z_r\in\mathcal{G}^t$ and $X_1=Y_1+(h_1-k_1t)fN\in\mathcal{G}$.

For the second part, if $\mathcal{J}=\mathcal{G}^t$, then $\mathcal{G}=\mathcal{J}\oplus\mathcal{G}^n$ and the result is obvious. Otherwise, let $X\in\mathcal{G}\setminus\mathcal{G}^n$, $X=X^t+(b-kt)fN$, where $\nabla_N X^t=kN$. But $X^t\in\mathcal{J}$, thus $X^t=a_1Y_1+\sum_{j=2}^r a_jZ_j$, $\nabla_N X^t=a_1k_1N$ and $k=a_1k_1$. We have $X=a_1Y_1+\sum_{j=2}^r a_jZ_j+(b-a_1k_1t)fN=a_1X_1+\sum_{j=2}^r a_jZ_j+(b-h_1)fN$. Then $X_1, Z_2, \cdots, Z_r$ and $\partial t=fN$ (if ∂t is a global field) gives a basis of $\mathcal{G}$. ■

From Propositions 2, 3, 6, we can give now a complete description of the Lie algebra $\mathcal{G}$:

THEOREM 2. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic (not bundle-like) foliation of dimension n. Let $(\tilde{M}, \tilde{\mathcal{F}})$ denote the universal cover of $(M, \mathcal{F})$. If $(\tilde{M}, \tilde{\mathcal{F}})$ has a warped product structure, then dimension $\mathcal{G}\leqq 2+(1/2)n(n-1)$ and:*

(1) *For $(\tilde{M}, \tilde{\mathcal{F}})$ there are the following possibilities:*

CASE	*dim* $\tilde{\mathcal{G}}^n$	*dim* $\tilde{\mathcal{G}}^t$	*dim* $\tilde{\mathcal{J}}$	*dim* $\tilde{\mathcal{G}}$
A	1	m	$m+1$	$m+2$
B	1	m	m	$m+1$

$\left(\textit{with } 0\leqq m\leqq \frac{1}{2}n(n-)\right)$

(2) *and for* $(M, \mathcal{F})$:

CASE	*CASE on* $\tilde{M}$	*dim* $\mathcal{G}^n$	*dim* $\mathcal{G}^t$	*dim* $\mathcal{J}$	*dim* $\mathcal{G}$
A_1	A	1	m'	$m'+1$	$m'+2$
A_2	A	1	m'	m'	$m'+1$
A_3	A	0	m'	$m'+1$	$m'+1$
A_4	A	0	m'	m'	m'
B_1	B	1	m'	m'	$m'+1$
B_2	B	0	m'	m'	m'

$\left(\text{with } 0 \leqq m' \leqq m \leqq \frac{1}{2} n(n-1)\right)$

We will give now some examples of all cases enumerated in Theorem 2.

(A) Let $M_1 = M' \times \boldsymbol{R} \times \boldsymbol{R}$, $\mathcal{F} \leftrightarrow (M' \times \boldsymbol{R}) \times \{point\}$, $ds^2 = ds^2_{M'} + dx^2 + e^{2x}dt^2$. Here, ∂t generates $\mathcal{G}^n$, whereas $\mathcal{G}^t = i(M')$ and $\partial x - t\partial t$ is a preserving Killing field neither tangent nor orthogonal to $\mathcal{F}$. Since $\boldsymbol{R}^2$ with the metric $ds^2 = dx^2 + e^{2x}dt^2$ is isometric to the hyperbollic plane, M_1 is complete when M' is complete.

(B) Let $M_2 = M' \times \boldsymbol{R} \times \boldsymbol{R}$, $\mathcal{F} \leftrightarrow (M' \times \boldsymbol{R}) \times \{point\}$, $ds^2 = ds^2_{M'} + dx^2 + e^{2(x+\sin x)}dt^2$. Also in this case, ∂t generates $\mathcal{G}^n$ and $\mathcal{G}^t = i(M')$; but now $\mathcal{J} = \mathcal{G}^t$. It is possible to see that $\boldsymbol{R}^2$ with this metric is a complete manifold, by solving differential equations of geodesics and aplying Theorem of Peano to extend these geodesics for any value of the parameter. Thus, M_2 will be complete when M' were a complete manifold.

(A_1) Let $M = M_1/\{\phi_1\}$ where $\phi_1(p, x, t) = (\tilde{\phi}(p), x, t)$ and $\tilde{\phi}$ is an isometry of M'. Let us consider in M metric and foliation induced by those in M_1.

(A_2) Let $M = M_1/\{\phi_2\}$, where now $\phi_2(p, x, t) = (p, x, t+1)$.

(A_3) Let $M = M_1/\{\phi_3\}$, where $\phi_3(p, x, t) = (p, x+1, e^{-1}t)$.

(A_4) Let $M = M' \times T^3_A$, where M' is a complete manifold and T^3_A is the so called "hyperbollic torus" and consider the product foliation $M' \times \mathcal{F}'$, $\mathcal{F}'$ the usual codimension-one folation of T^3_A (see for instance [6]). Here, $\mathcal{G}^t = i(M')$ and there are no Killing fields in the hyperbollic torus preserving $\mathcal{F}'$.

Remark. This case A_4 may not occur in surfaces (see [8]).

(B_1) Let $M = M_2/\{\psi_1\}$, where $\psi_1(p, x, t) = (p, x, t+1)$.

(B_2) Let $M = M_2/\{\psi_2\}$, where now $\psi_2(p, x, t) = (p, x+2\pi, te^{-2\pi})$.

§ 4. Bundle-like foliations

We shall consider now the special case when $Yf=0$, $\forall Y\in T(\mathcal{F})$. With a change of the parameter t, we may assume f to be constant and consequently $\tilde{M}$ to be a riemannian product. Then it is easyly seen that $\tilde{\mathcal{F}}$ (and $\mathcal{F}$ also) is a bundle-like (totally geodesic) foliation. Conversely, when $\mathcal{F}$ is totally geodesic and bundle-like, the transverse one-dimensional foliation $\mathcal{F}^{\perp}$ is also totally geodesic and bundle-like (see [5]). It follows from Theorem A of [1] that the universal cover of M is a riemannian product:

PROPOSITION 7. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic foliation. Then $\mathcal{F}$ is a bundle-like foliation if and only if the universal cover $(\tilde{M}, \tilde{\mathcal{F}})$ of $(M, \mathcal{F})$ is a riemannian product $\tilde{L}\times\boldsymbol{R}$, foliated with leaves of the form $\tilde{L}\times\{point\}$.*

Let us consider the Lie algebra $\mathcal{G}$ of Killing fields preserving the foliation. We will see that dimension $\mathcal{G}\leqq 1+(1/2)n(n+1)$, where n=dimension $\mathcal{F}$.

PROPOSITION 8. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic, bundle-like foliation. Then*

i) $\lambda\partial t\in\mathcal{G}^n \Leftrightarrow \lambda\partial t=hN$, *$h$ a constant.*

ii) $\mathcal{G}=\mathcal{G}^t\oplus\mathcal{G}^n$.

Proof. i) Suppose that $\lambda\partial t\in\mathcal{G}^n$. Thus, by Propositions 1 and 7 it follows that $\lambda f=h$ constant and $\lambda\partial t=\lambda fN=hN$. The converse follows by the same argument.

ii) Let $X=X^t+\lambda\partial t=X^t+\lambda fN\in\mathcal{G}$. Then, by Propositions 1 and 7, λf is constant and $\lambda\partial t\in\mathcal{G}^n$. So $X^t=X-\lambda fN\in\mathcal{G}\cap T(\mathcal{F})=\mathcal{G}^t$. ■

As an obvious consequence, we have:

COROLLARY 1. *Let $(M, \mathcal{F})$ be a complete manifold with a codimension-one, totally geodesic, bundle-like foliation of dimension n. Then $1\leqq$ dimension $\mathcal{G}\leqq 1+(1/2)n(n+1)$.*

Remark. In fact, the second inequality in Corollary 1 holds for any codimension-one, bundle-like foliation, also in the non-totally geodesic case (see [8]).

Let us give some examples which prove that inequalities in Corollary 1 are as fine as possible:

The $(n+1)$-dimensional euclidean space, foliated by parallel hyperplanes, gives us an example with dimension $\mathcal{G}=1+(1/2)n(n+1)$. Here $\mathcal{G}$ is generated by $\{i(\boldsymbol{R}^n)\cup\{\partial x_{n+1}\}\}$.

Now let G be the group generated by the isometries of the euclidean 3-space ϕ and ψ, where $\phi(x, y, t)=(-x, -y, t+1)$; and $\psi(x, y, t)=(x+1, y, t)$. Let $M=$

$(\boldsymbol{R}^2\times\boldsymbol{R})/G$, with the foliation induced by the one in $\boldsymbol{R}^3$ whose leaves are of the form $\boldsymbol{R}^2\times\{point\}$. Then, $\mathcal{G}$ is generated by $\{\pi_*(\partial t)\}$ and has dimension one.

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