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Unbounded Derivations Commuting with Compact Group Actions*

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Abstract. Let δ be a closed * derivation in a C^* algebra $\mathfrak{A}$ which commutes with an ergodic action of a compact group on $\mathfrak{A}$. Then δ generates a C^* dynamics of $\mathfrak{A}$. Similar results are obtained for non-ergodic actions on abelian C^* algebras and on the algebra of compact operators.

1. Introduction

In [9] Sakai showed that a non-zero closed * derivation in $C(\mathbb{T})$ commuting with translations is a constant multiple of the derivative. Following this, it was shown in [4, 7] that if G is a locally compact group and δ is a closed * derivation in $C_0(G)$ commuting with left translations by elements of G, then δ is the generator of a C^* dynamics (i.e., strongly continuous one-parameter group of * automorphisms) of $C_0(G)$. A like result holds for $C_0(G/H)$ (H a closed sub-group of G), provided that G is either separable or the projective limit of Lie groups. In this note, we assume G is compact and we obtain similar results for an ergodic action of G on an arbitrary C^* algebra (Theorem 2.1), and for an arbitrary action of G on an abelian C^* algebra (Theorem 3.2), or on the algebra of compact operators (Theorem 4.1).

We refer to [1, 3, 9] for background on unbounded derivations in C^* algebras.

2. Ergodic and G-Finite Actions

Before stating our first result we recall a few facts about Banach space representations of compact groups. Let V be a Banach space, G a compact group, and α a strongly continuous representation of G on V. For each $\pi \in \hat{G}$, define $P_\pi : V \to V$ by

$$P_\pi(x) = \int_G \dim(\pi)\,\mathrm{tr}(\pi(s))\alpha_s(x)ds.$$

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Then P_π is a continuous projection on V. The projection

$$P_0 : x \to \int_G \alpha_s(x) ds,$$

corresponding to the trivial one-dimensional representation has as its range the space of fixed points for the action α. A vector $x \in V$ lies in $\operatorname{span}\{P_\pi(V) : \pi \in \hat{G}\}$ if and only if $\operatorname{span}\{\alpha_s(x) : s \in G\}$ is finite dimensional. Such vectors are called G-finite. The G-finite vectors are norm dense in V.

Theorem 2.1. *Let $\mathfrak{A}$ be a C^* algebra with identity $\mathbb{1}$, G a compact group, and $\alpha : G \to \operatorname{Aut}(\mathfrak{A})$ a strongly continuous ergodic action. Let δ be a closed * derivation in $\mathfrak{A}$ commuting with the action of α.* (i.e., $\alpha_s \delta \alpha_s^{-1} = \delta$ *for all* $s \in G$.) *Then δ generates a C^* dynamics of $\mathfrak{A}$.*

Proof. Let P_π, P_0 be as above. Note that $P_0(\mathfrak{A}) = \mathbb{C}\mathbb{1}$ since α is ergodic. There is a unique faithful G-invariant state τ on $\mathfrak{A}$, defined by $\tau(x)\mathbb{1} = P_0(x)$. $\mathscr{D}(\delta)$ is a Banach *-algebra with the graph norm

$$\|x\|_\delta = \|x\| + \|\delta(x)\|.$$

Since δ commutes with α, $s \to \alpha_s|\mathscr{D}(\delta)$ is a strongly continuous representation of G on $\mathscr{D}(\delta)$. It follows that for each $x \in \mathscr{D}(\delta)$ and $\pi \in \hat{G}$,

$$P_\pi(x) \in \mathscr{D}(\delta), \quad \text{and} \quad \delta(P_\pi(x)) = P_\pi(\delta(x)). \tag{1}$$

Note in particular that for $x \in \mathscr{D}(\delta)$,

$$\begin{aligned} \tau(\delta(x))\mathbb{1} &= P_0(\delta(x)) = \delta(P_0(x)) \\ &= \delta(\tau(x)\mathbb{1}) = \tau(x)\delta(\mathbb{1}) \\ &= 0. \end{aligned} \tag{2}$$

Since $\mathscr{D}(\delta)$ is dense in $\mathfrak{A}$, $P_\pi(\mathscr{D}(\delta))$ is dense in $P_\pi(\mathfrak{A})$. But Hoegh-Krohn, Landstad and Størmer [5] showed that $P_\pi(\mathfrak{A})$ is finite-dimensional. Therefore, $P_\pi(\mathfrak{A}) = P_\pi(\mathscr{D}(\delta)) \subseteq \mathscr{D}(\delta)$. By (1), $P_\pi(\mathfrak{A})$ is invariant under δ and therefore consists of analytic vectors for δ. Thus the G-finite vectors are a dense set of analytic vectors for δ. Since $\mathfrak{A}$ has a faithful state τ satisfying $\tau(\delta(x)) = 0 (x \in \mathscr{D}(\delta))$ and δ has a dense set of analytic vectors, it follows that δ generates a C^* dynamics [2, Theorem 6]. ■

Remark. If G, in Theorem 1, is abelian, then there is a continuous one parameter subgroup $\{\gamma_t\}$ of G such that $\exp(t\delta) = \alpha_{\gamma_t}$. To see this, assume first that G acts faithfully on $\mathfrak{A}$. According to [8], for each $p \in \hat{G}$, the spectral subspace corresponding to p is one dimensional and is spanned by a unitary $u(p)$. The action of G is given by $\alpha_g(u(p)) = \langle g, p \rangle u(p)$. For each $p, q \in \hat{G}$, there is an $m(p, q) \in \mathbb{T}$ such that

$$u(p)u(q) = m(p, q)u(p + q). \tag{3}$$

Since $\mathbb{C} \cdot u(p)$ is invariant under $\exp(t\delta)$, there is for each $t \in \mathbb{R}$ and $p \in \hat{G}$ a $\lambda(t, p) \in \mathbb{T}$ such that

$$\exp(t\delta)u(p) = \lambda(t, p)u(p). \tag{4}$$

Using this and (3) one checks that $p \to \lambda(t, p)$ is a character of $\hat{G}$ for each $t \in \mathbb{R}$.

Hence there is a unique $\gamma_t \in G$ such that $\lambda(t, p) = \langle \gamma_t, p \rangle$. It follows from (4) that $t \to \gamma_t$ is a one-parameter subgroup of G. For fixed p, $\langle \gamma_t, p \rangle \mathbb{1} = [\exp(t\delta)(u(p))]u(p)^*$. Thus $t \to \langle \gamma_t, p \rangle$ is continuous. Since $\hat{G}$ is discrete, this means that $\{\gamma_t\}$ is a continuous one-parameter subgroup. Finally $\exp(t\delta)u(p) = \langle \gamma_t, p \rangle u(p) = \alpha_{\gamma_t}(u(p))$. Since span $\{u(p) : p \in G\}$ is dense in $\mathfrak{A}$, $\exp(t\delta) = \alpha_{\gamma_t}$.

The remark remains valid in case the action α has a non-trivial kernel N. This is because a continuous one-parameter subgroup of G/N always lifts to a continuous one-parameter subgroup of G. This follows, for instance, from [6, Lemma 3.12].

Theorem 2.1 can be generalized to C^* dynamical systems $(\mathfrak{A}, G, \alpha)$ where G is compact and $\mathfrak{A}$ is G-finite; that is $P_\pi(\mathfrak{A})$ is finite dimensional for all $\pi \in \hat{G}$. For this we need the following lemma, which may be known, since it is a straightforward generalization of [2, Theorem 6].

Lemma 2.2. *Let δ be a closed * derivation in a C^* algebra $\mathfrak{A}$. Suppose that ϕ is a completely positive map of $\mathfrak{A}$ into $\mathscr{B}(\mathscr{H})$, that the corresponding Stinespring representation of $\mathfrak{A}$ is faithful, and that $\phi(\delta(a)) = 0$ for all $a \in \mathscr{D}(\delta)$.*

Assume that either of the following conditions holds:

(1) *δ has a dense set of analytic elements, or*

(2) Range $(\delta + t1)$ *is dense in $\mathfrak{A}$ for all $t \in \mathbb{R} \backslash \{0\}$.*

Then δ generates a C^* dynamics of $\mathfrak{A}$.

Proof. Let $\{\pi, \mathscr{K}\}$ be the Stinespring representation of $\mathfrak{A}$ corresponding to the completely positive map ϕ and let $V : \mathscr{H} \to \mathscr{K}$ satisfy $V^*\pi(a)V = \phi(a) (a \in \mathfrak{A})$. Define an operator H in $\mathscr{K}$ by $\mathscr{D}(H) = \text{span}\{\pi(a)V\xi : a \in \mathscr{D}(\delta), \xi \in \mathscr{H}\}$, $iH\pi(a)V\xi = \pi(\delta(a))V\xi$. Using the derivation identity and the relation $\phi(\delta(a)) = 0$ one can check that H is well-defined and symmetric. Furthermore $\pi(\mathscr{D}(\delta))\mathscr{D}(H) \subseteq \mathscr{D}(H)$ and $\pi(\delta(a))\psi = i[H, a]\psi (a \in \mathscr{D}(\delta), \psi \in \mathscr{D}(H))$. One can now conclude that H is essentially self-adjoint and that δ is a generator as in the proof of [2, Theorem 6]. ■

Theorem 2.3. *Let $(\mathfrak{A}, G, \alpha)$ be a C^* dynamical system with G compact and $\mathfrak{A}$ G-finite, and let δ be a closed * derivation in $\mathfrak{A}$ commuting with α. Assume that if $a \in \mathscr{D}(\delta)$ is fixed by α, then $\delta(a) = 0$. It follows that δ generates a C^* dynamics of $\mathfrak{A}$.*

Proof. The proof of Theorem 2.1 shows that the G-finite elements of $\mathfrak{A}$ are a dense set of analytic elements for δ. If $\{\pi, \mathscr{H}\}$ is a faithful representation of $P_0(\mathfrak{A})$, then $\pi \circ P_0$ is a faithful completely positive map of a $\mathfrak{A}$ satisfying $\pi \circ P_0(\delta(a)) = 0$ for all $a \in \mathscr{D}(\delta)$. It follows from Lemma 2.2 that δ is a generator. ■

3. Compact Transformation Groups

Definitions. (1) Let X be a locally compact Hausdorff space and δ a closed * derivation in $C_0(X)$. A closed subset $M \subseteq X$ is called *self-determining* (or a *restriction set*) for δ if whenever $f \in \mathscr{D}(\delta)$ and $f|_M = 0$, it follows that $\delta(f)|_M = 0$. If M is self-determining, the formula $\delta_M(f|_M) = \delta(f)|_M$ defines a * derivation in $C_0(M)$ with domain $\{f|_M : f \in \mathscr{D}(\delta)\}$.

(2) A * derivation δ in a C^* algebra $\mathfrak{A}$ is said to be *well behaved* if for every $a \in \mathscr{D}(\delta)_{s.a}$, there is a state ϕ of $\mathfrak{A}$ such that $|\phi(a)| = \|a\|$ and $\phi(\delta(a)) = 0$. For a *

derivation δ in an abelian C^* algebra $C_0(X)$, the following is an equivalent condition [11, Proposition 7]: for all $x \in X$ and for all $f \in \mathscr{D}(\delta)_{\text{s.a.}}$, if $\|f\| = |f(x)|$, then $\delta(f)(x) = 0$.

Proposition 3.1. *Let δ be a closed * derivation in $C_0(X)$. Suppose ω is a state of $C_0(X)$ with support $M \subseteq X$ and $\omega(\delta(f)) = 0$ for all $f \in \mathscr{D}(\delta)$. Then M is self-determining and δ_M is closable.*

Proof. For all $f, g \in \mathscr{D}(\delta)$,

$$\omega(f\delta(g)) = \omega(\delta(fg)) - \omega(\delta(f)g) = -\omega(\delta(f)g).$$

If $f|_M = 0$, then for all $g \in \mathscr{D}(\delta)$, $\omega(\delta(f)g) = -\omega(f\delta(g)) = 0$. Since $\mathscr{D}(\delta)$ is dense in $C_0(X)$, it follows that $\delta(f)|_M = 0$. Thus M is self-determining.

Now suppose $\langle f_n \rangle \subseteq \mathscr{D}(\delta), f_n|_M \to 0$ and $\delta(f_n)|_M \to h$ uniformly on M. Then for all $g \in \mathscr{D}(\delta)$, $\omega(\delta(f_n)g) = -\omega(f_n\delta(g)) \to 0$. On the other hand, $\omega(\delta(f_n)g) \to \omega(hg)$. Therefore $\omega(hg) = 0$ for all $g \in \mathscr{D}(\delta)$, and $h = 0$. Thus δ_M is closable.

Theorem 3.2. *Let X be a locally compact Hausdorff space, G a compact group, and $(C_0(X), G, \alpha)$ a C^* dynamical system. Let $\theta: G \times X \to X$ be the corresponding continuous action of G on X. Suppose δ is a closed *-derivation in $C_0(X)$ commuting with α. Assume that if $f \in \mathscr{D}(\delta)$ is fixed by α, then $f \in \ker(\delta)$. Then δ generates a C^* dynamics of $C_0(X)$. If ψ is the action of $\mathbb{R}$ on X corresponding to the C^* dynamics $\exp(t\delta)$, then each orbit of ψ lies in an orbit of θ.*

Proof. We have assumed $P_0(\mathscr{D}(\delta)) \subseteq \ker(\delta)$. Since δ is closed, $\ker(\delta)$ is norm-closed, and it follows that $P_0(C_0(X)) \subseteq \ker(\delta)$.

Let M be an orbit of θ, and let ω be the unique G-invariant state of $C_0(X)$ with support M. Thus $\omega(f) = P_0(f)(x)$ for any $x \in M$. Since $P_0(\delta(f)) = \delta(P_0(f)) = 0$, $\omega(\delta(f)) = 0$ for all $f \in \mathscr{D}(\delta)$. Hence by the proposition, each orbit of M is self-determining and δ_M is closable.

Now $\bar{\delta}_M$ is invariant under the action of G on $C(M)$. By Theorem 1, or [4, Theorem 3.2], $\bar{\delta}_M$ generates a C^* dynamics of $C(M)$. It follows that δ_M is well behaved, and because this is true of each orbit M, δ is well behaved.

Although we do not strictly need this, we wish to observe that each δ_M is already closed. The argument is essentially due to Batty [1, Theorem 6.4]. Let $\mathscr{F}$ be the family of functions $f \in \mathscr{D}(\delta)$ such that $f|_M = 0$ and f is fixed by α. By assumption $\mathscr{F} \subseteq \ker(\delta)$. If $p \in X \backslash M$, there is a $g \in \mathscr{D}(\delta)$ such that $0 \leqq g \leqq 1$, $g(p) = 1$, and $g|_M = 0$, because $\mathscr{D}(\delta)$ is a Šilov subalgebra of $C_0(X)$. Let $f = P_0(g)$. Then $f \in \mathscr{F}$ and $f(p) > 0$. Consequently, $M = \cap\{f^{-1}(0); f \in \mathscr{F}\}$. Because δ is well behaved, it follows from [1, Corollary 4.5] that δ_M is closed.

Next we show that $(I \pm \delta)(\mathscr{D}(\delta))$ is dense in $C_0(X)$. It will then follow that δ is a generator [9, Proposition 4.7]. Let $F \in C_c(X)$ and let K be a saturated (that is, G-invariant) compact subset of X such that $F = 0$ outside K. If M is an orbit of θ, then there is an $f \in \mathscr{D}(\delta)$ such that $(f + \delta(f))|_M = F|_M$, because δ_M is a generator. Let $\varepsilon > 0$. By a compactness argument, there is a saturated open set $U \supseteq M$ such that $|F(x) - (f + \delta(f))(x)| < \varepsilon$ for $x \in U$. By compactness of K, there is a covering $\{U_1, \ldots, U_n\}$ of K by saturated open sets and functions $f_1, \ldots, f_n \in \mathscr{D}(\delta)$ such that $|F(x) - (f_i + \delta(f_i))(x)| < \varepsilon$ for $x \in U_i$.

Let $\{h_1, \ldots, h_n\}$ be a partition of unity over K subordinate to $\{U_i\}$, with each $h_i \in P_0(C(X)) \subseteq \ker(\delta)$. Define $f = \sum f_i h_i$. Then $f \in \mathscr{D}(\delta)$ and $\delta(f) = \sum \delta(f_i) h_i$. Now $F = \sum h_i F$ and $F - (f + \delta(f)) = \sum h_i(F - (f_i + \delta(f_i)))$. It follows that $\|F - (f + \delta(f))\|_\infty < \varepsilon$ and $(I + \delta)(\mathscr{D}(\delta))$ is dense. Similarly $(I - \delta)(\mathscr{D}(\delta))$ is dense, and δ is a generator.

Since δ_M is a generator for each orbit M, there is a one-parameter group $\{\phi_t^M : t \in \mathbb{R}\}$ of homeomorphisms of M such that

$$\delta(f)(p) = \frac{d}{dt}\bigg|_{t=0} f(\phi_t^M(p)) \quad (f \in \mathscr{D}(\delta),\ p \in M). \tag{1}$$

Define $\phi_t(p) = \phi_t^M(p)$ if $p \in M$. For any $f \in \mathscr{D}(\delta)$ and $p_1, p_2 \in X$,

$$\begin{aligned}|f(\phi_t(p_1)) - f(p_2)| &\leqq |f(\phi_t(p_1)) - f(p_1)| + |f(p_1) - f(p_2)| \\ &\leqq \|\delta(f)\|_\infty |t| + |f(p_1) - f(p_2)|.\end{aligned}$$

Thus $(t, p) \to f(\phi_t(p))$ is jointly continuous, and since $\mathscr{D}(\delta)$ is dense in $C_0(X)$, $\phi : \mathbb{R} \times X \to X$ is a jointly continuous action of $\mathbb{R}$ on X. Let $\{\beta_t : t \in \mathbb{R}\}$ be the corresponding C^* dynamics. It is evident from (1) (and a short compactness argument) that the generator D of $\{\beta_t\}$ extends δ. Therefore $D = \delta$ and $\exp(t\delta) = \beta_t$. The statement about orbits is then clear. ■

Remarks. (1) In modern potential theory there is a large literature on dissipative (dispersive) operators D commuting with compact group actions. Under general conditions numerous authors obtain dissipative group invariant extension operators $\tilde{D} \supseteq D$ which generate strongly continuous contractive (respectively, positive) semigroups $\{\exp t\tilde{D}\}_{t \geqq 0}$. But in the different settings the dissipative (respectively, dispersive) condition on the initial operator D is typically *not* a consequence of the group invariance, and is placed as a separate assumption. The relevant dissipativeness assumption for *derivations* is well-behavedness, and it is interesting that, in the present setting, this condition does follow from the assumed group invariance.

(2) We note that δ is implemented by a self-adjoint operator in a suitable representation. Let ω be any faithful G-invariant state of $C_0(X)$. Then ω satisfies $\omega(\delta(f)) = 0$ $(f \in \mathscr{D}(\delta))$. Let $C_0(X)$ act on $L^2(X, \omega)$ by multiplication. Define H in $L^2(X, \omega)$ by $\mathscr{D}(H) = \mathscr{D}(\delta)$, $iHf = \delta(f)$. Then H is essentially self-adjoint, and H implements δ in this representation. That is, $\delta(f)g = i[H, f]g$ $(f \in \mathscr{D}(\delta),\ g \in \mathscr{D}(H))$.

4. Actions on $\mathscr{K}(\mathscr{H})$

In this section we prove an analogue of Theorem 3.2 for the compact operators.

Theorem 4.1. *Let α be a strongly continuous action of a compact group G on the algebra $\mathscr{K}$ of compact operators on a separable Hilbert space $\mathscr{H}$. Let δ be a closed * derivation in $\mathscr{K}$ which commutes with α. Assume that if $a \in \mathscr{D}(\delta)$ is fixed by α, then $\delta(a) = 0$. It follows that δ generates a C^* dynamics.*

Proof. Let $\mathscr{U}$ denote the unitary operators on $\mathscr{H}$, $\mathscr{P}$ the projective unitary group $\mathscr{U}/\mathbb{T} \cdot \mathbb{1}$, and $\pi : \mathscr{U} \to \mathscr{P}$ the natural map. We give $\mathscr{U}$ the weak operator topology,

$\mathscr{P}$ the quotient topology and Aut$(\mathscr{K})$ the point-norm topology ($\alpha_n \to 1$ if $\|\alpha_n(x) - x\| \to 0$ for all $x \in \mathscr{K}$). Then there is a topological group isomorphism $\phi: \mathscr{P} \to$ Aut $(\mathscr{K})$ given by $\phi(\pi(u))(x) = uxu^*$.

According to [10, pp. 103–111], there is an exact sequence of compact groups

$$1 \to \mathbb{T} \to H \xrightarrow{j} G \to 1,$$

and there is a continuous homomorphism $\sigma: H \to \mathscr{U}$ such that the following diagram commutes:

$$\begin{array}{ccc} \mathscr{U} & \xrightarrow{\pi} & \mathscr{P} \\ \sigma \uparrow & & \uparrow \phi^{-1} \circ \alpha \\ H & \xrightarrow{j} & G \end{array}$$

Thus $\alpha \circ j(h)(x) = \sigma(h) x \sigma(h)^* (h \in H, x \in \mathscr{K})$. Replacing G by H and α by $\alpha \circ j$, we can assume that α is implemented by a continuous unitary representation σ of G.

A finite rank projection e is fixed by α if and only if $e\mathscr{H}$ is reducing for σ. Since G is compact, there is an increasing sequence $\langle e_n \rangle$ of such projections with strong-limit $(e_n) = \mathbb{1}$. By assumption $P_0(\mathscr{K}) \subseteq \ker(\delta)$; hence $e_n \in \ker(\delta)$ and $\delta(e_n a e_n) = e_n \delta(a) e_n (a \in \mathscr{D}(\delta))$. Since $e_n \mathscr{D}(\delta) e_n$ is dense in $e_n \mathscr{K} e_n$ and the latter algebra is finite dimensional, we have $e_n \mathscr{K} e_n = e_n \mathscr{D}(\delta) e_n \subseteq \mathscr{D}(\delta)$. The restriction of δ to $e_n \mathscr{K} e_n$ is necessarily a bounded derivation of this algebra. Let $\mathscr{K}_0 = \bigcup_n e_n \mathscr{K} e_n$. It follows that $\delta | \mathscr{K}_0$ is well behaved and $(I \pm \delta)\mathscr{K}_0 = \mathscr{K}_0$. But since $\|e_n a e_n - a\| \to 0$ for all $a \in \mathscr{K}$, it follows that $\mathscr{K}_0$ is a core for δ. Hence δ is a generator. ∎

We suspect that there should be a common generalization of Theorem 3.2 and 4.1 to a wider class of C^* algebras.

Notes added in proof: (1) After completing this paper we learned that C. Peligrad [13] and A. Ikunishi [12] have also studied closed * derivations commuting with compact group actions. Our Theorem 2.1 was first proved by Peligrad.

(2) One can give an example showing that the assumption that G is compact is essential in Theorem 2.1.

(3) Further progress has been made on the problem considered here.

Let $(\mathfrak{A}, G, \alpha)$ be a C^* dynamical system with G compact. Let δ be a closed * derivation in $\mathfrak{A}$ which commutes with α. Suppose the fixed point algebra $\mathfrak{A}^\alpha \subseteq \ker(\delta)$.

Theorem [14]. If G is abelian, then δ is a generator.
Theorem [15]. If G is separable and $\mathfrak{A}$ is type I separable, then δ is a generator.

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