BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 4, Number 3, May 1981

EINAR HILLE

(June 28, 1894–February 12, 1980)

BY NELSON DUNFORD[1]

Einar Hille died February 12, 1980 slightly more than a month after the conference, commemorating his eighty-fifth birthday, was held at the Irvine branch of the University of California.

He was born June 28, 1894, son of Professor Carl August Heuman and Edla Eckman. His mother later changed her name to Hille, a mistranslation of Heuman.

Hille's early interests were in mathematics, organic chemistry and linguistics. He entered Stockholm's Högskola in 1911 as a chemistry student and published a paper [**0**] in biochemistry jointly with his professor, Hans von Euler, who later was awarded the Nobel prize in chemistry and who was remotely related to the Swiss mathematician Leonard Euler. This paper was not Hille's first appearance in print. In December 1912 he reviewed the work which led to the two Nobel prizes in chemistry awarded to the Frenchmen (François Auguste) Victor Grignard and Paul Sabattier.

Since the laboratory part of chemistry did not appeal to Hille, he attended lectures on elliptic functions given by Ivar Bendixson (1861–1935). Bendixson was a liberal politician, a member of the city council, a pupil of Mittag-Leffler and an excellent lecturer. It was Bendixson who won Hille over to mathematics. The following year he attended lectures of Helge von Koch (1870–1924) on differential equations, a subject which later became an important part of Hille's life work. He also studied von Koch's specialty, infinite determinants, which he used ten years later.

In 1913 Hille received the AB (Cum Laude Approbatur) in mathematics, and in the fall of 1914 he started his mathematical studies in earnest. He also attended lectures of the economist Gustaf Cassel and, for a term, those of Torgny Segerstedt on Indian religions.

Of his teachers, I. Fredholm had little influence on him. Fredholm was uninspiring and shy, but he did give Hille a general interest in mathematical physics. Marcel Riesz had by far the greatest influence on him, not only as a teacher, but also as an inspiring associate for many years. Through him Hille was introduced to what was then modern analysis and developed an abiding interest in Fourier series, Dirichlet series and the theory of summability. Riesz suggested the topic of Hille's first mathematical investigation, a study of the

1980 *Mathematics Subject Classification*. Primary 01A70.

[1] I am indebted to Kirsti Hille for many of Einar Hille's personal documents and for her answers to my questions. Her help and my forty-four years of active association with him have provided material for this sketch of his life.

© 1981 American Mathematical Society
0002-9904/81/0000-0201/$05.25

variation in the arc-length of images of concentric circles under conformal mapping [1], earning him the Licentiate of Philosophy degree in 1916.

During 1916–1917 Hille did his military service as a typist in the Swedish Army. It was 325 miles to the nearest scientific library, but he had books of his own and could borrow what he needed through the local junior college. He had much free time and used it fully. Three investigations resulted: a continuation of [1], never published, a brief note on the relation between Dirichlet series and binomial series, later incorporated in [17], published in 1926, and an investigation on the solutions of Legendre's differential equation for arbitrary complex values of the parameter. This grew into his dissertation [2] of 1918 which gave him the Ph.D. degree and, half a year later, the title of docent and the right to lecture. He was awarded a Mittag-Leffler Prize for his dissertation. It contained many valuable ideas, not yet fully developed. One of the most important was the use of integral identities, later known as Green's transforms, attached to a differential equation. These served as a point of departure for his later work at Harvard. The study of certain orthogonal systems, introduced in [2], was further extended, almost 30 years later, by Harry Pollard, during and after his appointment as a Jowitt Fellow at Yale.

Hille obtained leave to act as one of the secretaries of the 4th Scandinavian Mathematical Congress in Stockholm. He was also able to attend the disputation of his lifelong friend, Harald Cramér.

The work in the Swedish Civil Service, 1918–1920, plus the teaching at the University, left him little time for research, but the basic idea of [9], Dirichlet series with complex exponents, hails from this period.

In 1920 he received a fellowship from the Swedish-American Foundation. He wanted to work at Harvard with George David Birkhoff, and in particular in the field of difference equations, in which Birkhoff was one of the leaders. In the fall of 1917, Hille had done some work in this field, but gave it up when he realized that R. D. Carmichael had already obtained most of the results. Birkhoff was also a leader in the field of differential equations. When Hille came to Harvard, it turned out that a systematic use of the integral identities mentioned above gave valuable information concerning the distribution in the complex plane of the zeros of solutions of linear second order equations. With the encouragement of Birkhoff, Hille took up this field. It resulted in papers [3]–[7], [10]–[13], [15], [16], published during 1921–1924. The resulting theory of oscillation theorems in the complex domain has been ably summarized by E. L. Ince in Chapter XXI of his *Ordinary differential equations*. Of his later papers [23], [20], [53], [86], [92] in part, and [94], belong to the same general range of ideas. The first deals with connections between his investigations and those of Rolf Nevanlinna on the distribution of values of meromorphic functions. Actually, it turned out that Hille's examples of meromorphic functions with preassigned equal defects, formed by quotients of solutions of a class of differential equations with polynomial coefficients, were the first valid ones in the literature. R. Nevanlinna in 1929 succeeded in showing that, in a sense, all meromorphic functions having these defect properties were obtainable in this manner. An ingenious use of the

Schwarzian derivative was the decisive step. In 1948 Zeev Nehari proved an important inequality for the Schwarzian derivative of a univalent function in which one of Hille's integral identities was useful. Later Hille proved that Nehari's inequality is the best possible by providing a counterexample in **[94]**.

Hille's stay at Harvard lasted two years, the second year as Benjamin Peirce Instructor. These years were decisive for his future development. Hille was always grateful for Birkhoff's helpful interest, an interest which lasted until Birkhoff's untimely death in 1945.

The association with G. D. Kellogg in 1921–1922 was fruitful, and Bouton, Coolidge and Osgood did much to round out Hille's mathematical education. In 1922 Hille moved to Princeton, where he was promoted to assistant professor after a year. At first Hille continued under the impetus received at Harvard. A couple of isolated papers on functional equations, **[8]**, **[14]**, show a variation of interest, and the two papers on Dirichlet series, **[9]** and **[17]**, though started much earlier, belong to the same period.

In 1925 Hille became interested in expansions in terms of Laguerre and Hermite polynomials, a topic to which he returned a number of times. The first papers were the notes **[19]**–**[21]** on Laguerre series and **[22]** on Hermite series. The latter contained basic results on Abel summability of such series and a study of Gauss-Weierstrass transforms and their inverses. The latter topic was later studied in great detail by A. González Domíngues who in 1941 wrote a long memoir on what he called Hille transforms.

In 1926 Oswald Veblen obtained a fellowship for Hille. He spent the time in Copenhagen, Stockholm and Göttingen. The publications **[23]**–**[25]** were from this period. The year began with a short sojourn in Copenhagen with Niels Erik Nörlund to work with partial difference equations. Hille then spent three months at the Mittag-Leffler Institute, where he prepared a bibliography, consisting of approximately 1000 papers (most of which he read) on the analytic representation of monogenic functions. Among the fruits of this work were **[26]**, **[33]** and various counterexamples in **[37]** and **[39]**. The references **[24]** and **[25]** are papers on various phases of the theory of the gamma function and **[25]** is joint with G. Rasch, at that time one of Nörlund's assistants. The latter two papers are closely related to the theory of difference equations, a field in which Nörlund is outstanding. Upon leaving the Mittag-Leffler Institute, he returned to Copenhagen, where he worked on the zeros of various functions connected with difference equations. He also constructed meromorphic functions with preassigned defects. These turned out to be the first valid examples; those constructed by Rolf Nevanlinna in his paper at the Copenhagen Scandinavian Mathematical Congress of 1924 turned out to be non-single-valued. Nevanlinna, a few years later, showed that Hille's methods could be used to construct a Riemann surface with a finite number of logarithmic branch-points, the inverse being a meromorphic function.

The mathematicians at Copenhagen were a friendly group and Hille made friends: Harald Bohr (also his famous brother Niels), Jakob Nielsen (a topologist and specialist on Fuchsian groups), Bonnesen, Steffensen, Mollerup and others.

Hille's fellowship year would have made a lasting impression on his

EINAR HILLE

June 28, 1894–February 12, 1980

mathematical production, even if it has been the only scientific influence he received. But in 1925 he had met J. D. Tamarkin, then fresh from Petrograd. In 1926 they began to correspond and in the fall of 1927 a close collaboration started. This lasted practically until Tamarkin's death in 1945, though their last joint paper, [**69**], dates from 1937. During these ten years they wrote twenty-three joint papers and another three of Hille's are a direct result of their collaboration. A. C. Offord, now one of the editors of the London Mathematical Society, collaborated with them on [**61**], G. Szegő on [**69**]. They started with the problem of the frequency of the characteristic values of linear integral equations [**29**], [**39**]; the latter represented for a number of years the last work on the subject.

Hille went to Göttingen for the summer semester of 1927 and made valuable contacts. At that time Göttingen was still the Mecca of mathematicians. Bernstein, Courant, Herglotz, Hilbert and Landau were on the faculty. The physicist and Nobel prize winner, Max Born, took part in a seminar run by Courant, and among the early speakers was John von Neumann, who presented his work on quantum mechanics. The algebraist Emmy Noether was there and occasionally van der Waerden. P. S. Aleksandrov was visiting lecturer and brought a number of visiting topologists: Heinz Hopf, Knaster, Kasimir Kuratowski. Hille met Lichtenstein and later in Berlin, Erhardt Schmidt. Göttingen was revisited in 1932 (just before Hitler came to power), 1953 and 1956. In Bonn he met Bessel-Hagen, Hausdorff and Otto Toeplitz. H. F. Bohnenblust, Hille's pupil, had just solved a problem of Bohr and Toeplitz on uniform convergence of Dirichlet series. In Hamburg, Hille met Emil Artin, Hecke and renewed his acquaintance with Wilhelm Blaschke. The Nazi storm broke loose a month later, and his friends had to flee for their lives.

Hille returned to Princeton as an associate professor in the fall of 1927. This began an enjoyable period of his life. Jean Dieudonné had graduated from École Normale and received a fellowship to work at Princeton. He asked Hille for a problem. What Hille suggested is still unsolved, but it suggested a problem to Dieudonné which he did solve and on the basis of which he became a pupil *honoris causa*. G. H. Hardy and Herman Weyl were visitors and Weyl brought H. F. Bohnenblust, who later became Hille's research assistant, collaborator and good friend.

The summer quarter of 1928 was spent at Stanford and the summer semester of 1931 at Chicago.

In the fall of 1932, on sabbatical leave, Hille attended the Zürich congress where he met Antoni Zygmund and Iovan Karamata. From Zürich he went briefly to Vienna and Budapest. The rest of the sabbatical was spent at Stockholm.

Hille moved to Yale in 1933 where he began his study of analytical semigroups in 1936. This proved to be a very fruitful subject which he pursued for the next twenty years. It produced many papers and the two treatises [**B1**] and [**B2**]. During this period his studies also yielded, as the bibliography shows, dozens of papers on other subjects.

The year 1936 was a very happy year for Hille. He met the lovely and vivacious Kirsti Ore, sister of Øystein Ore. They married in 1937 and had two sons, Harald in 1939 and Bertil in 1940. Harald is now a linguist in the United Nations and Bertil is professor of neurophysiology at the University of Washington. Kirsti was a loving and devoted wife–her suitcase was always packed. She went with him wherever he went.

Hille returned to Stanford for the academic year 1941–1942. Pólya and Szegő were there, and Tamarkin for the summer of 1941. They worked on various questions related to a problem in Fourier series already solved by Pólya and Norbert Wiener. Hille's collaboration with Max Zorn (then at UCLA) began in 1942.

The year 1936 was a happy one for me, too. I went to Yale as an instructor and met Hille for the first time. Our collaboration began immediately. Hille's knowledge of classical analysis was encyclopedic and proved invaluable to me. There was never a time when I needed some information that he did not have it immediately, and usually from several sources and different points of view.

He was a friendly man, kind to his students. He liked reading, especially history. Facts and dates on dynasties, fortifications and battles on land and sea, were at his fingertips.

He was mentally alert and mathematically active to the very end of his life. When he died, he had two papers at the proofreading stage and one in submission.

Hille was one of the very few mathematicians who brought a broad knowledge of classical analysis into fusion with modern operator theory. I shall attempt to give a brief notion of the great diversity of his contributions.

This attempt will be made by giving a short summary of the papers **[62]**, **[76]**, **[77]** and **[78]** in classical analysis and of **[63]**, **[73]** in modern analysis. The requirement of brevity precludes any discussion of many of his important papers.

In *On Laplace integrals* **[62]** Hille discusses two fundamental questions in the theory of Laplace integrals. They are the problem of representation and the problem of analytic continuation outside of the half-plane of convergence. It is shown that it is advantageous to connect the problem with that of determining those functions which can be represented as the quotient of two absolutely convergent Laplace integrals. This problem is solved completely. The result is: in order that $f(z)$ be representable as the quotient of two Laplace integrals, absolutely convergent for $x \geqslant a$, it is necessary that

(i) $f(z)$ is meromorphic for $x \geqslant a$, and those of its poles $\{b_n\}$ which lie outside of the circle $|z - a| = \alpha$ satisfy the condition $\Sigma\, \mathfrak{R}\{(b_n - a)^{-1}\} < \infty$,

(ii) $\int_{-\infty}^{\infty} \log^+|f(a + iy)|dy/(1 + y^2) < \infty$, and

(iii) $\beta^{-1}\int_{-\pi/2}^{\pi/2} \log^+|f(a + \beta e^{i\theta})|\cos\theta\, d\theta$

is bounded for all large values of β. Conversely, if these conditions are satisfied, then $f(z)$ is representable as the quotient of two Laplace integrals, absolutely convergent for $x > a$.

In **[76]**–**[78]** Hille studies the convergence of Fourier-Hermite series.

Among his many results are the following. Let $H_n(z)$ denote the nth normalized orthogonal function of Hermite, let $f(x)$ be a measurable function such that $x^n \exp(-x^2/2)f(x) \in L_1(-\infty, \infty)$ for $n = 0, 1, 2, \ldots$ and let $f_n = \int_{-\infty}^{\infty} f(t)H_n(t)\, dt$. It is shown if $f(z)$ is analytic, a necessary and sufficient condition that $\sum_{n=0}^{\infty} f_n H_n(z)$ converges to $f(z)$ in the strip S_τ, $-\tau < y < \tau$, is that $f(z)$ is holomorphic in S_τ and that to every given β, $0 \leqslant \beta < \tau$, there exists a finite positive $B(\beta)$ such that

$$|f(x+iy)| \leqslant B(\beta)\exp\left(-|x|(\beta^2 - y^2)^{1/2}\right), \qquad -\beta \leqslant y \leqslant \beta, -\infty < x < \infty.$$

Also, if $f(z)$ is holomorphic in the half-plane $y > -\alpha$ where $\alpha > 0$ and if

$$\limsup_{r\to\infty} (1/r)\log|f(re^{i\theta})| = h(\theta) \leqslant M, \qquad 0 \leqslant \theta \leqslant \pi,$$

then the Fourier-Hermite expansion of $f(z)$ can never converge outside of the real axis.

The results in *Notes on linear transformations*. I **[63]** clearly illustrate how Hille began his fusion of classical analysis into modern operator theory. Here it is shown that for suitably chosen Banach spaces for the vectors $x = x(u)$, the Gauss-Weierstrass singular integral

$$W(t)x \equiv (\pi t)^{-1/2}\int_{-\infty}^{\infty} \exp(-u^2/t)x(u + \cdot)\, du$$

and the Poisson integral for the upper half-plane

$$P(t)x \equiv \frac{t}{\pi}\int_{-\infty}^{\infty} \frac{x(u+\cdot)}{u^2+t^2}\, du$$

satisfy the semigroup equation

$$T(t)T(u) = T(t+u); \qquad t, u > 0. \tag{1}$$

The semigroup equation is a mathematical statement of the principle of determinism as applied to problems in classical mechanics which are defined by a linear homogeneous equation. This may be seen by observing that the major premise of Huygen's principle as stated by J. Hadamard in 1903 asserts that in such a physical system, with initial state x_0, the state $S(t, x_0)$ at any time $t > 0$ is linear in x_0 and is determined by its states at the two times $t_0 < t$ and $t - t_0$ by the equation

$$S(t, x_0) = S(t_0, S(t - t_0, x_0)); \qquad 0 < t_0 < t.$$

Hadamard, in applying this principle, was led to various functional equations satisfied by the solution of partial differential equations of *hyperbolic* type. In these cases the functional equations showed that the semigroup $T(t)$ of equation (1) is a group.

Hille's studies were primarily concerned with physical systems arising in the theories of heat conduction, diffusion problems, stochastic processes and potential theory. In such physical systems the state is not reversible and $T(t)$, $t > 0$, is strictly a semigroup. His studies led to an abundance of interesting and useful semigroups.

For example, the semigroup $W(t)$ arises in the theory of heat conduction, and $W(t)x_0$ is the solution of the equation for heat conduction in one dimension with the initial temperature $x_0 = x_0(u)$. Thus $W(t)x_0$; $0 < t$ is the solution for the *parabolic* initial value problem

$$\frac{\partial x}{\partial t} = \frac{1}{4}\frac{\partial^2 x}{\partial v^2}, \qquad x(0, u) = x_0(u); \qquad -\infty < u < \infty.$$

The semigroup $P(t)$ arises in the Dirichlet problem for the upper half-plane, and $P(t)x_0$ is the solution for this problem corresponding to the boundary values $x_0 = x_0(u)$. Thus $P(t)x_0$; $0 < t$ is the solution for the *elliptic* boundary value problem

$$\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 x}{\partial v^2} = 0, \qquad x(0, u) = x_0(u); \qquad -\infty < u < \infty.$$

Poisson's integral for the circle leads to (1) if the parameter r is replaced by e^{-t}. The semigroup equation is also encountered in the application of Abel's method of summation. An example of a different type leading to (1) is the theory of fractional integration. These examples are discussed in *Notes on linear transformations*. II. *Analyticity of semi-groups* [73].

The primary purpose of [73] is the investigation of the analyticity properties of the vectors $T(t)x$. The spectral properties of $T(t)$ are also discussed. The following comments give a very brief summary of these investigations.

It was observed in [63] that $W(t)x$ and $P(t)x$ are analytic in the right half-plane. The same property holds for the semigroups arising from Poisson's integral for the circle, Abel's method of summation and fractional integration.

One of the major problems discussed in [73] is that of determining conditions on the semigroup $T(t)$; $0 < t$ which are necessary, and others which are sufficient to establish its extension to a semigroup $T(\alpha)$ defined and holomorphic on a right half-plane.

In these studies it is convenient to define the notion of boundedness as follows. Let $T(\alpha)$; $\alpha = a + bi$, $0 < a$, $-\infty < b < \infty$ be a holomorphic extension of $T(t)$; $0 < t$. Then, for every $a_0 > 0$ there are constants A and B such that

$$|T(\alpha)| \leqslant A \exp(B|\alpha|); \qquad a_0 \leqslant a, \quad -\infty < b < \infty. \tag{2}$$

One set of sufficient conditions consists of the following. The norm $|T(t)|$ is a measurable function of t, for each x, $T(t)x$ is weakly continuous on the right at each point $t \geqslant 0$, where $T(0) = I$; the spectrum $\sigma(T(t))$ of $T(t)$ is located in the right half-plane for every t in some interval $0 < t \leqslant t_1$ and the resolvent $R(\alpha; T(t))$ is of subexponential order in a fixed sector for every t in an interval $0 < t \leqslant t_2$ and is of finite order for $t = t_2$.

Various modifications of these conditions are given.

The sufficient conditions imply the existence of a holomorphic semigroup $W(\alpha)$ defined on a half-plane $R(\alpha) > c \geqslant 0$ which satisfies (2) for $R(\alpha) \geqslant c + \delta$; $\delta > 0$ and with $W(t) = T(t)$; $t \geqslant c$.

Besides the extension problem, [73] contains discussions of the problem of determining domains which contain the spectrum $\sigma(T(t))$. One result states that if $|T(t)|$ is measurable on $0 < t$ and if the spectrum $\sigma(T(t))$ lies in the right half-plane for every t in some interval $0 < t \leq t_1$, then there is a $t_0 > 0$ with the following property. For t in the interval $2^{-n-1}t_0 < t \leq 2^{-n}t_0$, the spectrum $\sigma(T(t))$ is located in the domain of $\alpha = r\exp(i\theta)$ defined by the inequalities

$$r^{2n} \leq 2\cos(2^n\theta), \qquad |\theta| \leq \pi 2^{-n-1}; \qquad n = 0, 1, \ldots.$$

There are many other results concerning the spectrum of $T(t)$ as well as the existence of analytic extensions to a right half-plane.

The paper **[73]** was the first of Hille's major contributions to the general theory of semigroups of linear operators.

Among Hille's many other contributions to this field are the following. They may be found in the treatises **[B1]** and **[B2]**.

In this discussion it is assumed that $T(t)$; $0 \leq t$ is a semigroup with the properties

$$T(0) = I, \qquad T(t) \text{ is strongly continuous on } 0 \leq t < \infty. \tag{3}$$

Many results in **[B1]** and **[B2]** do not use the assumption that $T(0) = I$. The assumption (3) is made here because it permits a more lucid statement of the results.

The most basic problems in the theory of such semigroups are those concerned with the relations between $T(t)$ and its infinitesimal generator A. This generator is defined by the equations

$$A(h) \equiv h^{-1}(T(h) - I), \qquad D(A) = \left\{x\colon \lim_{h\to 0} A(h)x \text{ exists}\right\},$$
$$Ax \equiv \lim_{h\to 0} A(h)x; \qquad x \in D(A).$$

The infinitesimal generator is a closed, densely defined operator related to the semigroup by the equation

$$T(t)x = \lim_{h\to 0} \exp(tA(h))x; \qquad x \in X$$

uniformly on any finite interval. Many other equations of exponential type relating $T(t)$ and A or $A(h)$ are given in Chapter 10 of **[B2]**.

Relations of a different type are to be found in Chapter 11 of **[B2]**. One of the most useful of such relations may be stated as follows. The limit

$$\omega_0 \equiv \lim_{t\to\infty} \frac{1}{t}\log|T(t)|$$

exists and $-\infty \leq \omega_0 < \infty$. Every complex number α whose real part $R(\alpha) > \omega_0$ is in the resolvent set of A and, for such α,

$$R(\alpha; A) = \int_0^\infty e^{\alpha t}T(t)x\,dt; \qquad x \in X.$$

In Chapter XII of **[B2]** a converse problem is discussed. What properties should an operator A possess in order that it be the infinitesimal generator of a semigroup of bounded linear operators? The first solution to the problem was obtained independently by Hille and K. Yosida in 1948 and is known as the Hille-Yosida theorem. It states that if A is closed and densely defined, if

$R(t;A)$ exists for $t > 0$ and if

$$t|R(t;A)| \leqslant 1; \qquad 0 < t,$$

then A is the infinitesimal generator of a strongly continuous semigroup $T(t)$ with $|T(t)| \leqslant 1$; $t > 0$.

Hille's contribution to the theory of Lie semigroups as presented in Chapter XXV of **[B2]** is an extension of his work published in **[95]** and **[96]**.

Finally, I want to emphasize the enormous contribution made by his applications of semigroup theory to partial differential equations. These applications grew so rapidly after the publication of **[B1]** that a new treatise will be required to present them. They are nowhere discussed in **[B2]**.

Besides these fields of investigation, Hille made important contributions to the theories of ordinary and partial differential equations, integral equations, summability, Fourier series and ergodic theory.

Hille's honors were many. He was a member of the National Academy of Sciences, Royal Society of Sciences and Arts of Göthenburg, Physiographical Society of Lund, Royal Academy of Sciences of Stockholm and many more. He was president of the American Mathematical Society (1947–48) and colloquium speaker in 1944. He was editor of the Annals of Mathematics (1929–33) and of the Transactions of the American Mathematical Society (1937–43). He was made a Knight of the Swedish Order of the North Star and was awarded the John Ericsson Gold Medal by the American Society of Swedish Engineers. He taught at Harvard, Princeton, Yale, Stanford, Chicago, Stockholm, Uppsala, Nancy, Sorbonne, Mainz, Canberra, Haifa and the Tata Institute in Bombay.

Bibliography of Einar Hille

(Entries preceded by an asterisk are reprinted in **[B11]**.)

0. (with Hans von Euler), *Über die Primäre Umwandlumg der Hexosen bei der alkoholischen Gärung*, Z. Gärungsphysiologie **3** (1913), 235–240.

1. *Über die Variation der Bogenlänge bei konformer Abbildung von Kreisbereichen*, Ark. Mat. Astronom. Fys. no. 27 **11** (1917), 11 pp.

2. *Some problems concerning spherical harmonics*, Dissertation, Stockholm, 1918, Ark. Mat. Astronom. Fys. no. 17 **13** (1918), 76 pp.

3. *Oscillation theorems in the complex domain*, Trans. Amer. Math. Soc. **23** (1922), 350–385.

4. *An integral equality and its applications*, Proc. Nat. Acad. Sci. U.S.A. **7** (1921), 303–305.

*5. *On the zeros of Sturm-Liouville functions*, Ark. Mat. Astronom. Fys. no. 17 **16** (1922), 20 pp.

6. *Convex distribution of the zeros of Sturm-Liouville functions*, Bull. Amer. Math. Soc. **28** (1922), 261–265. A correction, ibid., p. 462.

7. *On the zeros of Legendre functions*, Ark. Mat. Astronom. Fys. no. 22 **17** (1923), 16 pp.

*8. *A Pythagorean functional equation*, Ann. of Math. (2) **24** (1922), 175–180.

9. *Note on Dirichlet's series with complex exponents*, Ann. of Math. (2) **25** (1924), 261–278.

10. *On the zeros of Mathieu functions*, Proc. London Math. Soc. (2) **23** (1924), 185–237.

11. *On the zeros of the functions of the parabolic cylinder*, Ark. Mat. Astronom. Fys. no. 26 **18** (1924), 56 pp.

*12. *An existence theorem*, Trans. Amer. Math. Soc. **26** (1924), 241–248.

13. *A note on regular singular points*, Ark. Mat. Astronom. Fys. no. 2 **19A** (1925), 21 pp.

*14. *A class of functional equations*, Ann. of Math. (2) **29** (1928), 215–222.

*15. *On the zeros of the functions defined by linear differential equations of the second order*, Proc. Internat. Congr. Math. (Toronto, 1924), vol. 1, pp. 511–519.

16. *A general type of singular point*, Proc. Nat. Acad. Sci. U.S.A. **10** (1924), 488–493.

*17. *Some remarks on Dirichlet series*, Proc. London Math. Soc. (2) **25** (1926), 177–184.

18. *Miscellaneous questions in the theory of differential equations*. I. *On the method of Frobenius*, Ann. of Math. (2) **27** (1926), 195–198.

19–21. *On Laguerre's series. First-third ncte*, Proc. Nat. Acad. Sci. U.S.A. **12** (1926), 261–265; 265–269; 348–352.

22. *A class of reciprocal functions*, Ann. of Math. (2) **27** (1926), 427–464.

*23. *Zero point problems for linear differential equations of the second order*, Mat. Tidsskrift **B** (1927), 25–44.

24. *On the logarithmic derivatives of the gamma function*, Danske Vid. Selsk. Mat.-Fys. Medd. no. 1 **8** (1927), 58 pp.

25. (with G. Rasch), *Über die Nullstellen der unvollständigen Gammafunktion $P(z, p)$*. II. *Geometrisches über die Nullstellen*, Math. Z. **29** (1928), 319–334.

26. *Note on the behavior of certain power series on the circle of convergence with application to a problem of Carleman*, Proc. Nat. Acad. Sci. U.S.A. **14** (1928), 217–220.

27. *Note on the preceding paper by Mr. Peek*, Ann. of Math. (2) **30** (1929), 270–271.

28. *Note on some hypergeometric series of higher order*, J. London Math. Soc. **4** (1929), 50–54.

*29. (with J. D. Tamarkin), *On the characteristic values of linear integral equations*, Proc. Nat. Acad. Sci. U.S.A. **14** (1928), 911–914.

30. (with J. D. Tamarkin), *On the summability of Fourier series*, Proc. Nat. Acad. Sci. U.S.A. **14** (1928), 915–918.

31. (with J. D. Tamarkin), *On the summability of Fourier series*, Second Note, Proc. Nat. Acad. Sci. U.S.A. **15** (1929), 41–42.

32. (with J. D. Tamarkin), *Remarks on a known example of a monotone continuous function*, Amer. Math. Monthly **36** (1929), 255–264.

33. *Note on a power series considered by Hardy and Littlewood*, J. London Math. Soc. **4** (1929), 176–182.

34. (with J. D. Tamarkin), *Sur une relation entre des résultants de MM. Minetti et Valiron*, C. R. Acad. Sci. Paris **188** (1929), 1142.

35. *On functions of bounded deviation*, Proc. London Math. Soc. (2) **31** (1930), 165–173.

36. *Bemerkung zu einer Arbeit des Herrn Müntz*, Math. Z. **32** (1930), 422–425.

37. (with J. D. Tamarkin), *On the theory of linear integral equations*. I, Ann. of Math. (2) **31** (1930), 479–528.

38. (with J. D. Tamarkin), *On the summability of Fourier series. Third note*, Proc. Nat. Acad. Sci. U.S.A. **16** (1930), 594–598.

39. (with J. D. Tamarkin), *On the characteristic values of linear integral equations*, Acta Math. **57** (1931), 1–76.

40. (with H. F. Bohnenblust), *Sur la convergence absolue des séries de Dirichlet*, C. R. Acad. Sci. Paris **192** (1931), 30–32.

*41. (with H. F. Bohnenblust), *On the absolute convergence of Dirichlet series*, Ann. of Math. (2) **32** (1931), 600–622.

42. (with J. D. Tamarkin), *On the summability of Fourier series. Fourth note*, Proc. Nat. Acad. Sci. U.S.A. **17** (1931), 376–380.

43. (with J. D. Tamarkin), *On the summability of Fourier series*. I, Trans. Amer. Math. Soc. **34** (1932), 757–783.

44. *Summation of Fourier series*, Bull. Amer. Math. Soc. **38** (1932), 505–528.

45. (with H. F. Bohnenblust), *Remarks on a problem of Toeplitz*, Ann. of Math. (2) **33** (1932), 785–786.

46. (with J. D. Tamarkin), *The summation of Fourier series by Hausdorff means*, Verh. Internat. Math. Kong. (Zürich, 1932), vol. 2, pp. 131–132.

47. (with J. D. Tamarkin), *On the summability of Fourier series*, Verh. Internat. Math. Kong. (Zürich, 1932), vol. 2, pp. 133–134.

*48. (with J. D. Tamarkin), *On the summability of Fourier series*. II, Ann. of Math. (2) **34** (1933), 329–348, 602–605.

49. (with J. D. Tamarkin), *On the summability of Fourier series*. III, Math. Ann. **108** (1933), 525–577.

*50. (with J. D. Tamarkin), *On a theorem of Paley and Wiener*, Ann. of Math. (2) **34** (1933), 606–614.

51. (with J. D. Tamarkin), *Questions of relative inclusion in the domain of Hausdorff means*, Proc. Nat. Acad. Sci. U.S.A. **19** (1933), 573–577.

52. *Über die Nullstellen der Hermiteschen Polynome*, Jber. Deutsch. Math.-Verein. **44** (1934), 162–165.

53. *On the complex zeros of the associated Legendre functions*, J. London Math. Soc. **8** (1933), 216–217.

*54. (with J. D. Tamarkin), *On the theory of Fourier transforms*, Bull. Amer. Math. Soc. **39** (1933), 768–774.

*55. (with J. D. Tamarkin), *A remark on Fourier transforms and functions analytic in a half-plane*, Compositio Math. **1** (1934), 98–102.

56. (with J. D. Tamarkin), *On moment functions*, Proc. Nat. Acad. Sci. U.S.A. **19** (1933), 902–908.

57. (with J. D. Tamarkin), *On the theory of Laplace integrals*, Proc. Nat. Acad. Sci. U.S.A. **19** (1933), 908–912.

58. (with J. D. Tamarkin), *On the theory of Laplace integrals*. II, Proc. Nat. Acad. Sci. U.S.A. **20** (1934), 140–144.

59. (with J. D. Tamarkin), *On the theory of linear integral equations*. II, Ann. of Math. (2) **35** (1934), 445–455.

60. (with J. D. Tamarkin), *On the summability of Fourier series. Fifth note*, Proc. Nat. Acad. Sci. U.S.A. **20** (1934), 369–372.

61. (with A. C. Offord and J. D. Tamarkin), *Some observations on the theory of Fourier transforms*, Bull. Amer. Math. Soc. **41** (1935), 427–436.

*62. *On Laplace integrals*, Ått. Skand. Matematikerkong., Stockholm, 1934, pp. 216–227.

*63. *Notes on linear transformations*. I, Trans. Amer. Math. Soc. **39** (1936), 131–153.

*64. (with J. D. Tamarkin), *On the absolute integrability of Fourier transforms*, Fund. Math. **25** (1935), 329–352.

65. (with Otto Szász), *On the completeness of Lambert functions*, Bull. Amer. Math. Soc. **42** (1936), 411–418.

66. (with Otto Szász), *On the completeness of Lambert functions*. II, Ann. of Math. (2) **37** (1936), 801–815.

*67. *A problem in "Factorisatio Numerorum"*, Acta Arith. **2** (1936), 134–144.

*68. *The inversion problem of Möbius*, Duke Math. J. **3** (1937), 549–568.

69. (with G. Szegö and J. D. Tamarkin), *On some generalizations of a theorem of A. Markoff*, Duke Math. J. **3** (1937), 729–739.

70. *Bilinear formulas in the theory of the transformation of Laplace*, Compositio Math. **6** (1938), 93–102.

71. *On the absolute convergence of polynomial series*, Amer. Math. Monthly **45** (1938), 220–226.

*72. *On semi-groups of transformations in Hilbert space*, Proc. Nat. Acad. Sci. U.S.A. **24** (1938), 159–161.

*73. *Notes on linear transformations*. II. *Analyticity of semi-groups*, Ann. of Math. (2) **40** (1939), 1–47.

74. *Analytical semi-groups in the theory of linear transformations*, Ått. Skand. Matematikerkong., Helsingfors, 1938, pp. 135–145.

75. *Remarks concerning group spaces and vector spaces*, Compositio Math. **6** (1939), 375–381.

76. *Contributions to the theory of Hermitian series*, Duke Math. J. **5** (1939), 875–936.

*77. *Sur les séries associées à une série d'Hermite*, C. R. Acad. Sci. Paris **209** (1939), 714–716.

*78. *Contributions to the theory of Hermitian series*. II. *The representation problem*, Trans. Amer. Math. Soc. **47** (1940), 80–94.

*79. *A class of differential operators of infinite order*. I, Duke Math. J. **7** (1940), 458–495.

80. (with H. L. Garabedian and H. S. Wall), *Formulations of the Hausdorff inclusion problem*, Duke Math. J. **8** (1941), 193–213.

81. *On the oscillation of differential transforms*. II. *Characteristic series of boundary value problems*, Trans. Amer. Math. Soc. **52** (1942), 463–497.

82. *Gelfond's solution of Hilbert's seventh problem*, Amer. Math. Monthly **49** (1942), 654–661.

*83. *Representation of one-parameter semi-groups of linear transformations*, Proc. Nat. Acad. Sci. U.S.A. **28** (1942), 175–178.

*84. *On the oscillation of differential transforms and the characteristic series of boundary-value problems*, Univ. Calif. Publ. Math. (N.S.) **2** (1944), 161–168.

*85. *On the analytical theory of semi-groups*, Proc. Nat. Acad. Sci. U.S.A. **28** (1942), 421–424.

86. (with G. Szegö), *On the complex zeros of the Bessel functions*, Bull. Amer. Math. Soc. **49** (1943), 605–610.

87. (with M. Zorn), *Open additive semi-groups of complex numbers*, Ann. of Math. (2) **44** (1943), 554–561.

*88. *On the theory of characters of groups and semi-groups in normed vector rings*, Proc. Nat. Acad. Sci. U.S.A. **30** (1944), 58–60.

*89. *Remarks on ergodic theorems*, Trans. Amer. Math. Soc. **57** (1945), 246–269.

90. (with W. B. Caton), *Laguerre polynomials and Laplace integrals*, Duke Math. J. **12** (1945), 217–242.

*91. (with N. Dunford), *The differentiability and uniqueness of continuous solutions of addition formulas*, Bull. Amer. Math. Soc. **53** (1947), 799–805.

*92. *Non-oscillation theorems*, Trans. Amer. Math. Soc. **64** (1948), 234–252.

93. *Sur les semi-groupes analytiques*, C. R. Acad. Sci. Paris **225** (1947), 445–447.

94. *Remarks on a paper by Zeev Nehari*, Bull. Amer. Math. Soc. **55** (1949), 552–553.

95. *Les semi-groupes linéaires*, C. R. Acad. Sci. Paris **228** (1949), 35–37.

*96. *Lie theory of semi-groups of linear transformations*, Bull. Amer. Math. Soc. **56** (1950), 89–114.

97. *On the differentiability of semi-group operators*, Acta Sci. Math. (Szeged) **12B** (1950),

19–24.

*98. *On the integration problem for Fokker Planck's equation in the theory of stochastic processes*, C. R. Onzième Congr. Math. Scand. (Trondheim, 1949), pp. 183–194.

99. *Les probabilités continues en chaîne*, C. R. Acad. Sci. Paris **230** (1950), 34–35.

*100. *"Explosive" solutions of Fokker-Planck's equation*, Proc. Internat. Congr. Math. (Cambridge, Mass., 1950), vol. 1, Amer. Math. Soc., Providence, R. I., 1952, p. 453.

101. *On the generation of semi-groups and the theory of conjugate functions*, Kungl. Fysiografiska Sällskapets i Lund Förhandlingar [Proc. Roy Physiog. Soc. Lund] **21**, no. 14 (1952), 13 pp.

*102. *Behavior of solutions of linear second order diferential equations*, Ark. Mat. **2** (1951), 25–41.

103. *A note on Cauchy's problem*, Ann. Polon. Math. **25** (1952), 56–68.

104. *Such stuff as dreams are made on–in mathematics*, Amer. Sci. **41** (1953), 106–112.

105. *Mathematics and mathematicians from Abel to Zermelo*, Math. Mag. **26** (1953); 127–146.

*106. *Une généralisation du problème de Cauchy*, Ann. Inst. Fourier (Grenoble) **4** (1952), 31–48.

107. *Sur le problème abstrait de Cauchy*, C. R. Acad. Sci. Paris **236** (1953), 1466–1467.

108. *Le problème abstrait de Cauchy*, Rend. Sem. Mat. Torino **12** (1953), 95–103.

109. *The abstract Cauchy problem and Cauchy's problem for parabolic differential equations*, J. Analyse Math. **3** (1954), 81–196.

110. *An abstract formulation of Cauchy's problem*, C. R. Douzième Congr. Math. Scand. (Lund, 1953), 1954, pp. 79–89.

111. *Some extremal properties of Laplace transforms*, Math. Scand. **1** (1953), 227–236.

112. (with George Klein), *Riemann's localization theorem for Fourier series*, Duke Math. J. **21** (1954), 587–591.

113. *On the integration of Kolmogoroff's differential equations*, Proc. Nat. Acad. Sci. U.S.A. **40** (1954), 20–25.

114. *Quelques remarques sur les équations de Kolmogoroff*, Bull. Soc. Math. Phys. R. P. Serbie **5** (1953), 3–14.

*115. *Some aspects of Cauchy's problem*, Proc. Internat. Congr. Math. (Amsterdam, 1954), vol. 3, Noordhoff, Groningen; North-Holland, Amsterdam, 1956, pp. 109–116.

116. *Sur un théorème de perturbation*, Rend. Sem. Mat. Torino **13** (1954), 169–184.

117. *Perturbation methods in the study of Kolmogoroff's equations*, Proc. Internat. Congr. Math. (Amsterdam, 1954), vol. 3, Noordhoff, Groningen; North-Holland, Amsterdam, 1956, pp. 365–376.

118. *On a class of orthonormal functions*, Rend. Sem. Mat. Univ. Padova **25** (1956), 214–249.

119. *Quelques remarques sur l'équation de la chaleur*, Rend. Mat. e Appl. (5) **15** (1956), 102–118.

120. *Über eine Klasse Differentialoperatoren vierter Ordnung*, S. B. Berlin. Math. Ges. (Jahrgang 1954/1955 u. 1955/1956), pp. 39–44.

121. *Problème de Cauchy: existence et unicité des solutions*, Bull. Math. Soc. Sci. Math. Phys. R. P. Roumanie (1) **49**, no. 2 (1957), 31–33.

121a. *Problema lui Cauchy: Existenta si unicitatea solutiilor*, Roumanian translation of **[121]**, ibid., pp. 143–145.

*122. *Remarques sur les systèmes des équations différentielles linéaires à une infinité d'inconnues*, J. Math. Pures Appl. (9) **37** (1958), 375–383.

123. *Green's transforms and singular boundary value problems*, Abstracts of Short Communications, Proc. Internat. Congr. Math. (Edinburgh, 1958), pp. 80–81.

*124. *On roots and logarithms of elements of a complex Banach algebra*, Math. Ann. **136** (1958), 46–57.

*125. *On the inverse function theorem in Banach algebras*, Calcutta Mathematical Society Golden Jubilee Commemoration Volume, 1958–1959, pp. 65–69.

126. *An application of Prüfer's method to singular boundary value problems*, Math. Z. **72** (1959), 95–106.

*127. *Sur les fonctions analytiques définies par des séries d'Hermite*, J. Math. Pures Appl. (9) **40** (1961), 335–342.

128. *What is a semi-group*? MAA Studies in Mathematics, vol. 3, Studies in Real and Complex Analysis (edited by I. I. Hirschman, Jr.), Prentice-Hall, Englewood Cliffs, N. J., 1965, pp. 55–66.

129. *Differential equations, ordinary*, Encyclopaedia Britannica, vol. 7, p. 407.

130. *Fekete polynomials and conformal mapping*, Abstract, Second Hungar. Math. Congr. (Budapest, 1960).

131. *Functions, analytic*, Encyclopaedia Britannica, vol. 9, pp. 1005–1007.

*132. *Linear differential equations in Banach algebras*, Proc. Internat. Sympos. Linear Spaces (Jerusalem, July 1960), Jerusalem Academic Press, Jerusalem, 1961, pp. 263–273.

132a. *Linear differential equations in Banach algebras*, abstract of **[132]** (Proc. Conf. Functional Analysis, Warsaw, September 1960), Studia Math. (Seria Specjalna) Zeszyt **1** (1963), pp. 53–54.

*133. *Pathology of infinite systems of linear first order differential equations with constant coefficients*, Ann. Mat. Pura Appl. (4) **55** (1961), 133–148.

134. *Remarks on transfinite diameters, general topology and its relations to modern analysis and algebra* (Proc. Sympos. Prague, September 1961), Czechoslovak Academy of Sciences, Prague, 1962, pp. 211–220. (Mimeographed corrections, 1962, reproduced in **[137]**, pp, 222–223.)

135. *Remarks on differential equations in Banach algebras*, Studies in Mathematical Analysis and Related Topics (essays in honor of George Pólya), Stanford Univ. Press, Stanford, Calif., 1962, pp. 140–145.

*136. *Green's transforms and singular boundary value problems*, J. Math. Pures Appl. (9) **42** (1963), 331–349.

137. *A note on transfinite diameters*, J. Analyse Math. **14** (1965), 209–224. (Theorems 4 and 8 are false. See **[139]**.)

138. *Topics in classical analysis*, Lectures on Modern Mathematics, vol. 3 (edited by T. L. Saaty), Wiley, New York, 1965, pp. 1–57.

*139. *Some geometric extremal problems*, J. Austral. Math. Soc. **6** (1966), 122–128.

*140. *Some aspects of differential equations in B-algebras*, Functional analysis (Proc. Conf., Univ. Calif., Irvine, 1966, edited by B. R. Gelbaum), Thompson Book Co., Washington, D. C., 1967, pp. 185–194.

*141. *Some properties of the Jordan operator*, Aequationes Math. **2** (1968), 105–110.

142. (with G. L. Curme), *Classical analytic representations*, Quart. Appl. Math. **27** (1969), 185–192.

*143. *On the Thomas-Fermi equation*, Proc. Nat. Acad. Sci. U.S.A. **62** (1969), 7–10.

144. *Some aspects of the Thomas-Fermi equation*, J. Analyse Math. **23** (1970), 147–170.

145. *Aspects of Emden's equation*, J. Fac. Sci. Univ. Tokyo Sect. I **17** (1970), 11–30.

146. *Remark on the Landau-Kallman-Rota inequality*, Aequationes Math. **4** (1970), 239–240.

147. *On a class of non-linear second order differential equations*, Revista de la Unión Matemática Argentina **25** (1971), 319–334.

148. *Introduction to the general theory of reproducing kernels*, (Santa Fe NSF-RMMS Summer

Symposium, 1970) Rocky Mountain J. Math. **2** (1972), 321–368.

149. *On the Landau-Kallman-Rota inequality*, J. Approximation **6** (1972), 117–122.

150. *Meanvalues and functional equations*, CIME, La Mendola, 1970. CIME, Ciclo III. Functional equations and Inequalities. Ediz. Cremonese, Roma, 1971, pp. 153–162.

151. *Vector meanvalues and related functional equations*, 9th Internat. Sympos. Functional Equations, Roma-Elba, 1971 (manuscript lost by Roman publisher).

152. *Generalizations of Landau's inequality to linear operators*, (Oberwolfach, 1971) Linear Operators and Approximations, ISNM, vol. 20, 1972, pp. 20–32.

153. *Quatre leçons sur des chapitres choicis d'analyse*, Conferences at the Universidad de La Laguna, Tenerife, Spain, November 1972.

154. *On a class of adjoint functional equations*, Acta Sci. Math. Szeged. **34** (1973), 141–161.

155a. *Letter to Lars Onsager*, in volume dedicated to him.

155b. *Pseudo-poles in the theory of Emden's equation*, Proc. Nat. Acad. Sci. **69** (1972), 1271–1272.

156. *On a class of series expansions in the theory of Emden's equation*, Proc. Roy Soc. Edinburgh (A) **71** (1972/73), 95–110.

157. *A note on quadratic systems*, Proc. Roy. Soc. Edinburgh (A) **72** (1972/73), 17–37.

158. *Finiteness of the order of meromorphic solutions of some non-linear ordinary differential equations*, Proc. Roy. Soc. Edinburgh (A) **72:29** (1973/74), 331–336.

159. *On Lappo-Danilevskij's solution of the Riemann problem for Fuchsian equations*, Aequationes Math. **12** (1975), 275–276, No. reprints, Cf. pp. 357–360 of **[B12]**.

160. *On some generalizations of Malmquist's theorem*, Math. Scand. **38** (1976), 59–79.

161. *Remarks on Briot-Bouquet differential equations*. I, Comment. Math. **21** (1978), 119–131.

162. *Mathematicians from A to Z whom I have known*, Ohio State University Lecture, Oct. 15, 1976 (not published).

162a. *Svenska matematiker, som jag känt*, Lecture to the K. Vetenskaps-o. Vitterhetssamhället i Göteborg, May 9, 1977.

163. *Some remarks on Briot-Bouquet differential equations*. II, J. Math. Anal. Appl. **65** (1978), 572–585.

164. *Non-linear differential equations: questions and some answers*, Abstracts of the Uppsala 1977 Internat. Conf. on Differential Equations, pp. 47–48.

165. *Non-linear differential equations: questions and some answers*, Acta Universitatis Upsaliensis (1977), 101–108.

166. *Second order Briot-Bouquet differential equations*, Acta Sci. Szeged. **40** (1978), 63–72.

167. *Higher order Briot-Bouquet equations*, Ark. Mat. **16** (1978), 271–286.

168. *On a special Weyl-Titchmarsh problem*, Integral Equations Operator Theory **1** (1978), 215–225.

169. *On a class of interpolation series*, Proc. Roy. Soc. Edinburgh Sect. A **35** (1979), 25 pp.

170. *Hypergeometric functions and conformal mapping*, J. Differential Equations **34** (1979), 147–152.

171. *Contributions to the theory of Hermitian series*. III. *Meanvalues*, Internat. J. Math. Math. Sci. (submitted).

172. *Differential operators of infinite order*, Proc. Roy. Soc. Edinburgh Sect. A (submitted).

173. *Remarks on surfaces in E^3 of negative Gaussian curvature*, The Pleijel Festskrift, Uppsala, 1979.

174. *Surfaces in E^3 of negative Gaussian curvature*, J. d'Analyse Math. Jerusalem (submitted).

175. (with Frank Cannonito), *An elementary proof of the Jordan normal form theorem based on*

the resolvent, Integral Equations Operator Theory (submitted).

Bibl. 1 *Essai d'une bibliographie de la représentation analytique d'une fonction monogene*, Acta Math. **52** (1929), 1–80.

Bibl. 2 (with J. Shohat and J. L. Walsh), *A bibliography of orthogonal polynomials*, Bull. Nat. Res. Council, no. 103, August 1940, ix + 204 pp.

Ob. 1. *Thomas Hakon Gronwall–In memoriam*, Bull. Amer. Math. Soc. **38** (1932), 775–786.

Ob. 2. *Jacob David Tamarkin–his life and work*, Bull. Amer. Math. Soc. **53** (1947), 440–457.

B1. *Functional analysis and semi-groups*, Amer. Math. Soc. Colloq. Publ., Vol. XXXI, Amer. Math. Soc., New York, 1948, xii + 528 pp.

B2. (with R. S. Phillips), *Functional analysis and semi-groups*, Amer. Math. Soc. Colloq. Publ., Vol. XXXI, rev. ed., Amer. Math. Soc., Providence, R. I., 1957, xii + 808 pp. (Revised, 1964).

B3. *Analytic function theory*, vol. I, Ginn and Co., Boston, 1959, xi + 308 pp.

B4. *Analytic function theory*, vol. II, Ginn and Co., Boston, 1962, xii + 496 pp.

B5. *Analysis*, Vol. I, Blaisdell, New York, 1964, xii + 626 pp.

B6. *Analysis*, Vol. II, Blaisdell, New York, 1966, xii + 672 pp.

B7. (with S. L. Salas), *First-year calculus*, Blaisdell, New York, 1968, xi + 415 pp.

B8. *Lectures on ordinary differential equations*, Addison-Wesley, Reading, Mass., 1969, xi + 723 pp.

B9. *Methods in classical and functional analysis*, Addison-Wesley, Reading, Mass., 1972, ix + 486 pp.

B10. (with S. L. Salas), *Calculus: One and several variables*, Xerox Publishing Co., Waltham, Mass., 1971, vii + 800 pp.

B11. *Classical analysis and functional analysis. Selected papers* (edited by Robert R. Kallman), M.I.T. Press, Cambridge, Mass., 1975 i xlvii. + 709 pp.

B12. *Ordinary differential equations in the complex domain*, Interscience, New York, 1976, xi + 484 pp.

Current address: 355 North Washington Drive, Sarasota, Florida 33577