Complementability of spaces of affine continuous functions on simplices*

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Abstract

We construct metrizable simplices X_1 and X_2 and a homeomorphism $\varphi : \overline{\operatorname{ext} X_1} \to \overline{\operatorname{ext} X_2}$ such that $\varphi(\operatorname{ext} X_1) = \operatorname{ext} X_2$, the space $\mathfrak{A}(X_1)$ of all affine continuous functions on X_1 is complemented in $\mathcal{C}(X_1)$ and $\mathfrak{A}(X_2)$ is not complemented in any $\mathcal{C}(K)$ space. This shows that complementability of the space $\mathfrak{A}(X)$ cannot be determined by topological properties of the couple $(\operatorname{ext} X, \overline{\operatorname{ext} X})$.

1 Introduction

A Banach space X is called an L^1*–predual* if X^* is isometric to some $L^1(\mu)$ space. A particular example of an L^1–predual is the space $\mathcal{C}(K)$ of all continuous functions on a compact space K. There was a question how "different" an L^1–predual can be from $\mathcal{C}(K)$–spaces which was answered by Y. Benyamini and J. Lindenstrauss in [3] where they constructed an ℓ^1–predual that is not complemented in any $\mathcal{C}(K)$–space.

The method of their construction was to find a suitable compact convex subset X of a locally convex space such that X is a simplex and the space $\mathfrak{A}(X)$ of all continuous affine functions on X is not complemented in any $\mathcal{C}(K)$–space (we refer reader to the next section for the notions not explained here). As it is known, the space $\mathfrak{A}(X)$ on a simplex X is an example of an L^1–predual space (see [6, Proposition 3.23]).

*Research was supported in part by the grants GA ČR 201/06/0018, GA ČR 201/07/0388, and in part by the Research Project MSM 0021620839 from the Czech Ministry of Education.

Received by the editors July 2007.

Communicated by F. Bastin.

2000 *Mathematics Subject Classification :* 46A55 (primary), 46B03 (secondary).

Key words and phrases : simplex, L^1–predual, complementability, affine functions.

Since some properties of $\mathfrak{A}(X)$ on a simplex X can be characterized by topological properties of the set $\operatorname{ext} X$ of all extreme points of X (see e.g. [6, Proposition 3.15] or [10, Theorem 1]), it seems natural to ask a similar question for the problem of complementability of $\mathfrak{A}(X)$ in a $\mathcal{C}(K)$–space. The aim of this note is to show that this is not the case.

We prove even more, namely that complementability of $\mathfrak{A}(X)$ on a simplex X cannot be determined by topological properties of the pair $(\operatorname{ext} X, \overline{\operatorname{ext} X})$. By a modification of the method of [3] we get the following theorem.

Theorem 1.1. *There exist metrizable simplices X_1 and X_2 and a homeomorphic mapping $\varphi : \overline{\operatorname{ext} X_1} \to \overline{\operatorname{ext} X_2}$ such that the sets $\operatorname{ext} X_1$, $\operatorname{ext} X_2$ are countable, $\varphi(\operatorname{ext} X_1) = \operatorname{ext} X_2$, $\mathfrak{A}(X_1)$ is complemented in $\mathcal{C}(X_1)$ and $\mathfrak{A}(X_2)$ is not complemented in any $\mathcal{C}(K)$ space.*

We remark that the simplices X_1, X_2 are constructed in such a way that the sets of extreme points are of type F_σ (i.e., it is a countable union of closed sets). This might be of some interest since the structure of simplices with extreme points being F_σ–set is more transparent (see e.g. [11, Théorème 80] or [9, Corollary 3.5]).

2 Preliminaries

All topological space will be considered as Hausdorff. If K is a compact space, we denote by $\mathcal{C}(K)$ the space of all continuous real–valued functions on K. We will identify the dual of $\mathcal{C}(K)$ with the space $\mathcal{M}(K)$ of all Radon measures on K. Let $\mathcal{M}^1(K)$ denote the set of all probability Radon measures on K and let ε_x stand for the Dirac measure at $x \in K$.

2.1 Function spaces

Throughout the paper we will consider a *function space* $\mathcal{H}$ on a compact space K. By this we mean a (not necessarily closed) linear subspace of $\mathcal{C}(K)$ containing the constant functions and separating the points of K. Let $\mathcal{M}_x(\mathcal{H})$ be the set of all *$\mathcal{H}$–representing measures* for $x \in K$, i.e.,

$$\mathcal{M}_x(\mathcal{H}) = \{\mu \in \mathcal{M}^1(K) : f(x) = \int_K f\, d\mu \text{ for any } f \in \mathcal{H}\}.$$

If $\mu \in \mathcal{M}_x(\mathcal{H})$, we say that x is a *barycenter of* μ and denote $x = r(\mu)$. Where no confusion can arise we simply say that μ represents x.

The set

$$\operatorname{Ch}_{\mathcal{H}} K = \{x \in K : \mathcal{M}_x(\mathcal{H}) = \{\varepsilon_x\}\}$$

is called the *Choquet boundary* of $\mathcal{H}$. It may be highly irregular from the topological point of view but it is a G_δ–set if K is metrizable (see [6, Proposition 2.9]).

Given a function space $\mathcal{H}$ on a compact space K we can define the set of *$\mathcal{H}$–affine continuous functions* as follows

$$\mathcal{A}^c(\mathcal{H}) = \{f \in \mathcal{C}(K) : f(x) = \int_K f\, d\mu \text{ for any } x \in K \text{ and } \mu \in \mathcal{M}_x(\mathcal{H})\}.$$

Clearly, $\mathcal{H} \subset \mathcal{A}^c(\mathcal{H})$.

We say that a function $h \in \mathcal{H}$ is *$\mathcal{H}$–exposing* for $x \in K$ if h attains its extremal value precisely at x. Obviously, any $\mathcal{H}$–exposed point is contained in the Choquet boundary of $\mathcal{H}$.

2.2 Examples of function spaces

We introduce the following main examples of function spaces.

In the *"convex case"*, the function space $\mathcal{H}$ is the linear space $\mathfrak{A}(X)$ of all continuous affine functions on a compact convex subset X of a locally convex space. In this example, the Choquet boundary of $\mathfrak{A}(X)$ coincides with the set of all extreme points of X (see [2, Theorem 6.3]) and is denoted by $\operatorname{ext} X$.

Further, the barycenter of a probability measure μ on X is the unique point $r(\mu) \in X$ for which $f(r(\mu)) = \mu(f)$ for any $f \in \mathfrak{A}(X)$, in other words, x is $\mathfrak{A}(X)$–represented by μ.

In the *"harmonic case"*, U is a bounded open subset of the Euclidean space $\mathbb{R}^m$ and the corresponding function space $\mathcal{H}$ is $\mathbf{H}(U)$, i.e., the family of all continuous functions on $\overline{U}$ which are harmonic on U. In the "harmonic case", the Choquet boundary of $\mathbf{H}(U)$ coincides with the set $\partial_{\text{reg}} U$ of all regular points of U (see [8, Theorem]).

2.3 Simplicial functions spaces

If $\mathcal{H}$ is a function space on a metrizable compact space K, for any $x \in K$ there exists a measure $\mu \in \mathcal{M}_x(\mathcal{H})$ such that $\mu(K \setminus \operatorname{Ch}_{\mathcal{H}} K) = 0$ (see e.g. [6, Theorem 2.10]).

If this measure is uniquely determined for every $x \in K$, we say that $\mathcal{H}$ is a *simplicial function space*. In the "convex case" it is equivalent to say that X is a *Choquet simplex*, briefly *simplex* (see [1, Theorem II.3.6], [2, Theorem 7.3] or [6]).

As another example of a simplicial function space serves the space $\mathbf{H}(U)$ from the "harmonic case" (see e.g. [8, Theorem]).

2.4 State space

By a standard technique briefly described below any function space can be viewed as the space $\mathfrak{A}(X)$ of affine continuous functions on a suitable compact convex set X. Details can be found in [1, Chapter 2, § 2], [2, Chapter 1, § 4] or [7, Section 6].

If $\mathcal{H}$ is a function space on a compact space K, we set

$$\mathbf{S}(\mathcal{H}) = \{\varphi \in \mathcal{H}^* : \|\varphi\| = \varphi(1) = 1\} .$$

Then $\mathbf{S}(\mathcal{H})$ endowed with the weak* topology is a compact convex set which is metrizable if K is metrizable. Let $\phi : K \to \mathbf{S}(\mathcal{H})$ be the evaluation mapping defined as $\phi(x) = s_x$, $x \in K$, where $s_x(h) = h(x)$ for $h \in \mathcal{H}$. Then ϕ is a homeomorphic embedding of K onto $\phi(K)$ and $\phi(\operatorname{Ch}_{\mathcal{H}} K) = \operatorname{ext} \mathbf{S}(\mathcal{H})$.

Let $\Phi : \mathcal{H} \to \mathfrak{A}(\mathbf{S}(\mathcal{H}))$ be the mapping defined for $h \in \mathcal{H}$ by $\Phi(h)(s) = s(h)$, $s \in \mathbf{S}(\mathcal{H})$. Then Φ serves as an isometric isomorphism of $\mathcal{H}$ into $\mathfrak{A}(\mathbf{S}(\mathcal{H}))$. Further,

Φ is onto if and only if the function space $\mathcal{H}$ is uniformly closed in $\mathcal{C}(K)$. In this case the inverse mapping is realized by

$$\Phi^{-1}(F) = F \circ \phi, \quad F \in \mathfrak{A}(\mathbf{S}(\mathcal{H})) .$$

In the sequel we will need the following theorem.

Theorem 2.1. *Let $\mathcal{H}$ be a closed function space on a metrizable compact space K. Then the following assertions are equivalent:*

(i) $\mathcal{H}$ is simplicial;

(ii) the state space $\mathbf{S}(\mathcal{A}^c(\mathcal{H}))$ is a simplex.

Proof. See [4, Theorem]. ■

By a projection, we always mean a bounded linear operator P on a Banach space such that $P = P^2$.

Without explicit mentioning, every Banach space is assumed to be a subspace of its second dual via its canonical embedding.

3 Construction

Definition 3.1. For a Banach space X we define

$$\lambda(X) = \inf \|T\| \, \|T^{-1}\| \, \|P\|,$$

where the infimum is taken over all isomorphisms T from X into a $\mathcal{C}(K)$ space and all projections $P : \mathcal{C}(K) \to TX$. If X is not isomorphic to a complemented subspace of any $\mathcal{C}(K)$–space, we put $\lambda(X) = \infty$.

Lemma 3.2. *Let X be a Banach space and B_{X^*} be its dual unit ball endowed with the weak* topology. Then*

$$\lambda(X) = \inf\{\|P\| : P \text{ is a projection of } \mathcal{C}(B_{X^*}) \text{ onto } X\} .$$

Proof. See [3, Lemma]. ■

Lemma 3.3. *Let Y be a 1–complemented subspace of a Banach space X. Then $\lambda(Y) \leq \lambda(X)$.*

Proof. Let $T : X \to Y$ be a projection of norm 1. We will show that for every projection $P : C(B_{X^*}) \to X$ we can find a projection $Q : C(B_{Y^*}) \to Y$ such that $\|P\| = \|Q\|$. Then, by Lemma 3.2, $\lambda(Y) \leq \lambda(X)$. If $\pi : X^* \to Y^*$ denotes the restriction operator, then $Q : C(B_{Y^*}) \to Y$ defined as

$$Q : f \mapsto TP(f \circ \pi) , \quad f \in \mathcal{C}(B_{Y^*}) ,$$

is a projection of norm $\|P\|$. This finishes the proof. ■

3.1 Construction

Let $\mathcal{H}$ be a simplicial function space on a compact space K such that $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$. Let

$$L = \bigcup_{i\in\mathbb{N}, j=1,2,3} K_{ij} \cup \{p, q\} \cup \{r_{ij} : i = -1, 0, 1, j = 1, 2, 3\} ,$$

where each K_{ij} is a copy of K. The topology on L is defined as follows: a basis of the neighborhoods of r_{0j}, $j = 1, 2, 3$, is given by the sets $\{r_{0j}\} \cup \bigcup_{i=n}^{\infty} K_{ij}$, $n \in \mathbb{N}$, each K_{ij} is both closed and open in L and all the remaining points are isolated.

Let

$$\mathcal{H}_1 = \{f \in \mathcal{C}(L) : f \restriction_{K_{ij}} \in \mathcal{H}, i \in \mathbb{N}, j = 1, 2, 3, \\ 2f(r_{0j}) = f(r_{-1j}) + f(r_{1j}), j = 1, 2, 3\} ,$$

and

$$\mathcal{H}_2 = \{f \in \mathcal{C}(L) : f \restriction_{K_{ij}} \in \mathcal{H}, i \in \mathbb{N}, j = 1, 2, 3, 2f(r_{01}) = f(p) + f(q), \\ 3f(r_{02}) = 2f(p) + f(q), 3f(r_{03}) = f(p) + 2f(q)\} .$$

It is straightforward to verify that $\mathcal{H}_1$, $\mathcal{H}_2$ are function spaces on L.

Lemma 3.4. *Let $\mathcal{H}$ be a simplicial function space on a compact space K such that $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$ and let and $\mathcal{H}_1, \mathcal{H}_2$ be the function spaces on a compact space L constructed above. Then*

(a) $\mathrm{Ch}_{\mathcal{H}_1} L = \mathrm{Ch}_{\mathcal{H}_2} L$, and if $\mathrm{Ch}_{\mathcal{H}} K$ is of type F_σ, then $\mathrm{Ch}_{\mathcal{H}_1} L$ is an F_σ–set as well;

(b) both $\mathcal{H}_1$ and $\mathcal{H}_2$ are simplicial;

(c) $\mathcal{A}^c(\mathcal{H}_1) = \mathcal{H}_1$, $\mathcal{A}^c(\mathcal{H}_2) = \mathcal{H}_2$;

(d) if $\mathcal{H}$ is C–complemented in $\mathcal{C}(K)$, then $\mathcal{H}_1$ is $\max\{C, 3\}$–complemented in $\mathcal{C}(L)$;

(e) $\lambda(\mathcal{H}_2) \geq \lambda(\mathcal{H}) + (500\lambda(\mathcal{H}))^{-1}$.

Proof. For the proof of (a) it is enough to show that both the sets $\mathrm{Ch}_{\mathcal{H}_1} L$ and $\mathrm{Ch}_{\mathcal{H}_2} L$ equal

$$\{r_{ij} : i = -1, 1, j = 1, 2, 3\} \cup \{p, q\} \cup \bigcup_{i\in\mathbb{N}, j=1,2,3} \mathrm{Ch}_{\mathcal{H}} K_{ij}.$$

Indeed, for a point $x \in K_{ij}$ we have

$$x \in \mathrm{Ch}_{\mathcal{H}_1} L \Leftrightarrow x \in \mathrm{Ch}_{\mathcal{H}_2} L \Leftrightarrow x \in \mathrm{Ch}_{\mathcal{H}} K_{ij} ,$$

as the characteristic function $\chi_{K_{ij}} \in \mathcal{H}_1 \cap \mathcal{H}_2$, and hence every measure $\mu \in \mathcal{M}_x(\mathcal{H}_1) \cup \mathcal{M}_x(\mathcal{H}_2)$ is supported by K_{ij}. For the points

$$\{r_{ij} : i = -1, 1, j = 1, 2, 3\} \cup \{p, q\},$$

it is easy to find $\mathcal{H}_1$–exposing and $\mathcal{H}_2$–exposing functions and thus all these points belong to $\operatorname{Ch}_{\mathcal{H}_1} L \cap \operatorname{Ch}_{\mathcal{H}_2} L$.

On the other hand, the points $\{r_{0j} : j = 1,2,3\}$ have $\mathcal{H}_1$–representing measures

$$\frac{1}{2}\left(\varepsilon_{r_{-1,1}} + \varepsilon_{r_{1,1}}\right), \quad \frac{1}{2}\left(\varepsilon_{r_{-1,2}} + \varepsilon_{r_{1,2}}\right), \quad \frac{1}{2}\left(\varepsilon_{r_{-1,3}} + \varepsilon_{r_{1,3}}\right), \tag{1}$$

respectively, and $\mathcal{H}_2$–representing measures

$$\frac{1}{2}\left(\varepsilon_p + \varepsilon_q\right), \quad \frac{1}{3}\left(2\varepsilon_p + \varepsilon_q\right), \quad \frac{1}{3}\left(\varepsilon_p + 2\varepsilon_q\right), \tag{2}$$

respectively, and hence they do not belong to the Choquet boundaries $\operatorname{Ch}_{\mathcal{H}_1} L$ and $\operatorname{Ch}_{\mathcal{H}_2} L$.

To show (b), let x be a point of L. If $x \in K_{ij}$ for some i, j, then x has a unique $\mathcal{H}_1$–representing measure and a unique $\mathcal{H}_2$–representing measure, both supported by the Choquet boundary of L, since $\mathcal{H}$ is simplicial and every $\mathcal{H}_1$ or $\mathcal{H}_2$–representing measure is supported by K_{ij}.

To finish the reasoning it is enough to notice that the points $r_{0j},\ j = 1,2,3$, have uniquely determined $\mathcal{H}_1$ and $\mathcal{H}_2$–representing measures carried by the Choquet boundary of L (see (1) and (2)).

For the proof of (c), let f be a function from $\mathcal{A}^c(\mathcal{H}_1)$. By the assumption, $f \restriction_{K_{ij}} \in \mathcal{H}$ for each K_{ij} and, obviously, f satisfies $2f(r_{0j}) = f(r_{-1j}) + f(r_{1j}),\ j = 1,2,3$. Hence $f \in \mathcal{H}_1$.

Analogously, $\mathcal{A}^c(\mathcal{H}_2) = \mathcal{H}_2$.

To verify (d), we assume that $P : \mathcal{C}(K) \to \mathcal{H}$ is a projection of the norm C. We define an operator $Q : \mathcal{C}(L) \to \mathcal{H}_1$ as

$$(Qf)(x) = \begin{cases} P(f \restriction_{K_{ij}})(x)\,, & x \in K_{ij}, \\ f(x)\,, & x = p, q, r_{ij},\ i = 0, -1, j = 1,2,3, \\ 2f(r_{0j}) - f(r_{-1j})\,, & x = r_{1j},\ j = 1,2,3. \end{cases}$$

It can be easily verified that Q is a projection of $\mathcal{C}(L)$ onto $\mathcal{H}_1$ and $\|Q\| = \max\{C, 3\}$.

For the proof of (e), we define a compact space $\widetilde{L} = L \setminus \{r_{ij}; i = -1, 1, j = 1,2,3\}$ and a function space $\widetilde{\mathcal{H}_2} = \{f \restriction_{\widetilde{L}} \colon\ f \in \mathcal{H}_2\}$. Then $\widetilde{\mathcal{H}_2}$ can be considered to be a subspace of $\mathcal{H}_2$ via the isometric isomorphism $E : \widetilde{\mathcal{H}_2} \to \mathcal{H}_2$ defined as

$$(Ef)(x) = \begin{cases} f(r_{0j})\,, & x = r_{ij},\ i = 1, -1, j = 1,2,3, \\ f(x)\,, & \text{elsewhere.} \end{cases}$$

By [3, Theorem], $\lambda(\widetilde{\mathcal{H}_2}) \geq \lambda(\mathcal{H}) + (500\lambda(\mathcal{H}))^{-1}$.

Since the operator $T : \mathcal{H}_2 \to \widetilde{\mathcal{H}_2}$ defined as

$$Tf = E(f \restriction_{\widetilde{L}})\,, \quad f \in \mathcal{H}_2\,,$$

is a projection of norm 1, we get from Lemma 3.3 that $\lambda(\widetilde{\mathcal{H}_2}) \leq \lambda(\mathcal{H}_2)$. Hence $\lambda(\mathcal{H}_2) \geq \lambda(\mathcal{H}) + (500\lambda(\mathcal{H}))^{-1}$, which completes the proof. ■

4 Proof of the theorem

We start with a simplicial function space $\mathcal{H}$ on a metrizable compact space L such that $\mathcal{H} = \mathcal{A}^c(\mathcal{H})$, $\mathcal{H}$ is 1–complemented in $\mathcal{C}(L)$ and $\operatorname{Ch}_{\mathcal{H}} L$ is of type F_σ (the simplest choice is to take L as a singleton and $\mathcal{H} = \mathcal{C}(L)$). We define two sequences $\{(L^n, \mathcal{H}_1^n)\}$, $\{(L^n, \mathcal{H}_2^n)\}$ of function spaces as follows: $(L^1, \mathcal{H}_1^1) = (L^1, \mathcal{H}_2^1) = (L, \mathcal{H})$, and for $n \in \mathbb{N}$, the space $(L^{n+1}, \mathcal{H}_1^{n+1})$ is the space $\mathcal{H}_1$ from Lemma 3.4 constructed from $(L^n, \mathcal{H}_1^n)$ and $(L^{n+1}, \mathcal{H}_2^{n+1})$ is the space $\mathcal{H}_2$ constructed from $(L^n, \mathcal{H}_2^n)$.

Finally, let

$$L_\infty = \bigcup_{n=1}^{\infty} L_n \cup \{x_\infty\}$$

be the one–point compactification of the topological sum of L^n's and

$$\mathcal{H}_i = \{f \in \mathcal{C}(L_\infty) : \ f \restriction_{L^n} \in \mathcal{H}_i^n, \ n \in \mathbb{N}\} , \quad i = 1, 2 .$$

Given $i \in \{1, 2\}$, it is easy to realize that $\mathcal{H}_i$ is a simplicial function space, $\mathcal{A}^c(\mathcal{H}_i) = \mathcal{H}_i$ and

$$\operatorname{Ch}_{\mathcal{H}_i} L_\infty = \{x_\infty\} \cup \bigcup_{n=1}^{\infty} \operatorname{Ch}_{\mathcal{H}_i^n} L^n .$$

In particular, $\operatorname{Ch}_{\mathcal{H}_1} L = \operatorname{Ch}_{\mathcal{H}_2} L$ and it is an F_σ–set (see Lemma 3.4(a)).

According to Lemma 3.4(d), $\mathcal{H}_1^n$ is 3–complemented in $\mathcal{C}(L^n)$ for each $n \in \mathbb{N}$. It follows that $\mathcal{H}_1$ is 3–complemented in $\mathcal{C}(L_\infty)$.

Indeed, if $P_n : \mathcal{C}(L^n) \to \mathcal{H}_1^n$ is a projection with $\|P_n\| \le 3$, the mapping $Q : \mathcal{C}(L_\infty) \to \mathcal{H}_1$ defined as

$$Qf(x) = \begin{cases} (P_n f)(x) , & x \in L^n, n \in \mathbb{N} , \\ f(x_\infty) , & x = x_\infty , \end{cases} \tag{3}$$

is a projection of $\mathcal{C}(L_\infty)$ onto $\mathcal{H}_1$.

On the other hand, by Lemma 3.4(e), $\lambda(\mathcal{H}_2^n) \to \infty$. Since each $\mathcal{H}_2^n$ is 1–complemented in $\mathcal{H}_2$, $\mathcal{H}_2$ is not complemented in any $\mathcal{C}(K)$ space (see Lemma 3.3).

The desired simplices X_1, X_2 will be the state spaces $\mathbf{S}(\mathcal{H}_1)$ and $\mathbf{S}(\mathcal{H}_2)$ (use Theorem 2.1). Let $\phi_i : L_\infty \to \mathbf{S}(\mathcal{H}_i)$, $i = 1, 2$, be the respective homeomorphic embeddings. Then $\phi = \phi_2 \circ \phi_1^{-1}$ is a homeomorphism of $\overline{\operatorname{ext} X_1}$ onto $\overline{\operatorname{ext} X_2}$ such that

$$\phi(\operatorname{ext} X_1) = \phi_2(\operatorname{Ch}_{\mathcal{H}_1} L_\infty) = \phi_2(\operatorname{Ch}_{\mathcal{H}_2} L_\infty) = \operatorname{ext} X_2 .$$

Since $\mathcal{H}_1$ is complemented in $\mathcal{C}(L_\infty)$, $\mathfrak{A}(X_1)$ is complemented in $\mathcal{C}(X_1)$ as well. Indeed, using (3) we can define the mapping

$$\tilde{Q} f = \Phi_1 Q(f \circ \phi_1) , \quad f \in \mathcal{C}(X_1) ,$$

to get a projection of $\mathcal{C}(X_1)$ onto $\mathfrak{A}(X_1)$ (we recall that Φ_1 is the isometric isomorphism of $\mathcal{H}_1$ onto $\mathfrak{A}(X_1)$.

As $\mathfrak{A}(X_2)$ is isometric with $\mathcal{H}_2$, $\mathfrak{A}(X_2)$ is not complemented in any $\mathcal{C}(K)$ space. This finishes the proof.

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