

DIRICHLET FINITE BIHARMONIC FUNCTIONS ON THE POINCARÉ N -BALL

BY D. HADA, L. SARIO, AND C. WANG¹

Communicated by W. T. Martin, February 8, 1973

On a Riemannian manifold R , let $\Delta = d\delta + \delta d$ be the Laplace-Beltrami operator. By definition, a sufficiently smooth function u on R is harmonic (biharmonic) if $\Delta u = 0$ ($\Delta^2 u = 0$). Denote by D the class of those functions f on R for which $D(f) = \int_R df \wedge * df$ is well defined and finite.

By the Poincaré N -ball we mean the ball

$$B_\alpha^N = \{x = (x^1, \dots, x^N) \mid |x| < 1\},$$

α constant, endowed with the Poincaré-type metric

$$ds_\alpha = \lambda(x) |dx|, \quad \lambda(x) = (1 - |x|^2)^\alpha.$$

The first purpose of this paper is to determine those values of the parameter α for which the class $H^2D(B_\alpha^N)$, $N \geq 3$, of Dirichlet finite nonharmonic biharmonic functions on B_α^N is nonvoid. In Sario-Wang [3] it was proved that $H^2D(B_\alpha^N) \neq \emptyset$ for $N = 3$ if and only if $\alpha > -3/5$, and the question was raised whether the same is true for every N if and only if $\alpha > -3/(N + 2)$. We show that this is indeed so if $3 \leq N \leq 6$. However, quite unexpectedly, for $N > 6$ it turns out that $H^2D(B_\alpha^N) \neq \emptyset$ if and only if $\alpha \in (-3/(N + 2), 5/(N - 6))$.

The above result has interesting applications to the classification theory. Let Q be the class of quasiharmonic functions u , defined by $\Delta u = 1$, and denote by QD the subclass of Dirichlet finite functions in Q . The classes O_G , O_{QD} , and O_{H^2D} of Riemannian manifolds without Green's functions, QD -functions, and H^2D -functions, respectively, have the following properties:

- (i) For every N , the classes O_{QD} and O_{H^2D} decompose the totality of Riemannian N -manifolds into three nonempty disjoint subclasses.
- (ii) For every N , the class $O_G - O_{H^2D}$ is nonvoid.
- (iii) For $N > 6$, the classes O_G and O_{H^2D} decompose the totality of Riemannian N -manifolds into four nonempty disjoint subclasses.
- (iv) The unit N -ball with the natural metric $(1 - |x|^2) |dx|$ belongs to O_{H^2D} if and only if $N > 10$.

The proofs will appear in [1].

AMS (MOS) subject classifications (1970). Primary 31B30.

¹ This work was sponsored by the U.S. Army Research Office—Durham, Grant DA-ARO-D-31-124-71-G181, University of California, Los Angeles.

BIBLIOGRAPHY

1. D. Hada, L. Sario and C. Wang, *Dirichlet finite biharmonic functions on the Poincaré N -ball*, J. Reine Angew. Math. (to appear).
2. L. Sario and C. Wang, *Quasiharmonic functions on the Poincaré N -ball*, Rend. Mat. (to appear).
3. ———, *Existence of Dirichlet finite biharmonic functions on the Poincaré 3-ball*, Pacific J. Math. (to appear).

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, LOS ANGELES, CALIFORNIA
90024