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On 2-microhyperbolicity at the boundary

By

Francesco TONIN and Giuseppe ZAMPIERI

§1. Statement of the result

Let M be a real analytic manifold, X a complexification of M, S a real analytic hypersurface of M, $M^{\pm}$ the two open components of $M\backslash S$. Let T^*X denote the complex tangent bundle to X endowed with the canonical 1-form α and 2-form $\sigma = d\alpha$, and let H be the Hamiltonian isomorphism. Let $T^*X^{\mathbf{R}}$ (resp. $T^*X^{\mathbf{I}}$) denote the real underlying manifold to T^*X endowed with the forms $\alpha^{\mathbf{R}} = \Re\alpha$ and $\sigma^{\mathbf{R}} = \Re\sigma$ (resp. $\alpha^{\mathbf{I}} = \Im\alpha$ and $\sigma^{\mathbf{I}} = \Im\sigma$), and let $H^{\mathbf{R}}$ (resp $H^{\mathbf{I}}$) denote the corresponding Hamiltonian isomorphisms. Let V be a smooth regular (i. e. $\alpha|_V \neq 0$) involutive submanifold of T^*_MX and denote by $\widetilde{V}$ the union of the complexifications of the bicharacteristic leaves of V. Assume there are analytic functions r, s on T^*_MX such that.

$$s|_V = r|_{S\times_M T^*_MX} = 0, \quad \{s, r\} \equiv 1.$$

Let $\widetilde{V}^{\theta}$ be the union of the integral leaves of $\Re\ (e^{\sqrt{-1}\left(\frac{\pi}{2}+\theta\right)}H_{rc})$ issued from $\widetilde{V}$ and let W denote the union of the leaves of $\Re\ (e^{\sqrt{-1}\frac{\pi}{2}}H_{rc})$ issued from V. Let $\mathscr{B}^2_{M^{\pm}|X} = \mathscr{B}^{2,W}_{M^{\pm}|X}$ be the complex of 2-hyperfunctions at the boundary along W in the sense of [U-Z]. Let $\mathcal{M}$ be a coherent $\mathscr{E}_X$-module (a pseudo-differetial system).

Theorem 1.1. *Assume that there exists $\theta \in [-\pi, \pi]$, $\theta \neq \pm\frac{\pi}{2}$ such that*

$$\pm\Re(e^{\sqrt{-1}\theta}H_{rc}) \notin C(\mathrm{char}\ \mathcal{M}, \widetilde{V}^{\theta}) \tag{1.1}$$

(with $C(\cdot,\cdot)$ being the normal cone by [K-S]*). Then*

$$\mathrm{R}\Gamma_{\pi^{-1}(S)}\mathrm{R}\mathscr{H}om(\mathcal{M}, \mathscr{B}^2_{M^{\pm}|X}) = 0. \tag{1.2}$$

Remark that $\mathscr{C}_{M^{\pm}|X}|_W \to \mathscr{B}^2_{M^{\pm}|X}$ is injective when restriced to solutions of $\mathcal{M}$. In fact (1.1) implies non-characteristicity of S for the system $\mathcal{M}$, so that [U-Z] can be applied. This gives $M^{\pm}$-regularity in the sense of [S]:

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Corollary 1.2. *Let* (1.1) *hold with* $\theta\neq\pm\frac{\pi}{2}$. *Then*

$$\Gamma_{\pi^{-1}(S)}\mathcal{H}om(\mathcal{M},\mathcal{C}_{M^{\pm}|X})|_W=0. \tag{1.3}$$

Example (a). Let $z=x+\sqrt{-1}\,y$ (resp. $(z,\zeta)=(x+\sqrt{-1}\,y,\ \xi+\sqrt{-1}\,\eta)$, resp. $(x,\sqrt{-1}\,\eta)$) be the variable in X (resp. T^*X, resp. T^*_MX), and write also $z=(z_1,z',z'')$. Let $P\ (z,\frac{\partial}{\partial z})$ be a differential operator whose principal symbol $\sigma\ (P)$ is a quadratic form of the type:

$$\sigma(P)=\zeta_1^2+A\ (z,\zeta')-B(z'',\zeta'') \tag{1.4}$$

with A, B homogeneous of degree 2 in ζ' and ζ'' respectively, B real on T^*_MX, $B|_{T^*_MX}\leq 0$, $B|_{z''=0}=0$. Set $r=x_1$, $V=\{\eta_1=\eta'=0\}$. We claim that (1.1) holds. To see this, it is enough to show that for some positive constant c:

$$|\Re\zeta_1|\leq c\,[|\zeta'|+|\xi''|+|y''|]\ \text{if}\ \sigma(P)=0 \tag{1.5}$$

In fact if $\xi''=y''=0$ than $\Re\zeta_1^2-\Im\zeta_1^2+\Re A\ (z',x'',\zeta')-B\ (x'',\sqrt{-1}\,\eta'')$ $(=\Re\sigma(P))=0$ implies $|\Re\zeta_1|\leq|\Im\zeta_1|+c|\zeta'|$. If in addition one assumes $\zeta'=0$ then $2\Re\zeta_1\Im\zeta_1\,(=\Im\sigma(P))=0$ implies $\Re\zeta_1=0$. By applying the local Bochner's tube theorem one then gets (1.5).

Thus for instance for $X=\mathbf{C}^3$, $S=\{x_1=0\}$, $r=x_1$, $V=\{\eta_1=\eta_2=0\}$ and for $P\ (z,\frac{\partial}{\partial z})=\frac{\partial^2}{\partial z_1^2}\pm\frac{\partial^2}{\partial z_2^2}-z_3^2\frac{\partial^2}{\partial z_3^2}$, (1.1) is satisfied and then (1.2) follows for both $M^{\pm}$. In particular according to (1.3) the two traces over S of an analytic solution of P on $M^{\pm}$ are microanalytic at $(0,\sqrt{-1}\ dx_3)$.

Example (b). Let us write $\zeta=(\zeta_1,\zeta',\zeta'',\zeta''')$, set $C(\zeta_1,\zeta'')=-\zeta_1^4+4\sqrt{-1}\sum_i\zeta''^2_i\zeta_1^2+\sum_i\zeta''^4_i$, take any $D\,(z,\zeta')$ homogeneous (in ζ') of degree 4, and define:

$$\sigma(P)=C(\zeta_1,\zeta'')+D(z,\zeta')$$

By the change $w_1=e^{-\sqrt{-1}\frac{\pi}{4}}\zeta_1$, C becomes $w_1^4-4\sum_i\zeta''^2_iw_1^2+\sum_i\zeta''^4_i$. This polynomial is hyperbolic (irreducible) with distinct roots $w_1=\pm\,2^{\frac{1}{2}}\,|\zeta''|\left(1\pm\left(1-\frac{\sum_i\zeta''^4_i}{4(\sum_i\zeta''^2_i)^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$ (for real ζ'').

It follows that (1.1) is verified with $V=\{\eta_1=\eta'=0\}$ and $\theta=\frac{\pi}{4}$. In particular the four traces over S of any real analytic solution of $Pu=0$ on $M^{\pm}$ are microanalytic at any $p=(0,\sqrt{-1}\,\eta''')$. Note that for the above $\sigma\ (P)$ one could not apply neither [S-Z], [U-Z], nor [D'A-T-Z].

§2. Proof of Theorem 1.1

We take symplectic coordinates $(z,\zeta)=(x+\sqrt{-1}\,y;\ \xi+\sqrt{-1}\,\eta)\in T^*X$, $(x,\sqrt{-1}\,\eta)\in T^*_MX$ such that $r=x_1$, $s=\eta_1$, V: $\eta_1=\eta'=0$. We put:

$$X=\mathbf{C}\times X'\times X''$$
$$M=\mathbf{R}\times M'\times M''$$
$$S=\{0\}\times M'\times M''$$
$$\widetilde{M}=\mathbf{C}\times X'\times M''$$
$$S^{\theta}=\mathbf{R}e^{\sqrt{-1}\theta}\times X'\times M''$$
$$\widetilde{M}_1=\mathbf{R}\times X'\times M''$$
$$\widetilde{S}_1=\{0\}\times X'\times M''.$$

We also set:

$$\widetilde{M}_1^{\pm}=\mathbf{R}^{\pm}\times X'\times M''$$
$$M^{\pm\theta}=(\widetilde{M}\setminus S^{\theta})^{\pm}.$$

It follows:

$$V=M\times_{\widetilde{M}}T^*_{\widetilde{M}}X$$
$$\widetilde{V}=T^*_{\widetilde{M}}X$$
$$W=M\times_{\widetilde{M}_1}T^*_{\widetilde{M}_1}X$$
$$\widetilde{V}^{\theta}=\left(\mathbf{C}_{z_1}\times\left(\Re e^{\sqrt{-1}\left(\frac{\pi}{2}+\theta\right)}\right)_{\zeta_1}\right)\times(X'\times\{0\})\times T^*_{M''}X''.$$

For a locally closed set $A\subset X$ and for the sheaf $\mathcal{O}_X$ of holomorphic functions on X, we shall denote by $\mu_A(\mathcal{O}_X)$ the "microfunctions along A" in the sense of [K-S].

Theorem 2.1. *Let* (1.1) *hold with* $\theta\neq k\pi$, $k\in\mathbf{Z}$; *then*

$$\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{\widetilde{M}_1}(\mathcal{O}_X))\xrightarrow{\sim}\mathrm{R}\Gamma_{\pi^{-1}(\widetilde{M}_1)}\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{\widetilde{M}}(\mathcal{O}_X))\tag{2.1}$$
$$\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{\widetilde{S}_1}(\mathcal{O}_X)\xrightarrow{\sim}\mathrm{R}\Gamma_{\pi^{-1}(\widetilde{S}_1)}\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{S^{\theta}}(\mathcal{O}_X))\tag{2.2}$$

whence

$$\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{\widetilde{M}_1^{\pm}}(\mathcal{O}_X)\xrightarrow{\sim}\mathrm{R}\Gamma_{\pi^{-1}(\widetilde{M}_1)}\mathrm{R}\mathcal{H}om(\mathcal{M},\mu_{M^{\pm\theta}}(\mathcal{O}_X))\tag{2.3}$$

Proof. According to [K-S, Th. 5.4.1] it is enough to prove that:

$$H^{\mathbf{R}}(\pi^*\dot{T}^*_{\widetilde{M}_1}\widetilde{M})\cap C(\mathrm{char}\,\mathcal{M},T^*_{\widetilde{M}}X)=\emptyset\tag{2.4}$$
$$H^{\mathbf{R}}(\pi^*\dot{T}^*_{\widetilde{S}_1}S^{\theta})\cap C(\mathrm{char}\,\mathcal{M},T^*_{S^{\theta}}X)=\emptyset\tag{2.5}$$

But (2.4) is equivalent for some $c>0$ and for $(z,\zeta)\in\mathrm{char}\,\mathcal{M}$ to:

$$|\eta_1|\leq c\,[|\xi_1|+|\zeta'|+|\xi''|+|y''|];\tag{2.4'}$$

(2.5) is equivalent to:

$$|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|\leq c\,[|\Re(e^{\sqrt{-1}\left(\frac{\pi}{2}+\theta\right)}\overline{z_1})|+|\zeta'|+|\xi''|+|y''|];\tag{2.5'}$$

(1.1) is equivalent to:

$$|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|\leq c\,[|\zeta'|+|\xi''|+|y''|].\tag{1.1'}$$

Obviously (1.1′)⇒(2.5′). Finally (1.1′)⇒(2.4′) due to the following Lemma (applied for $\psi=\frac{\pi}{2}$ and with $\theta\in[0,\pi[$).

Lemma 2.2. *Let* $\psi\neq\theta\pm\frac{\pi}{2}$. *Then* (1.1′) *implies, for some* $c>0$ *and for* $(z,\zeta)\in$ char $\mathcal{M}$:

$$|\Re e^{\sqrt{-1}\psi}\bar{\zeta}_1|\leq c[|\Re e^{\sqrt{-1}(\frac{\pi}{2}+\psi)}\bar{\zeta}_1|+|\zeta'|+|\xi''|+|y''|].$$

Proof. We have

(2.6)
$$\begin{aligned}\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\psi)}\bar{\zeta}_1)&=\cos(\tfrac{\pi}{2}+\psi-\theta)\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)+\sin(\tfrac{\pi}{2}+\psi-\theta)\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)\\&=-\sin(\psi-\theta)\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)+\cos(\psi-\theta)\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)\\&=\cos(\psi-\theta)(\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1-\mathrm{tg}(\psi-\theta)\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)),\end{aligned}$$

(the last equality follows from $\psi\neq\theta\pm\frac{\pi}{2}$). Assume that (1.1′) is fulfilled. Let $|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)|\leq 2\,\mathrm{tg}\,(\psi-\theta)|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|$; then $|\bar{\zeta}_1|\leq|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|+|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)|\leq c[|\zeta'|+|\xi'|+|y''|]$. On the other hand assume $|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)|\geq 2\,\mathrm{tg}\,(\psi-\theta)|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|$; then by (2.6):

$$|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\psi)}\bar{\zeta}_1)|\geq\frac{\cos(\psi-\theta)}{2}|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)|,$$

which implies:

$$\begin{aligned}|\bar{\zeta}_1|&\leq|\Re(e^{\sqrt{-1}\theta}\bar{\zeta}_1)|+|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\theta)}\bar{\zeta}_1)|\\&\leq c[|\zeta'|+|\xi''|+|y''|]+\frac{2}{\cos(\psi-\theta)}|\Re(e^{\sqrt{-1}(\frac{\pi}{2}+\psi)}\bar{\zeta}_1)|.\end{aligned}$$

Let now $\tilde{X}=(\mathbf{C}\times\mathbf{C})\times X'\times X''\ni(x_1^{\mathbf{C}},y_1^{\mathbf{C}},z',z'')$ be a partial complexification of X and let $T^*\tilde{X}\ni(\tilde{z},\tilde{\zeta})=(x_1^{\mathbf{C}},y_1^{\mathbf{C}},z',z'';\xi_1^{\mathbf{C}},\eta_1^{\mathbf{C}},\zeta',\zeta'')$ be the cotangent bundle to $\tilde{X}$. Let us cosider the embedding j: $X\hookrightarrow\tilde{X}$ defined by $x_1+\sqrt{-1}\,y_1\mapsto(x_1,y_1)$ and the submersion ϕ: $\tilde{X}\to X$ defined by $(x_1^{\mathbf{C}},y_1^{\mathbf{C}})\mapsto x_1^{\mathbf{C}}+\sqrt{-1}\,y_1^{\mathbf{C}}$. We remark that $\phi\circ j=\mathrm{id}_X$ and that

(2.7)
$$j^{-1}(\mathcal{O}_{\tilde{X}}^{\mathcal{M}\otimes\bar{\partial}z_1})=\mathcal{O}_X^{\mathcal{M}}$$

and

(2.8) char $(\mathcal{M}\otimes\bar{\partial}z_1)={}^t\phi'$ char $(\mathcal{M})$

$$=\{(\tilde{z},\tilde{\zeta})\in T^*\tilde{X};\ (x_1^{\mathbf{C}}+\sqrt{-1}y_1^{\mathbf{C}},z',z'';\xi_1^{\mathbf{C}},\zeta',\zeta'')\in\text{char}\,\mathcal{M},\ \eta_1^{\mathbf{C}}=\sqrt{-1}\xi_1^{\mathbf{C}}\}.$$

Put

$$\begin{aligned}&\tilde{M}^\theta=\mathbf{C}(\cos\theta,\sin\theta)+\mathbf{R}(-\sin\theta,\cos\theta)\times X'\times M''\\&\tilde{S}^\theta=\mathbf{C}(\cos\theta,\sin\theta)\times X'\times M''\\&\tilde{M}^{\pm\theta}=(\tilde{M}^\theta\backslash\tilde{S}^\theta)^\pm.\end{aligned}$$

We have a commuting diagram

$$\begin{array}{ccc} X & \to & \tilde{X} \\ \uparrow & & \uparrow \\ \tilde{M} & \to & \tilde{M}^\theta \\ \uparrow & & \uparrow \\ S^\theta & \to & \tilde{S}^\theta \end{array}$$

where all the arrows are injective. Let $T^*X \overset{\rho}{\leftarrow} X \times_{\tilde{X}} T^*\tilde{X} \overset{\tilde{\omega}}{\to} T^*\tilde{X}$ be the mappings canonically associated to the embedding $X \hookrightarrow \tilde{X}$. Note that ρ is injective over $\tilde{\omega}^{-1}(\text{char}\ \bar{\partial}_{z_1})$.

Theorem 2.3. *Assume* (1.1) *holds with* $\theta \neq k\frac{\pi}{2}$, $k \in \mathbf{Z}$. *Then*

(2.9)

$$\mathrm{R}\mathscr{H}om(\mathcal{M}, \mu_{\tilde{M}}(\mathcal{O}_X)) \otimes or_{X|\tilde{X}}[-2] \simeq \mathrm{R}\rho_*\tilde{\omega}^{-1}\mathrm{R}\Gamma_{\pi^{-1}(\tilde{M})}\mathrm{R}\mathscr{H}om(\mathcal{M}\otimes\bar{\partial}z_1, \mu_{\tilde{M}^\theta}(\mathcal{O}_{\tilde{X}}))$$

(2.10)

$$\mathrm{R}\mathscr{H}om(\mathcal{M}, \mu_{S^\theta}(\mathcal{O}_X)) \otimes or_{X|\tilde{X}}[-2] \simeq \mathrm{R}\rho_*\tilde{\omega}^{-1}\mathrm{R}\Gamma_{\pi^{-1}(S^\theta)}\mathrm{R}\mathscr{H}om(\mathcal{M}\otimes\bar{\partial}z_1, \mu_{\tilde{S}^\theta}(\mathcal{O}_{\tilde{X}}))$$

which implies

(2.11)

$$\mathrm{R}\mathscr{H}om(\mathcal{M}, \mu_{M^{*\theta}}(\mathcal{O}_X)) \otimes or_{X|\tilde{X}}[-2] \simeq \mathrm{R}\rho_*\tilde{\omega}^{-1}\mathrm{R}\Gamma_{\pi^{-1}(\tilde{M}_1)}\mathrm{R}\mathscr{H}om(\mathcal{M}\otimes\bar{\partial}z_1, \mu_{\tilde{M}^{*\theta}}(\mathcal{O}_{\tilde{X}})).$$

Proof. We shall assume $\theta \in]0, \pi[$, $\theta \neq \frac{\pi}{2}$ in the proof. By [K-S, Th. 5.4.1] one needs to show that

(2.12) $$H^{\mathbf{R}}(\pi^*\dot{T}^*_{\tilde{M}}\tilde{M}^\theta) \cap C(\text{char}\ \mathcal{M}\otimes\bar{\partial}z_1, T^*_{\tilde{M}^\theta}\tilde{X}) = \emptyset$$

(2.13) $$H^{\mathbf{R}}(\pi^*\dot{T}^*_{\tilde{S}^\theta}\tilde{S}^\theta) \cap C(\text{char}\ \mathcal{M}\otimes\bar{\partial}z_1, T^*_{\tilde{S}^\theta}\tilde{X}) = \emptyset.$$

But (2.12) is equivalent, for $(\tilde{z}, \tilde{\zeta}) \in \text{char}\ (\mathcal{M}\otimes\bar{\partial}z_1)$, to:

(2.12′)

$$|\cos\theta\Im\xi_1^{\mathbf{C}}+\sin\theta\Im\eta_1^{\mathbf{C}}| \le c\,[|\cos\theta\Re\xi_1^{\mathbf{C}}+\sin\theta\Re\eta_1^{\mathbf{C}}| \\ +|-\sin\theta\Re\xi_1^{\mathbf{C}}+\cos\theta\Re\eta_1^{\mathbf{C}}|+|-\sin\theta\Im x_1^{\mathbf{C}}+\cos\theta\Im y_1^{\mathbf{C}}|+|\zeta'|+|\zeta''|+|y''|],$$

and (2.13) to:

(2.13′)

$$|\cos\theta\Im\xi_1^{\mathbf{C}}+\sin\theta\Im\eta_1^{\mathbf{C}}| \le c\,[|\cos\theta\Re\xi_1^{\mathbf{C}}+\sin\theta\Re\eta_1^{\mathbf{C}}|+ \\ |-\sin\theta x_1^{\mathbf{C}}+\cos\theta y_1^{\mathbf{C}}|+|\zeta'|+|\xi''|+|y''|].$$

Recall (2.8); in particular $\xi_1^{\mathbf{C}}$ and $\eta_1^{\mathbf{C}}$ are related by $\eta_1^{\mathbf{C}} = \sqrt{-1}\,\xi_1^{\mathbf{C}}$ and thus (2.12′) is trivial. As for (2.13′), this easily follows from

(2.14) $$\pm\Re(e^{\sqrt{-1}(\frac{\pi}{2}-\theta)}\partial_{\zeta_1}) \notin C(\text{char}\ \mathcal{M}, T^*_{\tilde{M}}X)$$

which holds by Lemma 2.2 applied with $\theta \in [0, \pi[$ and for $\psi = \frac{\pi}{2} - \theta \neq \theta \pm \frac{\pi}{2}$ (due to $\theta \neq 0, \frac{\pi}{2}$).

End of Proof of Theorem 1.1 (a) Let $\theta \in [0, \pi[$, $\theta \neq 0, \frac{\pi}{2}$. Using (2.3), (2.11) we get:

(2.15)

$$R\mathcal{H}om(\mathcal{M}, \mu_{\tilde{M}_1^\pm}(\mathcal{O}_X)) \otimes or_{X|\tilde{X}}[-2] \simeq R\rho_* \bar{\omega}^{-1} R\Gamma_{\pi^{-1}(\tilde{M}_1)} R\mathcal{H}om(\mathcal{M} \otimes \bar{\partial} z_1, \mu_{\tilde{M}^{\pm\theta}}(\mathcal{O}_{\tilde{X}})).$$

On the other hand, set $\pm w = \pi^* \dot{T}^*_{\tilde{S}^\theta} \tilde{M}^\theta$; then we have:

$$H_{\mathbf{R}}(\pm w) \notin C(\text{char } \mathcal{M} \otimes \bar{\partial} z_1, \text{SS } \mathbf{Z}_{\tilde{M}^{\pm\theta}}) \tag{2.16}$$

(with SS denoting the microsupport in the sense of [K-S]). In fact we have SS $\mathbf{Z}_{\tilde{M}^{\pm\theta}} = (T^*_{\tilde{M}^\theta} \tilde{X})^{\pm} \mp \mathbf{R}^+ H^{\mathbf{R}}(w)$ where we have put $(T^*_{\tilde{M}^\theta} \tilde{X})^{\pm} \overset{\text{def.}}{=} \tilde{M}^{\pm\theta} \times_{\tilde{M}^\theta} T^*_{\tilde{M}^\theta} \tilde{X}$. On the other hand, if Γ is an open convex conic neighborhood of $H^{\mathbf{R}}(w)$ such that $((T^*_{\tilde{M}^\theta} \tilde{X})^{\pm} \mp \Gamma) \cap \text{char}(\mathcal{M} \otimes \bar{\partial} z_1) = \emptyset$, then also $(((T^*_{\tilde{M}^\theta} \tilde{X})^{\pm} \mp \mathbf{R}^+ H^{\mathbf{R}}(w)) \mp \Gamma) \cap \text{char}(\mathcal{M} \otimes \bar{\partial} z_1) = \emptyset$ due to $\mathbf{R}^+ H^{\mathbf{R}}(w) + \Gamma \subset \Gamma$.

By the theory of the propagation by [K-S] one gets from (2.16):

$$R\Gamma_{\pi^{-1}(\tilde{S}^\theta)} R\mathcal{H}om(\mathcal{M} \otimes \bar{\partial} z_1, \mu_{\tilde{M}^{\pm\theta}} \mathcal{O}_{\tilde{X}}) = \emptyset. \tag{2.17}$$

Thus applying the functor $R\Gamma_{\pi^{-1}(M)}(\cdot) \otimes or_{M|\tilde{X}}[n+2]$, using (2.15), and recalling that $\mathcal{B}^2_{M^+|X} \overset{\text{def.}}{=} R\Gamma_{\pi^{-1}(M)} \mu_{\tilde{M}_1}(\mathcal{O}_X) \otimes or_{M|X}[n]$ one gets (1.2).

(b) Let $\theta = 0$ (cf. [U-Z]). By the theory of propagation of [K-S] we get immediately in this case:

$$R\Gamma_{\pi^{-1}(\tilde{S}_1)} R\mathcal{H}om(\mathcal{M}, \mu_{\tilde{M}_1^\pm}(\mathcal{O}_X)) = 0.$$

DIP DI MATEMATICA,
UNIV. DI PADOVA,
VIA BELZONI 7,
35131 PADOVA,
ITALY

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