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HOMOGENEOUS FACTORS OF RIEMANNIAN MANIFOLDS

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Given a connected Riemannian manifold M and a closed connected subgroup G of the Lie group of isometries of M, we look for necessary and sufficient conditions that we can write M as the Riemannian product of an orbit and some other Riemannian manifold. More precisely, we ask: do there exist a Riemannian manifold P with a transitive action of G, a Riemannian manifold N and a G-equivariant isometry $f: P \times N \to M$, where N has the trivial G-action and $P \times N$ the diagonal action. We call this a global equivariant splitting.

Of course the corresponding local condition (Definition 2.3.) is necessary, but in general it is not sufficient. The obstruction is a homomorphism from the fundamental group of M to the group of G-isometries of an orbit. The main result is that M has a global equivariant splitting iff it has a local one and the obstruction vanishes (Theorem 2.9). The obstruction vanishes if M is simply connected, trivially, or if $H^1(M; \mathbf{R}) = 0$ and G is simply connected solvable (Corollary 1.16). We show that every homomorphism qualifying for being an obstruction is in fact an obstruction (Lemma 2.10). Note that the Riemannian manifolds under consideration need not be complete, and also that we need no assumptions about holonomy.

The local condition is not easy to check. So we give three conditions (IGC, NI, KC) whose conjunction is equivalent to the local condition. These three conditions should be easier to check, since two of them (NI, KC) are differential conditions and then it remains to check that the number of connected components of isotropy groups is constant.

This paper contains two sections. In §1 we discuss the consequences of IGC and NI (or equivalently, of the existence of local orthogonal cross sections), and in §2 we apply this to our problem and discuss the existence of local equivariant splittings.

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This paper has a twofold motivation. In the differentiable category we have the following result, which is a special case of the main result in [1]: Suppose G is a closed connected subgroup of the Lie group of isometries of the Riemannian manifold M, and all isotropy groups of G are maximal compact subgroups of G. Then there is a G-equivariant diffeomorphism $G/K \times S \xrightarrow{\sim} M$, where K is a maximal compact subgroup of G, and S is some differentiable manifold with trivial G-action. (E.g., if G is homeomorphic to some euclidean space, or equivalently, if G is simply connected solvable, then every compact subgroup of G is trivial. For $G = \mathbf{R}$ see [5]). In this paper we ask the corresponding question in the category of Riemannian manifolds.

In [6] the existence of a Riemannian $\mathbf{R}$-factor of a Riemannian manifold M was discussed, provided that $\mathbf{R}$ acts properly and isometrically on M, and a local Riemannian splitting of M exists, compatible with the action. This special case of our question was a second motivation for our paper.

Having the global equivariant splitting one can apply it to other situations by looking at the covering transformation groups. This will be done in a forthcoming paper by the second author.

1. Local orthogonal cross sections

1.1. We make the following hypotheses throughout this paper: Let M be a connected Riemannian manifold. The group of isometries of M endowed with the compact-open topology is a Lie group. Let G be a closed connected subgroup thereof. Stated differently, we assume that we have a proper effective differentiable action of the connected Lie group G on M by isometries, [4], [3]. We ask for conditions that a (*global*) *equivariant splitting* exist, i.e., that there be a Riemannian manifold P with a transitive G-action, a Riemannian manifold N, and a G-equivariant isometry $f\colon P \times N \to M$, where N has the trivial G-action, and $P \times N$ the diagonal action. The following conditions are obviously necessary.

1.2. (*IGC*) Any two isotropy groups are conjugate.

In more detail: If x is a point of M, let $G_x = \{g \in G;\ gx = x\}$ be the isotropy group of x. We require that the isotropy groups G_x and G_y of any two points x and y of M are conjugate in G. Every G-orbit Gx is a closed submanifold of M, G-diffeomorphic with G/G_x, [4]. For $x \in M$ let T_xGx be the tangent space at x of the orbit of x, and N_x be its orthogonal complement in M_x, the tangent space of M at x.

1.3. (NI) $\mathfrak{N} = \{N_x; x \in M\}$ is an integrable distribution on M.

This condition is equivalent to the following two conditions: (1) the dimension of N_x (or Gx or G_x) is independent of x (which follows from IGC) and (2) there is a (local) integral manifold of $\mathfrak{N}$ through every point of M. By a theorem of Frobenius, (2) is equivalent to $\mathfrak{N}$ being involutive. IGC and NI are obviously necessary conditions for our existence problem. In this section we explore the consequences of IGC and NI. These two conditions do not suffice to give a positive answer for our problem. There are both local and global obstructions.

1.4. NI implies that for every $x \in M$ there is a local integral manifold S of $\mathfrak{N}$ through x. As G acts properly on M we can apply a lemma of Palais ([4, 2.2 and 2.1.7]; cf. [2]) to conclude that we may assume that S is a slice at x for the G-action, i.e., there is a differentiable G-retraction f: $GS \to Gx$ of the neighborhood GS of the orbit Gx to Gx such that $f^{-1}(x) = S$. In particular the isotropy group of every point $y \in S$ is a subgroup of G_x, whence the isotropy group of every point y of the neighborhood GS of x is conjugate to a subgroup of G_x. As the compact Lie groups G_x and G_y have the same dimension by NI, we have

1.5. Corollary. *Suppose NI holds and the number of connected components of isotropy groups is constant. Then IGC holds.*

Now let $N(z)$ be the maximal integral manifold of $\mathfrak{N}$ containing $z \in M$. Let $x \in N(z)$. Then $N(z)$ contains an open subset S which is a slice at x, hence $G_y \subseteq G_x$ for $y \in S$. If a closed subgroup H_1 of a compact Lie group H_2 is isomorphic to H_1, then $H_1 = H_2$. Hence

1.6. Lemma. *If NI and IGC hold, then all points of a maximal integral manifold of $\mathfrak{N}$ have the same isotropy group.*

1.7. Definition. A *local orthogonal cross section* is a local submanifold S of M such that

(a) the tangent space of S at every one of its points y is the orthogonal complement of the tangent space of the orbit through y, and

(b) every orbit intersects S in at most one point.

1.8. Lemma. *The following conditions are equivalent.*

(1) *Every point of M is contained in a local orthogonal cross section.*

(2) *IGC and NI.*

Proof. (1) $\Rightarrow$ (2). The existence of local orthogonal cross sections (even without Definition 1.7(b)) implies NI. So for every $x \in M$ there is a local orthogonal cross section which is a slice at x. Then for $y \in S$ we have $\#(Gy \cap S) = \# G_x/G_y = 1$ by property 1.7(b). So the isotropy groups of all points of the neighborhood GS of x are conjugate to G_x, which easily implies IGC.

(2) $\Rightarrow$ (1). Let S be a connected slice at x, with $T_y S = (T_y Gy)^{\perp}$ for $y \in S$. Let $f\colon GS \to Gx$ be the G-retraction with $f^{-1}(x) = S$. We have $gS \cap S \neq \varnothing$ iff $g \in G_x$, since f is a G-map. This implies $\# Gy \cap S = \# G_x y \cap S = \# G_x/G_y = 1$ for $y \in S$ by Lemma 1.6, whence property 1.7(b).

1.9. For every point $z \in M$ let $\mathrm{Stab}_G(N(z))$ be the group of elements $g \in G$ such that $gN(z) = N(z)$ or equivalently $gN(z) \cap N(z) \neq \varnothing$. This group acts on $N(z)$ with ineffective kernel G_z by Lemma 1.6. The group $H(z) = \mathrm{Stab}_G(N(z))/G_z$ acts freely on $N(z)$. We consider $H(z)$ as a discrete group. Note that $H(z) = H(x)$ if $x \in N(z)$.

1.10. Lemma. *Suppose IGC and NI hold. Let $p\colon M \to G\backslash M$ be the natural map to the orbit space. Then $p|N(z)\colon N(z) \to G\backslash M$ is a regular covering map with group of deck transformations $H(z)$ for every point $z \in M$.*

Note that M need not be a complete Riemannian manifold.

Proof. We drop z from the notation. Let $x \in N$, and let S be a connected open submanifold of N which is a slice at x. We prove that the inverse image $N \cap GS$ of the neighborhood $p(S)$ of $p(x)$ for $p|N$ is H-homeomorphic to $H \times S$ via $\varphi\colon H \times S \to N \cap GS$, $\varphi(h, s) = hs$. This implies the lemma. The map φ is an injective H-map. It is open because S is open in N, and it is surjective by the definition of $H = \mathrm{Stab}_G(N(z))/G_z$.

In particular, $p|N(z) \to G\backslash M$ is surjective for every z; in other words, we have

1.11. Corollary. *Every maximal integral manifold of $\mathfrak{N}$ intersects every orbit.*

1.12. Let $N_G(H)$ be the normalizer in G of a subgroup H of G. Then we have a group isomorphism of $N_G(G_z)/G_z$ with the group of G-homeomorphisms of G/G_z given by $h \cdot G_z \to \{g\, G_z \to g\, h\, G_z\}$. Since $\mathrm{Stab}_G(N(z)) \subset N_G(G_z)$ by Lemma 1.6, $H(z)$ acts effectively on G/G_z. The diagonal action of $H(z)$ on $G/G_z \times N(z)$ is free, properly discontinuous and differentiable. The map $G/G_z \times N(z) \to M$, $(g\, G_z, n) \mapsto gn$, induces a diffeomorphism

1.13. $G/G_z \times_{H(z)} N(z) \xrightarrow{\sim} M.$

The surjectivity of this map follows from Corollary 1.11, and everything else is clear. More explicitely, we have

1.14. Lemma. *Suppose IGC and NI hold. Then $p\colon M \to G\backslash M$ is the (locally trivial) fibre bundle with fibre G/G_z, structure group $H(z)$ and associated principal fibre bundle $p|N(z)\colon N(z) \to G\backslash M$.*

If we have a global equivariant splitting $f\colon P \times N \xrightarrow{\sim} M$ as in §1.1, then f induces isometries $P \xrightarrow{\sim} G/G_z$, $N \xrightarrow{\sim} N(z)$ for some (every) point $z \in M$, hence $H(z) = 1$ by comparing with Lemma 1.10. So a necessary condition for our existence problem is $H(z) = \{1\}$ for one (every) $z \in M$.

Let us give a different description of $H(z)$. Let c be a path in M starting at $z \in M$. By Lemma 1.10 there is a unique path c' in $N(z)$ starting at z such that $p(c') = p(c)$. We call c' the projection of c normal to the orbits.

1.15. Lemma. *$H(z)$ is the image of the following homomorphism $\varphi_z\colon \pi_1(M, z) \to N_G(G_z)/G_z = \{G\text{-homeomorphisms of } Gz\}$, $\varphi_z([c]) = n \cdot G_z$ if $n\,z$ is the endpoint of the projection c' of c normal to the orbits.*

Proof. The image of φ_z is contained in $H(z)$. Conversely, for every $n \cdot G_z \in H(z)$ there is a path c_1 in $N(z)$ from z to nz and a path c_2 from nz to z in Gz. Let c be the composite loop based at z. Obviously $c' = c_1$ hence $\varphi_z([c]) = n \cdot G_z$.

It is easy to see how φ_z depends on z: For $g \in G$ we have $\varphi_{gz} = I_g \circ \varphi_z$, where I_g is the isomorphism $N_G(G_z)/G_z \to N_G(G_{gz})/G_{gz}$ induced by the inner automorphism with g. If $y \in N(z)$, let d be a path in $N(z)$ connecting z and y. Then $\varphi_y([d^- \circ c \circ d]) = \varphi_z([c])$ for $[c] \in \pi_1(M, z)$. In particular, for any two points y and z in M the groups $H(y)$ and $H(z)$ are isomorphic. An example of an application of Lemma 1.15 is

1.16. Corollary. *If G is $\mathbf{R}^n$ or more generally a simply connected solvable Lie group, and $H^1(M; \mathbf{R}) = 0$, then φ_z is trivial, i.e., $H(z) = \{1\}$ for every $z \in M$.*

Proof. Any compact subgroup of G is trivial, hence $G_z = \{1\}$. Since $H^1(M; \mathbf{R}) = \mathrm{Hom}(\pi_1(M; z); \mathbf{R}) = 0$, by induction on the length of the derived series, every homomorphism φ of $\pi_1(M, z)$ to G is trivial.

1.17. The necessary condition $H(z) = \{1\}$–or equivalently φ_z trivial–is not sufficient for a positive answer to our existence problem stated in §1.1. E.g., let M be the Poincaré model of the hyperbolic plane, the upper half plane in $\mathbf{C}$. The additive group $\mathbf{R}$ acts isometrically on M by $\mathbf{R} \times M \to M$, $(r, z) \to e^r \cdot z$. The orbits are the rays in M starting at 0. There are global orthogonal cross sections $N(z)$ through every point $z \in M$, namely those parts of circles with center 0, which lie in M. Now $G \times N(z) \to M$, $(r, y) \to e^r \cdot y$, is a global diffeomorphism. Comparing with Lemma 1.10 yields $H(z) = \{1\}$, which also follows (quicker but less instructively) from Corollary 1.16. But M is not isometric to the product of $N(z)$ and a transitive $\mathbf{R}$-manifold, because otherwise its curvature would be zero.

2. Local equivariant splitting

2.1. After the discussion of the preceding section it is clear that we need, for the existence of a global equivariant splitting, a local equivariant splitting which follows if we know that the G-retraction of §1.4 induces an isometry

$Gz \to Gx$ for every $z \in S$. This latter condition can be translated into a differential condition (Definition 2.5). The differential conditions of our final theorem should be easier to check than the existence statements. Our hypotheses from §1.1 are still in force.

2.2. Existence. A *local splitting* at $x \in M$ is an open subset V of Gx, a Riemannian manifold S and an isometry f of $V \times S$ onto an open neighborhood U of x in M such that $f(V \times \{s\})$ is the intersection of one G-orbit with U for every $s \in S$.

2.3. Definition. A *local equivariant splitting* at $x \in M$ is a G-neighborhood U of x, a Riemannian submanifold S of M containing x, and a G-equivariant isometry $f\colon Gx \times S \to U$ such that $f(x, s) = s$ for $s \in S$, where G acts trivially on S and diagonally on $Gx \times S$.

This is the local version of the global equivariant splitting of §1.1. The apparently weaker Definition 2.2 is in fact equivalent to Definition 2.3.

2.4. Lemma. *There is a local splitting at $x \in M$ iff there is a local equivariant splitting at $x \in M$.*

Proof. Let U, V, S and f be as in the definition of a local splitting. We first show that $f(\{v\} \times S)$ is a local orthogonal cross section for every $v \in V$. The tangent space of this submanifold of U at any one of its points, $y = f(v, s)$ say, is the orthogonal complement of the tangent space of the orbit $Gy \cap U = f(V \times \{s\})$. Every orbit intersects $f(\{v\} \times S)$ in at most one point, because suppose an orbit intersects $f(\{v\} \times S)$ in $y = f(v, s)$, then the intersection of this orbit with $f(\{v\} \times S)$ is $Gy \cap f(\{v\} \times S) = f(V \times \{s\}) \cap f(\{v\} \times S) = y$.

So we can apply the results of §1 to GU, assuming without loss of generality that V and S are connected. Secondly, we show that we may assume that $x \in V \subset M$, $x \in S \subset M$, S is a connected slice, and $f(x, v) = v$ and $f(x, s) = s$ for $v \in V$, $s \in S$. Let $x = f(w, t)$ for some $w \in V$, $t \in S$, and define $V' = f(V \times \{t\}) = Gx \cap U$, $S' = f(\{w\} \times S)$. As in §1.4 we may assume that S' is a connected slice at x. Let us define the isometries $f_1\colon V \to V'$, $f_1(v) = f(v, t)$ and $f_2\colon S \to S'$, $f_2(s) = f(w, s)$. Then $f' := f \circ (f_1 \times f_2)^{-1}\colon V' \times S' \to U$ has the required properties.

Thirdly, we show that every f with these properties is a G-map, as far as this notion makes sense, i.e., $f(gv, s) = g f(v, s)$ if v and gv are in V. Let us fix v and look at the two maps $f_1(s) := f(gv, s)$ and $f_2(s) := g f(v, s)$ from S to U. For $s = x$ we have $f_1(x) = gv = f_2(x)$. The images are local orthogonal cross sections by the first step of our proof, hence both are contained in $N(gv)$, the maximal integral manifold in GU of the distribution $\mathfrak{N}$ of normal spaces of the orbits. Now f_1 and f_2 both fit into the commutative diagram

$$\begin{tikzcd} & N(gv) \arrow[d, "p"] \\ S \arrow[ur, "f_1, f_2"] \arrow[r, "p"'] & G\backslash M \end{tikzcd}$$

and have the same value at x, so they are equal since $p|N(gv)$ is a covering map by Lemma 1.10.

Finally, it follows from the third step that there is a unique G-equivariant extension $\bar{f}$: $Gx \times S \to GU$ of f. Since G acts isometrically on both sides and f is an isometry, $\bar{f}$ is an isometry. So $\bar{f}$ is the desired local equivariant splitting at $x \in M$.

It does not seem easy to check the existence of local (equivariant) splittings. So we give an equivalent differential condition which should be easier to check.

The Lie group G acts on the Riemannian manifold M, differentiably and by isometries. Let φ be the corresponding homomorphism of G into the group of isometries of M. For every element X of the Lie algebra $\mathfrak{g}$ of G the *Killing vector field* $\varphi(X)$ is by definition the infinitesimal generator of the one-parameter-group $t \mapsto \varphi \circ \exp tX$ of isometries of M, where $t \to \exp tX$ is the one-parameter-subgroup of G with tangent vector X at $t = 0$. A vector field of the form $\varphi(X)$, $X \in \mathfrak{g}$, is simply called a Killing vector field for the action. For the tangent space $T_x Gx$ of the orbit through x we have $T_x Gx = \{\varphi(X)(x);\ X \in \mathfrak{g}\}$. Its orthogonal complement in M_x is again denoted by N_x and called the normal space of the orbit at x.

2.5. Definition (*Killing vector fields constant in normal direction of the orbit*). We say (KC) holds if for every $X \in \mathfrak{g}$ and every $Y \in N_x$ we have

$$Y\|\varphi(X)\|^2 = 0,$$

where $\|\varphi(X)\|(x)$ is the length of the vector $\varphi(X)(x) \in M_x$ with respect to the Riemannian metric. So we can apply any tangent vector $Y \in M_x$ to the differentiable function $\|\varphi(X)\|^2$, the square of the length of the Killing vector field $\varphi(X)$.

2.6. Remark. If NI holds, KC is equivalent to the following: The length of every Killing vector field of the action is constant on every connected integral manifold of $\mathfrak{N}$.

2.7. Proposition. *Under our hypotheses* (*see* §1.1) *the following conditions are equivalent*:

(1) *There is a local equivariant splitting at every point of* M.

(2) *IGC, NI and KC hold.*

Proof. (1) $\Rightarrow$ (2). Since the existence of local equivariant splittings implies the existence of local orthogonal cross sections, IGC and NI follow from Lemma 1.8. To prove KC we use the notation of Definition 2.3. Let $P = Gx$. Let $\varphi(X)(x)$ be a tangent vector of P at x, and $\psi(X)$ the Killing vector field on $P \times S$ corresponding to $X \in \mathfrak{g}$. Then $\psi(X)(x, s)$ is the image of $\varphi(X)(x)$ under the obvious isometry $P \xrightarrow{\sim} P \times \{s\}$, $s \in S$. So $\|\psi(X)(x, s)\|$ does not depend on s, which implies KC after applying f.

(2) $\Rightarrow$ (1). Let S be a connected local orthogonal cross section which is a slice at x. Then $f\colon Gx \times S \to GS$, $f(gx, s) = gs$, is a G-diffeomorphism, [4]. We endow Gx with the Riemannian metric induced from M. We prove that f is an isometry. Since G acts isometrically on both sides it is enough to show that the differential $df_{(x,s)}$ of f at (x, s) is a linear isometry for every $s \in S$. Obviously, the restriction of $df_{(x,s)}$ to the tangent space of S at (x, s) is an isometry. For $X \in \mathfrak{g}$ let $\psi(X)$ be the corresponding Killing vector field on $Gx \times S$ for the action of G on $Gx \times S$. We have $df_{(x,s)}\psi(X)(x, s) = \varphi(X)(s)$, since f is a G-map. So $df_{(x,s)}\colon T_{(x,s)}G(x, s) = \{\psi(X)(x, s); X \in \mathfrak{g}\} \to T_sGs = \{\varphi(X)(s); X \in \mathfrak{g}\}$, which are both the orthogonal complement of the tangent space of S in their respective tangent spaces. So it remains to prove that $df_{(x,s)}\colon T_{(x,s)}G(x, s) \to T_sGs$ is a linear isometry. For $X \in \mathfrak{g}$ the lengths of $\psi(X)(x, s)$ and $\varphi(X)(s) = df_{(x,s)}\psi(X)(x, s)$ are independent of s; the first claim is obvious, and the second claim follows from KC. For $s = x$ we have $\|\psi(X)(x, x)\| = \|\varphi(X)(x)\|$ by the definition of the Riemannian metric on Gx, which proves our claim.

2.8. Proposition. *If there is a local equivariant splitting at every point of M, the homomorphism φ_z of Lemma* 1.15 *takes its values in the group $I(z)$ of G-isometries of the orbit of z:*

$$\varphi_z\colon \pi_1(M, z) \to I(z).$$

The image of $\pi_1(Gz, z) \to \pi_1(M, z)$ is contained in the kernel of φ_z, hence φ_z induces a homomorphism $\pi_1(G\backslash M, p(z)) \to I(z)$.

Proof. Let c be a path in $G\backslash M$ starting at $p(x)$ and ending at $p(y)$. For any point $x' \in Gx$ there is a unique path c' in $N(x')$ with $p(c') = c$ starting at x'. Let its endpoint be y'. We thus obtain a G-map $l(c)\colon Gx \to Gy$. We may regard l as a functor from the fundamental groupoid of $G\backslash M$ to the category of G-spaces and G-maps. We claim that $l(c)$ is an isometry. This is true for paths in sufficiently small open subsets of $G\backslash M$ because of the existence of local equivariant splittings. The general statement follows by breaking c up into small enough pieces. Now if $[c]$ is the homotopy class of a closed path based at z, then $\varphi_z[c] = l \circ p(c)$.

2.9. Theorem. *M has a global equivariant splitting iff it has a local splitting at every point, and φ_z is trivial for one (every) point z of M. More generally, if a local (equivariant) splitting exists at every point of M, the diffeomorphism* 1.13 *is an isometry $Gz \times_{\mathrm{im}\,\varphi_z} N(z) \xrightarrow{\sim} M$.*

One cannot improve Proposition 2.8, because every homomorphism $\varphi_z: \pi_1(M, z) \to I(z)$, whose kernel contains the image of $\pi_1(Gz, z) \to \pi_1(M, z)$, occurs as in Proposition 2.8. More precisely, we have

2.10. Lemma. *Given a connected Lie group G, a connected Riemannian manifold P with a transitive differentiable isometric action of G, a connected Riemannian manifold M', and a homomorphism $\varphi: \pi_1(M', z') \to I = \{G$-isometries of $P\}$. Then there are a Riemannian manifold M with an isometric action of G with orbits isometric to P, local equivariant splittings at every point of M, and an isometry ψ of the orbit space $G \backslash M$ with M' such that $\varphi_z = \mu_* \circ \varphi \circ \psi_* \circ p_*$, where $\mu: P \to Gz$ is a G-isometry, $p: M \to G \backslash M$ is the natural map, and $\psi \circ p(z) = z'$.*

Proof. Let $\pi: N \to M'$ be the covering space of M' corresponding to $\ker \varphi$. Pick a basepoint $n \in N$ over z'. There is a unique Riemannian tensor on N such that π is a local isometry. The group G acts isometrically on the Riemannian manifold $P \times N$, trivially on N. The group $H = \mathrm{im}\,\varphi$ acts G-equivariantly, isometrically and properly discontinuously on $P \times N$, namely on P as subgroup of I and on N by deck transformations. Define $M = H \backslash P \times N$ and check.

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