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# On blocks of twisted group algebras

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**0. Introduction.** Let $G$ be a finite group, let $F$ be a field and let $\alpha: G\times G\rightarrow F^{*}$ be a cocycle. Denote by $F^{\alpha}G$ the corresponding twisted group algebra of $G$ over $F$. The algebra $F^{\alpha}G$ has an $F$-basis $\{\bar{g}|g\in G\}$ and the multiplication in $F^{\alpha}G$ is determined by

$$\bar{x}\bar{y}=\alpha(x, y)\overline{xy} \qquad \text{for all } x, y\in G$$

Observe that $F^{\alpha}G$ is isomorphic to the ordinary group algebra $FG$ if and only if $\alpha$ is a coboundary. By an *$\alpha$-representation* of $G$ over $F$ (or simply projective representation of $G$ over $F$. if $\alpha$ is not pertinent to the discussion), one understands any map $\rho: G\rightarrow GL(n, F)$ (for some $n\geq 1$) with $\rho(1)=1$ and $\rho(x)\rho(y)=\alpha(x, y)\rho(xy)$ for all $x, y\in G$. Two $\alpha$-representations $\rho_{i}: G\rightarrow GL(n, F)$, $i=1, 2$, are said to be *linearly equivalent* if there exists $M\in GL(n, F)$ such that

$$\rho_{2}(g)=M^{-1}\rho_{1}(g)M \qquad \text{for all } g\in G$$

The modular $\alpha$-representation theory of $G$ is concerned with the study of blocks of $F^{\alpha}G$, for the isomorphism classes of $F^{\alpha}G$-modules correspond bijectively, in a well-known manner, with the linear equivalent classes of $\alpha$-representations of $G$ over $F$.

The intention of the present paper is to apply Külshammer's theorem [4] in order to provide ring-theoretic information on the structure of blocks of $F^{\alpha}G$ whose defect groups are central. The corresponding result for ordinary group algebras is due to Külshammer [3]. Because it costs us no effort, we shall prove our result under slightly more general circumstances. Namely, we shall consider those blocks $B$ of $F^{\alpha}G$ for which $G=DC_{G}(D)$ where $D$ is a defect group of $B$. Our result is as follows

THEOREM. *Let $B$ be a block of the twisted group algebra $F^{\alpha}G$ of a finite group $G$ over the field $F$ of characteristic $p>0$. Assume that $G=DC_{G}(D)$ where $D$ is a defect group of $B$ and that the values of $\alpha$ on $D\times D$ belong to a perfect subfield of $F$. Then*

(i) *$B\cong B/J(B)\otimes_{F}FD$, where $J(B)$ is the Jacobson radical of $B$. In particular, $B$ has a unique irreducible $F^{\alpha}G$-module.*

(ii) *$dim_{F}J(B)=(1-|D|^{-1})\ (dim_{F}B)$*

(iii) *If $F$ is algebraically closed, and $n$ is the dimension of the* *irreducible $F^{\alpha}G$-module of B, then B is isomorphic to a full matrix algebra $M_n(FD)$ with entries in the group algebra FD.*

## 1. Notation and terminology.

Throughout this paper, $F$ denotes a field of characteristic $p>0$, $G$ a finite group and $\alpha$ an element of $Z^2(G, F^*)$. An element $g\in G$ is said to be $\alpha$-regular if $\alpha(x, g)=\alpha(g, x)$ for all $x\in C_G(g)$. If $\cdot$ is $\alpha$-regular, then so is any conjugate of $G$ and therefore we may speak about $\alpha$-regular classes of $G$. Given $x=\sum x_g\bar{g}\in F^{\alpha}G$, the support of $x$, written Supp$x$, is defined by

$$\mathrm{Supp}x=\{g\in G\mid x_g\neq 0\}$$

Let $e$ be a block idempotent of $F^{\alpha}G$ and write

$$\mathrm{Supp}e=C_1\cup\cdots\cup C_t$$

for some $\alpha$-regular classes $C_1, \cdots, C_t$ of $G$. Then the largest of the defect groups of $C_i$, $1\leq i\leq t$, is called a defect group of $e$ (or of the block $B(e)$ containing $e$). As in the ordinary case, it can be shown that a defect group of $e$ is uniquely determined up to conjugacy in $G$. If $p^d$ is the order of defect groups of $e$, then $d$ is called the defect of $e$ (or of $B(e)$). Let $H$ be a subgroup of $G$. In order to prevent our expressions from becoming too cumbersome, we shall use the same symbol $\alpha$ for an element of $Z^2(G, F^*)$ and its restriction in $Z^2(H, F^*)$. With this convention, $F^{\alpha}H$ is just the $F$-linear span of the elements $\bar{h}$, $h\in H$.

## 2. An F-basis for $Z^{\alpha}(G:H)$.

Throughout this section, $H$ denotes a subgroup of $G$ and $Z^{\alpha}(G:H)$ the centralizer of $F^{\alpha}H$ in $F^{\alpha}G$, i. e.

$$Z^{\alpha}(G:H)=\{x\in F^{\alpha}G\mid xy=yx \text{ for all } y\in F^{\alpha}H\}$$

Our aim here is to exhibit an $F$-basis for $Z^{\alpha}(G:H)$. The following terminology is due to Reynolds [6].

*By a monomial space over F* we mean a triple $(V, S, V_s)$ where $V$ is a vector space over $F$, $S$ is a finite set and $(V_s)$ is a family of one-dimensional subspaces of $V$ indexed by $S$ such that $V=\oplus_{s\in S}V_s$.

*By a monomial representation* of $G$ on $(V, S, (V_s))$ we mean a homomorphism:

$$\Gamma: G\rightarrow GL(V)$$

such that for each $g\in G$, $\Gamma(g)$ permutes the $V_s$, $s\in S$. It follows that $\Gamma$ determines a homomorphism $\gamma$ from $G$ to the permutation group of the set $S$, where for all $g\in G$ and $x$, $y\in S$

$$\gamma(g)x=y \text{ if and only if } \Gamma(g)V_x=V_y$$

For each $s\in S$, let $G(s)$ be the *stabilizer* of $s$, i. e.

$$G(s)=\{g\in G \mid \gamma(g)s=s\}$$

We say that an element $s$ of $S$ is $\Gamma$-*regular* if for all $g\in G(s)$, $\Gamma(g)$ is the identity mapping on $V_s$. We shall refer to a $G$-orbit of $S$ as being $\Gamma$-*regular* if each element of this orbit is $\Gamma$-regular. By the *fixed-point space* of $\Gamma$ we understand the set of those $v\in V$ for which $\Gamma(g)v=v$ for all $g\in G$.

Next we generalize the notion of $\alpha$-regularity. Two elements $x, y\in G$ are called *H-conjugate* if $y=hxh^{-1}$ for some $h\in H$. It is clear that the $H$-conjugacy is an equivalence relation and so $G$ is a union of $H$-conjugacy classes. For a given $g\in G$, let $C_H(g)$ denote the centralizer of $g$ in $H$, i. e. $C_H(g)=\{h\in H \backslash hg=gh\}$. We say that an element $g\in G$ is $(\alpha, H)$-regular if for all $h\in C_H(g)$, $\alpha(h, g)=\alpha(g, h)$. Thus $g$ is $\alpha$-regular if and only if $g$ is $(\alpha, G)$-regular. It follows from the definition of $(\alpha, H)$-regularity that if $g$ is $(\alpha, H)$-regular, then so is any $H$-conjugate of $g$ and so we may speak about $(\alpha, H)$-*regular H-conjugacy classes of G.*

LEMMA 1. *Let $(V, S, (V_s))$ be a monomial space over an arbitrary field $F$, and let $\Gamma: G\rightarrow GL(V)$ be a monomial representation of $G$ on $(V, S, (V_s))$. Let $X$ be a set of all representatives for the $\Gamma$-regular orbits of $S$, and for each $x\in X$, let $w_x$ be a nonzero element of $V_x$. Set*

$$v_x=\sum_{g\in T_x}\Gamma(g)w_x$$

*where $T_x$ is a left transversal for $G(x)$ in $G$. Then $\{v_x \mid x\in X\}$ is an F-basis for the fixed-point space of $\Gamma$.*

PROOF. Let $Y$ denote a set of all representatives for the nonregular orbits of $S$, let $Z=XUY$ and, for each $z\in Z$, let $U_z$ be the sum of one dimensional subspaces of $V$ indexed by the elements of the orbit containing $z$. Then $V=\bigoplus_{z\in Z}U_z$ is a decomposition of $V$ into direct sum of invariant subspaces. It follows that if $W$ is the fixed-point space of $\Gamma$ and

$$W_z=W\cap U_z,\ z\in Z$$

then $W=\bigoplus_{z\in Z}W_z$. Let $v=\sum_{s\in S}\lambda_s v_s$, $\lambda_s\in F$, $0\neq v_s\in V_s$, belong to $W$ and suppose that there is an $s\in S$ such that $\lambda_s\neq 0$. Then, for a given $g\in G(s)$,

$$\Gamma(g)v_s=\mu_s v_s \qquad \text{for some } \mu_s\in F,$$

and hence the equality $\Gamma(g)v=v$ forces $\lambda_s=\lambda_s\mu_s$. It follows that $\mu_s=1$, so $s$ is $\Gamma$-regular and hence $W=\bigoplus_{x\in X}W_s$.

Fix $x\in X$ and, for each $g\in T_x$, set $v_{g,x}=\Gamma(g)w_x$. Then the elements $\{v_{g,x} \mid g\in T_x\}$ form an $F$-basis of $U_x$ and hence an arbitrary element $v\in W_x$ can be uniquely written in the form

$$v=\sum_{g\in T_x}\lambda_{g,x}v_{g,x}$$

Since for all $y\in G$, $\Gamma(y)$ permutes the $v_{g,x}$, $g\in T_x$, it follows that $v_x\in W_x$

and that all the coefficients $\lambda_{g,x}$ of $v$ are equal. So the lemma is true.

LEMMA 2. *Let $X$ be a set of representatives for the $(\alpha, H)$-regular $H$-conjugacy classes of $G$, and for each $x \in X$, let*

$$v_x = \sum_{h \in T_x} \bar{h}\bar{x}\bar{h}^{-1},$$

*where $T_x$ is a left transversal for $C_H(x)$ in $H$. Then the elements $v_x$, $x \in X$, constitute an $F$-basis for the $F$-algebra $Z^\alpha(G:H)$.*

PROOF. Let $V = F^\alpha G$ and, for each $g \in G$, let $V_g = \{\lambda \bar{g} \mid \lambda \in F\}$. Then $(V, G, (V_g))$ is a monomial space over $F$. Moreover, the mapping

$$\Gamma : H \rightarrow GL(V)$$

defined by

$$\Gamma(h)v = \bar{h}\bar{v}\bar{h}^{-1} \qquad \text{for all } v \in V,\ h \in H$$

is easily seen to be a monomial representation of $H$ on $(V, G, (V_h))$. The homomorphism $\gamma$ from $H$ to the permutation group of the set $G$ determined by $\Gamma$ is given by

$$\gamma(h)g = hgh^{-1} \qquad \text{for all } h \in H,\ g \in G$$

and hence $G(g) = C_H(g)$ for all $g \in G$. It therefore follows that $g \in G$ is $\Gamma$-regular if and only if for all $g \in C_H(g)$, $\bar{h}\bar{g} = \bar{g}\bar{h}$. *Thus* $g \in G$ is $\Gamma$-regular if and only if $g$ is $(\alpha, H)$-regular. Hence a typical $\Gamma$-regular $H$-orbit of $G$ is an $(\alpha, H)$-regular $H$-conjugacy class of $G$. Since $Z^\alpha(G:H)$ is the fixed-point space of $\Gamma$, the result follows by virtue of Lemma 1.

As an immediate consequence of Lemma 1, we derive

COROLLARY 3. *Let $X$ be a set of all representatives for the $\alpha$-regular classes of $G$. For each $x \in X$, put $k_x = \sum_{g \in T_x} \bar{g}\bar{x}\bar{g}^{-1}$, where $T_x$ is a left transversal for $C_G(x)$ in $G$. Then*

(i) $\{k_x \mid x \in X\}$ *is an $F$-basis for the centre $Z(F^\alpha G)$ of $F^\alpha G$*

(ii) *For each $x \in Z(F^\alpha G)$, Supp$x$ is a union of a certain number of $\alpha$-regular classes of $G$.*

## 3. Subsiduary results.

In this section we establish some subsiduary results required for the proof of the Theorem.

Let $P$ be a $p$-subgroup of $G$ and let $C_1, \cdots, C_r$ be all $\alpha$-regular classes of $G$ whose defect groups are conjugate to subgroups of $P$. Owing to Corollary 3, for each $i \in \{1, \cdots, r\}$, we may choose $z_i \in Z(F^\alpha G)$ with Supp$z_i = C_i$. We denote by $Z^\alpha(P)$ the $F$-linear span of $z_1, \cdots, z_r$. It follows from the definition of $Z^\alpha(P)$ that $Z^\alpha(P)$ consists of all central elements $x$ in $F^\alpha G$ with Supp$x \subseteq C_1 U \cdots UC_r$. Note that every block idempotent of $F^\alpha G$ with $P$ as a defect group is contained in $Z^\alpha(P)$ (see Karpilovsky [2]).

LEMMA 4. *Let $S\subseteq H$ be subgroups of $G$ and let $X$ be a left transversal for $S$ in $H$. For each $z\in Z^{\alpha}(G:S)$, put*

$$T_{H:S}(z)=\sum_{x\in X}\bar{x}z\bar{x}^{-1}$$

*Then the following properties hold*:

(i) *$T_{H:S}$ is an $F$-linear map from $Z^{\alpha}(G:S)$ to $Z^{\alpha}(G:H)$ which is independent of the choice of $X$*

(ii) *For all $y\in Z^{\alpha}(G:H)$ and $z\in Z^{\alpha}(G:S)$.*

$$T_{H:S}(yz)=y\ T_{H:S}(z)\ \text{and}\ T_{H:S}(zy)=T_{H:S}(z)y$$

(iii) *If $D$ is a subgroup of $S$, then for all $z\in Z^{\alpha}(G:D)$,*

$$T_{H:D}(z)=T_{H:S}(T_{S:D}(z))$$

(iv) *If $P$ is a $p$-subgroup of $G$, then for any $z\in Z^{\alpha}(P)$ there exists $w\in Z^{\alpha}(G:P)$ such that $z=T_{G:P}(w)$.*

PROOF (i). If $y=xs$ for some $x\in X$ and $s\in S$ then

$$\bar{y}z\bar{y}^{-1}=\overline{xs}z\overline{xs}^{-1}=\alpha^{-1}(x,\ s)\bar{x}\bar{s}z\alpha(x,\ s)\bar{s}^{-1}\bar{x}^{-1}=\bar{x}z\bar{x}^{-1}$$

*proving that* $T_{H:S}(z)$ is independent of the choice of $X$. To prove that $T_{H:S}(z)\in Z^{\alpha}(G:H)$, it suffices to verify that $T_{H:S}(z)$ commutes with all $\bar{h}$, $h\in H$. Since $hX$ is another transversal for $S$ in $H$, we have:

$$\bar{h}T_{H:S}(z)\bar{h}^{-1}=\sum_{x\in X}\bar{h}\bar{x}z\bar{x}^{-1}\bar{h}^{-1}=\sum_{x\in X}\overline{hx}z\overline{hx}^{-1}=T_{H:S}(z),$$

as required. The fact that $T_{H:S}$ is $F$-linear being obvious the assertion follows.

(ii) For all $x\in X$, we have $y\bar{x}=\bar{x}y$ and hence

$$T_{H:S}(yz)=\sum_{x\in X}\bar{x}yz\bar{x}^{-1}=y\sum_{x\in X}\bar{x}z\bar{x}^{-1}=yT_{H:S}(z)$$

The second equality is proved similarly.

(iii) Let $Y$ be a left transversal for $D$ in $S$. Then $XY$ is a left transversal for $D$ in $H$. Hence

$$\begin{aligned}T_{H:D}(z)&=\sum_{\substack{x\in X\\ y\in Y}}\overline{xy}\ z\ \overline{xy}^{-1}=\sum_{x\in X}\Big(\sum_{y\in Y}\bar{x}\,\bar{y}\,z\,\bar{y}^{-1}\bar{x}^{-1}\Big)\\&=\sum_{x\in X}\bar{x}\Big(\sum_{y\in Y}\bar{y}\,z\,\bar{y}^{-1}\Big)\bar{x}^{-1}\\&=T_{H:S}(T_{S:D}(z)),\end{aligned}$$

as required.

(iv) We may harmlessly assume that $\mathrm{Supp}z=C$ where $C$ is an $\alpha$-regular class of $G$. Fix $g\in C$, set $L=C_G(g)$ and $Q$ a Sylow $p$-subgroup of $L$. Since $z\in Z^{\alpha}(P)$ we may assume that $Q\subseteq P$. Because $g$ is $\alpha$-reqular, $\bar{g}\in Z^{\alpha}(G:Q)$. Owing to Corollary 3, $T_{G:L}(\bar{g})=\lambda z$ for some $\lambda\in F$. Hence, we may also assume that $z=T_{G:L}(\bar{g})$. Using (iii) to compute $T_{G:Q}(\bar{g})$ in two different ways, we find

(*) $$T_{G:L}(T_{L:Q}(\bar{g}) = T_{G:P}(T_{P:Q}(\bar{g}))$$

Because $T_{L:Q}(\bar{g}) = (L:Q)\bar{g}$ and $(L:Q)$ is prime to $p$, we may define

$$W = (L:Q)^{-1}T_{P:Q}(\bar{g})$$

Then $w \in Z^{\alpha}(G:P)$ and, by (*), $T_{G:P}(w) = z$, as asserted.

LEMMA 5. *Let $B = B(e)$ be a block of $F^{\alpha}G$ with $D$ as a defect group. Then $F^{\alpha}De \cong F^{\alpha}D$ as $F$-algebras.*

PROOF. The map $F^{\alpha}D \to F^{\alpha}De$ defined by $x \to xe$, $x \in F^{\alpha}D$, is obviously a surjective homomorphism of $F$-algebras. Since $|D| = \dim_F F^{\alpha}D$, it therefore suffices to verify that the set $\{e\bar{d} \mid d \in D\}$ is linearly independent over $F$. The result being trivial for $D = 1$, assume that $|D| > 1$. Suppose that $\sum_{d \in D} \lambda_d e\bar{d} = 0$ for some $\lambda_d \in F$. Because $D$ is a group, we may harmlessly assume that $\lambda_1 = -1$ if not all $\lambda_d = 0$. Hence

$$e = \sum_{d \in D - \{1\}} \lambda_d e\bar{d},$$

so writing $e = \sum_{g \in G} e_g \bar{g}$, it follows that

$$\begin{aligned}\sum_{g \in G} e_g \bar{g} &= \sum_{d \in D - \{1\}} \lambda_d \Big( \sum_{g \in G} e_g \bar{g}\bar{d} \Big) \\ &= \sum_{\substack{d \in D - \{1\} \\ \in G}} \lambda_d e_g \bar{g}\bar{d} \\ &= \sum_{\substack{d \in D - \{1\} \\ g \in G}} \lambda_g e_g \alpha(g, d)\overline{gd}\end{aligned}$$

Thus, for every $x \in \mathrm{Supp}e$, there exists $y \in \mathrm{Supp}e$ and $d \in D - \{1\}$ such that $x = yd$. Since $D$ is a defect group of $B$, $\mathrm{Supp}e \cap C_G(D) \neq \phi$, for otherwise the image of $e$ under the Brauer homomorphism

$$Z(F^{\alpha}G) \to Z(F^{\alpha}N_G(D))$$

would be zero, a contradiction. Hence, we may assume that $x \in G_G(D)$, in which case $y = xd^{-1} = d^{-1}x$. Owing to Passman [5], both $x$ and $y$ are $p'$-elements and so $d = 1$, a contradiction. So the lemma is true.

LEMMA 6. *Let $H$ be a subgroup of $G$ and let $\pi : F^{\alpha}G \to F^{\alpha}H$ be the natural projection. Then $\pi$ is a homomorphism of (left and right) $F^{\alpha}H$-modules.*

PROOF. Let $T$ be a right transversal for $H$ in $G$ containing 1. Then $F^{\alpha}G$ is a free left $F^{\alpha}H$-module with the elements $\{\bar{t} \mid t \in T\}$ as a basis. Hence the mapping $\bar{1} \to \bar{1}$, $\bar{t} \to 0$, $1 \neq t$, extends to an $F^{\alpha}H$-homomorphism $\psi : F^{\alpha}G \to F^{\alpha}H$. Fix $g \in G$ and write $g = ht$ for some $h \in H$, $t \in T$. If $t \neq 1$, then

$$\begin{aligned}\psi(\bar{g}) &= \psi(\alpha^{-1}(h, t)\bar{h}\bar{t}) \\ &= \alpha^{-1}(h, t)\bar{h}\psi(\bar{t}) \\ &= 0\end{aligned}$$

On the other hand, if $t=1$, then $\psi(\bar{g})=\bar{g}\psi(\bar{1})=\bar{g}$ and hence $\pi=\psi$. A similar argument shows that $\pi$ is a homomorphism of right $F^{\alpha}H$-modules, as required.

## 4. Proof of the Theorem.

Suppose that the isomorphism

(i) $B\cong B/J(B)\otimes_F FD$

is true. Then taking the $F$-dimensions of both sides of (1) yields (ii) Since $B$ is indecomposable, it follows from (1) the $B/J(B)$ must be simple and therefore $B$ has a unique irreducible $F^{\alpha}G$-module. In particular, if $F$ is algebraically closed, then $B/J(B)\cong M_n(F)$ where $n$ is the dimension of the irreducible $F^{\alpha}G$-module of $B$. Hence

$$B\cong M_n(F)\otimes_F FD\cong M_n(FD),$$

proving (iii). We are therefore left to verify (1).
Set $A=F^{\alpha}G$ and $A'=F^{\alpha}D$ so that $B=Ae$ and that, by Lemma 5,

$$A'e=F^{\alpha}De\cong F^{\alpha}D$$

Let $E$ be a perfect subfield of $F$ with $\alpha(x, y)\in E$ for all $x, y\in D$. Since $E$ is perfect, we have $E^{\alpha}D\cong ED$ (see Conlon [1]) and therefore

$$A'e\cong F^{\alpha}D\cong F\otimes_E E^{\alpha}D\cong F\otimes_E ED\cong FD$$

Let $\psi: F^{\alpha}G\rightarrow F^{\alpha}D$ be the natural projection. Then, by Lemma 6,

(2) $\psi$ is a homomorphism of $(F^{\alpha}D, F^{\alpha}D)$-bimodules

Let $\{g_1, \cdots g_n\}$ be a left transversal for $D$ in $G$. Because $F^{\alpha}G$ is a free right $F^{\alpha}D$-module with the elements $\bar{g}_1, \cdots, \bar{g}_n$ as a basis, each $a\in F^{\alpha}G$ can be uniquely written in the form

$$a=\sum_{i=1}^{n}\bar{g}_i x_i$$

with $x_i\in F^{\alpha}D$. For any $k\in\{1, \cdots, n\}$, we have

$$\bar{g}_k^{-1}a=x_k+\sum_{\substack{i=1\\ i\neq k}}^{n}\bar{g}_k^{-1}\bar{g}_i x_i$$

Since for $k\neq i$, $\psi(\bar{g}_k^{-1}\bar{g}_i)=0$, we deduce that $x_k=\psi(\bar{g}_k^{-1}a)$ and hence that

(3) $$a=\sum_{i=1}^{n}\bar{g}_i\psi(\bar{g}_i^{-1}a) \qquad \text{for all } a\in A$$

A similar argument shows that

(4) $$a=\sum_{i=1}^{n}\psi(a\bar{g}_i)\bar{g}_i^{-1} \qquad \text{for all } a\in A$$

By Lemma 4. 4(iv), we have that

(5) $$e=T_{G:D}(w) \qquad \text{for some } w\in Z^{\alpha}(G:D)$$

Because $F^{\alpha}D\cong FD$, we also have that

(6) $$\dim_F(F^{\alpha}D/J(F^{\alpha}D))=1.$$

Invoking (2)-(6) together with Külshammer's theorem (Külshammer [4]), we are left to verify that

(7) $$F^{\alpha}G=F^{\alpha}D.\ Z^{\alpha}(G:D)$$

To prove (7), we need only show that $\bar{g}\in F^{\alpha}D.\ Z^{\alpha}(G:D)$ for all $g\in G$. Fix $g\in G$ and write $g$ in the form $g=xy$ with $x\in D$ and $y\in C_G(D)$.

Then

$$\bar{g}=(\alpha^{-1}(x,\ y)\bar{x})\bar{y}$$

and $\bar{y}\in Z^{\alpha}(G:D)$ since any $p$-element of $G$ is $\alpha$-regular (see Conlon [1]). So the theorem is true.

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