On ideal transforms of noetherian rings, II*

By

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2. Finiteness or integrality of global transforms of local rings and its applications

Proposition (2.1) *Let $(A, \mathfrak{m}_1, \dots, \mathfrak{m}_r)$ be a semi-local ring with radical $\mathfrak{m} = \mathfrak{m}_1 \cdots \mathfrak{m}_r$ (containing a non-zero-divisor). Then*

(2.1.1) *$A_{\text{red}}(\mathfrak{m})$ is integral over $A(\mathfrak{m})_{\text{red}}$.*

(2.1.2) *$A_{\text{red}}(\mathfrak{m})$ is integral over A_{red} if and only if $A(\mathfrak{m})$ is integral over A .*

Let $(B, \mathfrak{n}_1, \dots, \mathfrak{n}_s)$ be a semi-local A -algebra with radical $\mathfrak{n} = \mathfrak{n}_1 \cdots \mathfrak{n}_s$, contained in $Q(A)$. Suppose $\text{rad}(\mathfrak{m}B) = \mathfrak{n}$. Then

(2.1.3) *$B_{\text{red}}(\mathfrak{n})$ is integral over $A_{\text{red}}(\mathfrak{m})$ if and only if $B(\mathfrak{n})$ is integral over $A(\mathfrak{m})$.*

Proof. Take any $\bar{x}$ in $A_{\text{red}}(\mathfrak{m})$. Since $\mathfrak{m}$ contains a non-zero-divisor in A , there exists y in $Q(A)$ such that $\bar{y}$ (=the image of y in $Q(A_{\text{red}})$) = $\bar{x}$. As $\mathfrak{m}^\nu \bar{x}$ is contained in A_{red} for some ν , $\mathfrak{m}^\nu y$ is contained in $A + \text{nil-rad}(Q(A))$. Hence, if $\mathfrak{m}^\nu = a_1 A + \cdots + a_k A$, $a_j y = y_j$ is integral over A . Let $A_1 = A[y_1, \dots, y_k]$, then there exists a non-zero-divisor d in A such that $\frac{1}{d}A$ contains A_1 . Hence y^μ is in $A_1(\mathfrak{m}A_1) \subset \frac{1}{d}A(\mathfrak{m})$ for any μ . Therefore $\bar{y}^\mu = \bar{x}^\mu$ is in $\frac{1}{d}A_{\text{red}}(\mathfrak{m})$ for any μ . Since $A(\mathfrak{m})_{\text{red}}$ is noetherian [3], $\bar{x}$ is integral over $A(\mathfrak{m})_{\text{red}}$. This proves (2.1.1).

(2.1.2) and (2.1.3) can be proved easily by (2.1.1).

Proposition (2.2) *Let $(A, \mathfrak{m}_1, \dots, \mathfrak{m}_r)$, $(B, \mathfrak{n}_1, \dots, \mathfrak{n}_s)$ be semi-local rings with common total quotient ring. If B is integral over A , then $B(\mathfrak{n})$ is integral over $A(\mathfrak{m})$.*

By (2.1.3), to prove Proposition (2.2), it is sufficient to prove the following:

Proposition (2.2') *Let $(A, \mathfrak{m}_1, \dots, \mathfrak{m}_r)$, $(B, \mathfrak{n}_1, \dots, \mathfrak{n}_s)$ be semi-local domains with common field of quotients. If B is integral over A , then $B(\mathfrak{n})$ is integral over $A(\mathfrak{m})$.*

Proof. Let $B = \varinjlim_{\lambda} B_{\lambda}$, where B_{λ} is a finite A -algebra (with radical $\mathfrak{n}_{\lambda}$).

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Then, since $\mathfrak{n}$ is finitely generated, $B(\mathfrak{n}) = \varinjlim B_\lambda(\mathfrak{n}_\lambda)$. Therefore, as $B_\lambda(\mathfrak{n}_\lambda)$ is a finite $A(\mathfrak{m})$ -module (cf. Lemma (1.4)), $B(\mathfrak{n})$ is integral over $A(\mathfrak{m})$.

Proposition (2.3) *Let $(A, \mathfrak{m})$ be a semi-local ring. Then*

(2.3.1) *the following are equivalent:*

- a) $A(\mathfrak{m})$ is integral over A .
- b) $\dim \hat{A}/\hat{\mathfrak{p}} > 1$ for any minimal prime ideal $\hat{\mathfrak{p}}$ in $\hat{A}$.

(2.3.2) *the following are equivalent:*

- c) $A(\mathfrak{m})$ is a finite A -module.
- d) $\dim \hat{A}/\hat{\mathfrak{p}} > 1$ for any associated prime $\hat{\mathfrak{p}}$ of $\hat{A}$.

Proof. By Lemma (1.1), we may assume A is complete.

1) a) $\Leftrightarrow$ b): By (2.1.2), we may assume A is reduced. The equivalence follows from Corollary (1.7) (cf. [6, Proposition 4]).

2) c) $\Rightarrow$ d): i) Since A and $A(\mathfrak{m})$ have common total quotient ring, there exists a canonical one-to-one correspondence between $\text{Ass}(A)$ and $\text{Ass}(A(\mathfrak{m}))$. Hence, to prove c) $\Rightarrow$ d), it is sufficient to show that $\dim A(\mathfrak{m})/\mathfrak{p}' > 1$ for any $\mathfrak{p}'$ in $\text{Ass}(A(\mathfrak{m}))$.

ii) Since $A(\mathfrak{m})$ is a finite A -module, $A(\mathfrak{m})(\mathfrak{m}) = A(\mathfrak{m})$. Hence $\text{prof}(A(\mathfrak{m})_{\mathfrak{m}'}) > 1$ for any maximal ideal $\mathfrak{m}'$ in $A(\mathfrak{m})$. Therefore $\dim A(\mathfrak{m})/\mathfrak{p}' > 1$ for any $\mathfrak{p}'$ in $\text{Ass}(A(\mathfrak{m}))$ (cf. [4, (15. F)]).

3) d) $\Rightarrow$ c): i) Let x be a non-zero-divisor of A contained in $\mathfrak{m}$. Since A is $(x$ -adic) complete, to prove d) $\Rightarrow$ c), it is sufficient to show that $A(\mathfrak{m})/xA(\mathfrak{m})$ is a finite A -module and that $\bigcap_{n=1}^{\infty} x^n A(\mathfrak{m}) = (0)$ (cf. [4, Lemma, p. 212]).

ii) $A(\mathfrak{m})/xA(\mathfrak{m})$ is a finite A -module by THEOREM.

iii) Since $\dim A/\mathfrak{p}_i > 1$ for any $\mathfrak{p}_i$ in $\text{Ass}(A)$ ($i=1, \dots, s$), there exists a non-maximal prime ideal $\mathfrak{q}_i$ such that $\mathfrak{q}_i \supset (\mathfrak{p}_i, xA)$ for any i . Hence the canonical homomorphism $A(\mathfrak{m}) \rightarrow \prod_{i=1}^s A(\mathfrak{m})_{\mathfrak{q}_i}$ is injective. So, to prove $\bigcap_{n=1}^{\infty} x^n A(\mathfrak{m}) = (0)$, it is sufficient to show that $\bigcap_{n=1}^{\infty} x^n A(\mathfrak{m})_{\mathfrak{q}_i} = (0)$ for any i .

iv) Since $\mathfrak{q}_i$ is non-maximal, $A(\mathfrak{m})_{\mathfrak{q}_i} = A_{\mathfrak{q}_i}$ by Lemma (1.1). Therefore $\bigcap_{n=1}^{\infty} x^n A(\mathfrak{m})_{\mathfrak{q}_i} = \bigcap_{n=1}^{\infty} x^n A_{\mathfrak{q}_i} = (0)$.

Corollary (2.4) *Let $(A, \mathfrak{m}_1, \dots, \mathfrak{m}_r)$ be a semi-local ring with radical $\mathfrak{m} = \mathfrak{m}_1 \cdots \mathfrak{m}_r$ (containing a non-zero-divisor). Then*

(2.4.1) *the following are equivalent:*

- a) $A(\mathfrak{m})$ is integral over A .
- b) $\dim \hat{A}/\hat{\mathfrak{p}} > 1$ for any minimal prime ideal $\hat{\mathfrak{p}}$ in $\hat{A}$.

(2.4.2) *the following are equivalent:*

- c) $A(\mathfrak{m})$ is a finite A -module.
- d) $\dim \hat{A}/\hat{\mathfrak{p}} > 1$ for any associated prime $\hat{\mathfrak{p}}$ of $\hat{A}$.

Proposition (2.5) ([5, (33.11)], cf. [6, Proposition 7]) *Let $\bar{\mathfrak{p}}$ be a height one prime ideal of the derived normal ring of a noetherian domain A . Then $\mathfrak{p} = \bar{\mathfrak{p}} \cap A$*

is an associated prime ideal of a principal ideal of A (i.e. $\text{prof } A_{\mathfrak{p}}=1$).

Proof. We may assume A is local with maximal ideal $\mathfrak{p}$. Then $A(\mathfrak{p})$ is not integral over A by Corollary (1.7). Hence $\hat{A}$ has a minimal prime ideal $\hat{\mathfrak{p}}$ such that $\dim \hat{A}/\hat{\mathfrak{p}}=1$ by (2.3.1). Therefore $\text{prof } A=1$ (cf. [4, (15. F)]).

Theorem (2.6) *Let A be a noetherian ring and I an ideal of A (containing a non-zero-divisor x). Then*

(2.6.1) *the following are equivalent:*

- a) $A(I)$ is integral over A .
- b) $A_{\mathfrak{p}}(\mathfrak{p})$ is integral over $A_{\mathfrak{p}}$ for any prime ideal $\mathfrak{p}$ of A containing I .

(2.6.2) *the following are equivalent;*

- c) $A(I)$ is a finite A -module.
- d) $A_{\mathfrak{p}}(\mathfrak{p})$ is a finite $A_{\mathfrak{p}}$ -module for any prime ideal $\mathfrak{p}$ of A containing I .
- e) $A_{\mathfrak{p}}(\mathfrak{p})$ is a finite $A_{\mathfrak{p}}$ -module for any prime $\mathfrak{p}$ in $\text{Ass}(A/xA)$ containing I .

Proof. 1) b) $\Rightarrow$ a): i) Take any a in $A(I)$. We construct a sequence (A_i, I_i) ($i=0, 1, 2, \dots$) of pairs of a finite A -algebra A_i and a semi-prime ideal I_i of A_i such that $A_0=A$, $I_0=\text{rad}(I)$, $A_{i-1} \subset A_i \subset Q(A)$, $a \in A_i(I_i)$, A_i satisfies b) for I_i and that, if $I_{i-1} \cap A \not\subseteq A$, $I_{i-1} \cap A \subseteq I_i \cap A$ ($i=1, 2, \dots$).

ii) Suppose $I_{i-1} \not\subseteq A_{i-1}$. Let $S = A_{i-1} - \bigcup_{j=1}^s \mathfrak{p}_j$, where $I_{i-1} = \mathfrak{p}_1 \cap \dots \cap \mathfrak{p}_s$. By inductive construction, $S^{-1}A_{i-1}(I_{i-1})$ is integral over $S^{-1}A_{i-1}$ and $a \in S^{-1}A_{i-1}(I_{i-1})$. Hence, there exists a finite A_{i-1} -algebra A_i such that $a \in S^{-1}A_i$. Put $I_i = \text{rad}((A_i : a)_{A_i})$. Then $a \in A_i(I_i)$, A_i satisfies b) for I_i and $I_{i-1} \cap A \subseteq I_i \cap A$ by Proposition (2.2) and Lemma (1.1).

iii) As A is noetherian, there exists a natural number $k > 0$ such that $I_k = A_k$. Since $a \in A_k(I_k) = A_k$, a is integral over A .

2) e) $\Rightarrow$ c): By Lemma (1.1), $A(I) \subset A_x$ and $A(I) \otimes_A A^* = A^*(IA^*)$, where A^* is the x -adic completion of A . Hence, to prove e) $\Rightarrow$ c), we may assume A is x -adic complete (cf. [4, Lemma 1, p. 7], [5, (8.9)]).

We claim: If A satisfies e), $A(I)/xA(I)$ is a finite A -module and $\bigcap_{n=0}^{\infty} x^n A(I) = (0)$ (cf. [4, Lemma, p. 212]).

i) $\bigcap_{n=0}^{\infty} x^n A_{\mathfrak{p}}(I) = (0)$ for any prime ideal $\mathfrak{p}$ of A containing x (consequently, as $A(I)$ is contained in $Q(A)$, $\bigcap_{n=0}^{\infty} x^n A(I) = (0)$): It is sufficient to show that, for any associated prime $\mathfrak{q}_0$ of A , there exists a prime $\mathfrak{q}$ of A , containing $\mathfrak{q}_0$ and x , such that $\bigcap_{n=0}^{\infty} x^n A_{\mathfrak{q}}(I) = (0)$.

Take a minimal prime $\mathfrak{q}$ of $(\mathfrak{q}_0, xA)$. Then $\mathfrak{q}$ is in $\text{Ass}(A/xA)$ by [4, Lemma 4, p. 99]. Suppose $\mathfrak{q} \supset I$, then $\dim A_{\mathfrak{q}}/\mathfrak{q}_0 > 1$ by (2.3.2). This contradicts the choice of $\mathfrak{q}$ (Hauptidealsatz), so $\mathfrak{q} \not\supset I$, then $A_{\mathfrak{q}}(I) = A_{\mathfrak{q}}$, and we get the assertion.

ii) $A(I)/xA(I)$ is a finite A -module: (We imitate PROOF)

1°) For any a in $A(I)$, there exists a $k > 0$ such that $a \in Ax^{-k} + xA(I)$: Because $A(I)$ is contained in A_x .

2°) Let $J_k = (x^k A(I) \cap A, xA)$ be a descending chain of ideals of A . We

claim:

(*) For $h \gg 0$, $J_h = xA$.

To prove (*), it is sufficient to show:

(*)_p For $h \gg 0$, $x^h A(I)_p \cap A_p \subset xA_p$, whenever p is in $\text{Ass}(A/xA)$.

If $p \supset I$, then $A(I)_p = A_p$. Hence (*)_p is clear. Suppose $p \not\supset I$. We may assume (*)_q is valid for any prime $q(\subsetneq p)$. Then $A_q(I)/xA_q(I)$ is a finite A_q -module (cf. PROOF) and, if q contains x , $\bigcap_{n=1}^{\infty} x^n A_q(I) = (0)$. Hence we may assume, moreover, $A_q(I)$ is a finite A_q -module for any prime $q(\not\supseteq p)$.

Now by assumption, there exists a finite A -algebra B such that $B_p = A_p(p)$. Then, since $A_p(p)$ is a finite A_p -module, $\text{prof}(B_{p'}) > 1$ for any maximal ideal p' of B_p . Hence $q' \cap A = q \not\supseteq p$ for any q' in $\text{Ass}(B_p/xB_p)$. Then, as $A_q(I)$ is a finite A_q -module, $B_{q'}(IB)$ is a finite $B_{q'}$ -module. Therefore, for any given $i > 0$, there exists a $h(i)$ such that $x^{h(i)} B_p(I) \cap B_p \subset x^i B_p$. Since B_p is a finite A_p -module, there exists a $j > 0$ such that $x^j B_p \cap A_p \subset xA_p$ (Artin-Rees). Therefore $x^{h(j)} A_p(I) \cap A_p \subset x^{h(j)} B_p(I) \cap B_p \cap A_p \subset xA_p$.

3°) By 1°) and 2°), $A(I)/xA(I)$ is a finite A -module.

Proposition (2.7) (cf. [1, Proposition 1.1]) *Let (A, m) be a local ring and I an ideal of A containing a non-zero-divisor x . Then*

(2.7.1) *the following are equivalent:*

- a) $A(I)$ is integral over A .
- b) $\dim \hat{A}_{\hat{q}}/\hat{p} > 1$ for any prime ideal $\hat{q}$ of $\hat{A}$ containing $I\hat{A}$ and for any minimal prime $\hat{p}$ of $\hat{A}$ contained in $\hat{q}$.

(2.7.2) *the following are equivalent:*

- c) $A(I)$ is a finite A -module.
- d) $\dim \hat{A}_{\hat{q}}/\hat{p} > 1$ for any prime ideal $\hat{q}$ of $\hat{A}$ containing $I\hat{A}$ and for any associated prime $\hat{p}$ of $\hat{A}$ contained in $\hat{q}$.

Proof. 0) To prove (2.7.1) and (2.7.2), we may assume A is complete (cf. Lemma 1.1)).

1) a) $\Rightarrow$ b) follows from (2.6.1) and (2.3.1).

2) c) $\Rightarrow$ d) follows from (2.6.2) and (2.3.2).

3) Since a complete local domain satisfies the second chain condition (cf. [5, (34.4)]), b) $\Rightarrow$ a) follows from (2.6.1), (2.1.2) and Corollary (1.7) (cf. Lemma (1.1)).

4) Moreover, since any localization (with respect to a prime ideal) of a complete local domain is analytically unramified (cf. [5, (36.3)]), d) $\Rightarrow$ c) follows from (2.6.2) and 3) (cf. [4, (9.B)]).

3. Derived normal rings of noetherian domains

Proposition (3.1) (cf. [2, p. 74]) *Let A be a noetherian domain and x a non-zero element of A . Suppose A_x is normal and $\bar{A}_p$ is a finite A_p -module for any p in $\text{Ass}(A/xA)$, then $\bar{A}$ is a finite A -module.*

Proof. Let $M = \bar{A} \otimes_A A^*$, where A^* is the x -adic completion of A . Since

A_x is normal, to prove the assertion, it is sufficient to show that M is a finite A^* -module (cf. [4, Lemma 1, p. 7], [5, (8.9)]).

We claim: $M/xM(=\bar{A}/x\bar{A})$ is a finite $A^*/xA^*(=A/xA)$ -module and $\bigcap_{n=0}^{\infty} x^n M = (0)$ (cf. [4, Lemma, p. 212]).

1) $\bar{A}/x\bar{A}$ is a finite (A/xA) -module: (We imitate PROOF)

1°) For any a in $\bar{A}$, there exists a $k > 0$ such that $a \in Ax^{-k} + x\bar{A}$: Because A_x is normal.

2°) Let $I_h = (x^h \bar{A} \cap A, xA)$. Then the descending chain of ideals I_h stabilizes for some h : By assumption, there exists a $m_0 > 0$ such that $x^m \bar{A}_{\mathfrak{p}} \cap A_{\mathfrak{p}} \subset xA_{\mathfrak{p}}$ for any $m > m_0$ and for any associated prime $\mathfrak{p}$ of A (Artin-Rees). Hence $I_h = xA$ for any $h > m_0$.

3°) By PROOF, 1°), 2°) give the assertion.

2) $\bigcap_{n=0}^{\infty} x^n M = (0)$: Since M is contained in the integral closure of A^* in its total quotient ring, to get the claim, it is sufficient to show that A^* is reduced (cf. [5, (33.10)]). This follows from the canonicol injection:

$$A/x^n A \hookrightarrow \prod_{\mathfrak{p} \in \text{Ass}(A/xA)} A_{\mathfrak{p}}/x^n A_{\mathfrak{p}} \quad \text{for any } n.$$

Proposition (3.2) *Let A be a noetherian domain and x a non-zero element of A . Suppose A_x is normal and $A_{\mathfrak{p}}^* = \varprojlim_n A_{\mathfrak{p}}/x^n A_{\mathfrak{p}}$ (=the x -adic completion of $A_{\mathfrak{p}}$) is reduced for any $\mathfrak{p}$ in $\text{Ass}(A/xA)$. Then $\bar{A}$ is a finite A -module.*

Proof. By Proposition (3.1), it is enough to show that $\bar{A}_{\mathfrak{p}}$ is a finite $A_{\mathfrak{p}}$ -module for any $\mathfrak{p}$ in $\text{Ass}(A/xA)$.

If $\text{ht } \mathfrak{p} = 1$, then $\bar{A}_{\mathfrak{p}}$ is a finite $A_{\mathfrak{p}}$ -module by [5, (32.2)]. Suppose $\text{ht } \mathfrak{p} > 1$. We may assume that A is local with maximal ideal $\mathfrak{p}$ and $\bar{A}_{\mathfrak{q}}$ is a finite $A_{\mathfrak{q}}$ -module for any non-maximal prime $\mathfrak{q}$ of A . Let $(B, \mathfrak{n}_1, \dots, \mathfrak{n}_t)$ be a well-branched overring of A . Suppose $\dim B_{\mathfrak{n}_i} > 1$ iff $i \leq s (\leq t)$. Put $C = B_S$, where $S = B - \bigcup_{i \leq s} \mathfrak{n}_i$. Then $(C, \mathfrak{n}_1, \dots, \mathfrak{n}_s)$ is a semi-local domain as in Proposition (1.2). We have

$B^* = B \otimes_A A^* = \varprojlim_n B/x^n B = \varprojlim_n C/x^n C \oplus \varprojlim_n B_{\mathfrak{n}_{s+1}}/x^n B_{\mathfrak{n}_{s+1}} \oplus \dots \oplus \varprojlim_n B_{\mathfrak{n}_t}/x^n B_{\mathfrak{n}_t}$
 $= C^* \oplus B_{\mathfrak{n}_{s+1}}^* \oplus \dots \oplus B_{\mathfrak{n}_t}^*$. Since A^* is reduced, $B^*, C^*, B_{\mathfrak{n}_j}^* (j > s)$ are all reduced. Consequently, $\bar{B}_{\mathfrak{n}_j}$ is a finite $B_{\mathfrak{n}_j}$ -module for any $j (> s)$ (cf. [5, (32.2)]). Since C^* is reduced and $C(\mathfrak{n})/xC(\mathfrak{n})$ is a finite C -module (cf. THEOREM), $C(\mathfrak{n})$ is a finite C -module (cf. Proof of Proposition (3.1)).

Hence, to prove $\bar{A}$ is a finite A -module, it is sufficient to show $\overline{C(\mathfrak{n})}$ is a finite $C(\mathfrak{n})$ -module (cf. [4, Lemma 1, p. 7], [5, (8.9)]). Therefore, it is sufficient to prove $\overline{C(\mathfrak{n})_{\mathfrak{q}'}}$ is a finite $C(\mathfrak{n})_{\mathfrak{q}'}$ -module for any $\mathfrak{q}'$ in $\text{Ass}(C(\mathfrak{n})/xC(\mathfrak{n}))$. We have $\text{prof}(C(\mathfrak{n})_{\mathfrak{n}'}) > 1$ for any maximal ideal $\mathfrak{n}'$ of $C(\mathfrak{n})$ by Proposition (1.2). Hence $\mathfrak{q}' \cap A = \mathfrak{q}$ is non-maximal in A . By assumption, $\bar{A}_{\mathfrak{q}}$ is a finite $A_{\mathfrak{q}}$ -module. Therefore, $\overline{C(\mathfrak{n})_{\mathfrak{q}'}}$ is a finite $C(\mathfrak{n})_{\mathfrak{q}'}$ -module.

Remark (3.3) (3.3.1) In Propositions (3.1) and (3.2), we cannot weaken the assumption "for any $\mathfrak{p}$ in $\text{Ass}(A/xA)$ " by "for any $\mathfrak{p}$ in minimal $\text{Ass}(A/xA)$ " (cf. [1, Proposition 3.3]).

(3.3.2) (Seydi) If $(A_m)^{(1)} = \bigcap_{\substack{\text{ht } \mathfrak{p}=1 \\ \mathfrak{p} \subset m}} A_{\mathfrak{p}}$ is a finite A_m -module for any maximal ideal m of A , then we can weaken the assumption “for any $\mathfrak{p}$ in $\text{Ass}(A/xA)$ ” in Propositions (3.1) and (3.2) by “for any $\mathfrak{p}$ in minimal $\text{Ass}(A/xA)$ ” (Proof is easy).

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References

- [1] D. Ferrand-M. Raynaud: Fibres formelles d'un anneau local noethérien, Ann. Sci. Ecole Norm. Sup. (4) **3** (1970), 295-311.
- [2] H. Kurke, G. Pfister und M. Roczen: Henselsche Ringe und algebraische Geometrie, VEB Deutscher Verlag der Wissenschaften, 1975.
- [3] J. Matijevic: Maximal ideal transforms of noetherian rings, Proc. Amer. Math. Soc. **54** (1976), 49-52.
- [4] H. Matsumura: Commutative algebra, Benjamin, 1970.
- [5] M. Nagata: Local rings, Interscience, 1962.
- [6] J. Nishimura: Note on integral closures of a noetherian integral domain, Jour. Math. Kyoto Univ. **16** (1976), 117-122.
- [7] J. Querré: Sur un théorème de Mori-Nagata, C.R. Acad. Sc. Paris, t. **285** (1977), 323-324.
- [8] H. Seydi: Sur la transformation de Nagata (preprint).