Brazilian Journal of Probability and Statistics
2010, Vol. 24, No. 1, 78–90
DOI: 10.1214/08-BJPS022

Measuring inequality and social welfare from any arbitrary distribution

V. T. Beena[a] **and M. Kumaran**[b]

[a]*Govt. College for Women*
[b]*Nehru Arts and Science College*

Abstract. Different measures of inequality such as the Lorenz curve, the generalised Lorenz curve (GLC) and the cumulated mean income curve (COMIC) are obtained for any univariate continuous distribution. GLC and COMIC are used to identify the best income distribution on welfare grounds when the ordinary Lorenz curves fail to work. Explicit expressions for the moments of a given Lorenz curve are also derived. The proposed method selects the appropriate generalised lambda distribution (GLD) representation corresponding to a given distribution under consideration and computes the different measures of inequality. A numerical illustration of the results, using per capita domestic product at current prices for various states/union territories of India for two periods 1994–95 and 2000–01, is also provided.

1 Introduction

Inequality measures have been used extensively in studies of the distribution of income, regional disparities in household consumption, partial ordering of social welfare states, distributions of sizes of firms, reliability on survival times of leukemia patients, fishermens' luck regarding the number of fishes caught, the distribution of scientific grants obtained, the distribution of political power, employment and educational opportunities. Many practitioners of disciplines such as economics, demography, political science and sociology, therefore, heavily rely on these measures as important tools in undergoing concentration analysis.

Gastwirth (1971, 1974) has analysed the methods of estimation of the Lorenz curve and Gini index and obtained the large sample distribution of the mean deviation and related measures of income inequality. Generalising the Gini index, Kakwani (1977, 1980) has discussed applications of Lorenz curves in economic analysis. Methods of tabulating the Lorenz functions of lognormal and Pareto distributions and their properties are included in Moothathu (1981, 1983). Shorrocks (1983) has proposed GLC and COMIC as useful analytical devices for measuring inequality. A sufficient condition for two nonintersecting Lorenz curves is established in Moothathu (1991). Arnold and Nagaraja (1991) and Kleiber (1999) have

Key words and phrases. Cumulated mean income curve, generalised lambda distribution, generalised Lorenz curve, Lorenz curve, social welfare function.

Received June 2008; accepted December 2008.

discussed Lorenz ordering with almost all commonly considered families of income distributions. Properties of COMIC and COMIC indices are described in Arora, Jain and Pudir (2005/06).

A wide variety of functional forms have been considered as possible models for a given income distribution. It is well known that the Pareto distribution is a good fit for the variables such as income, size of city population, natural resources, stock price fluctuations, etc., which follow statistical distributions with very long tails. But the fit is rather poor over the entire range. On the other hand, the lognormal distribution fits well over a large part of the distribution, but diverges markedly at the extremities. The generalised lambda distribution (GLD), a family of hypothetical statistical distributions with four independent parameters, can be suggested as superior for describing the distribution of income. The family contains unimodal, symmetric, asymmetric, truncated, heavy tail distributions and a variety of distributions including all those commonly used by practitioners. The strength of the GLD lies in its ability to approximate many distributions, represent data when the underlying distribution is unknown and fit or generate random variates.

Important measures of inequality and social welfare rankings of the income distribution for any univariate continuous distribution are discussed in this article. Section 2 defines and discusses the GLD family and its characteristics. Section 3 focuses on important inequality measures such as the Lorenz curve, the Gini index, notion of Lorenz dominance, the GLC, the concept of generalised Lorenz dominance, COMIC, COMIC ordering and explicates social welfare functions. Explicit expressions for these measures and moments of the Lorenz curve based on the GLD are also proposed in the same section. A numerical illustration of the results based on per capita domestic product at current prices for various states/union territories of India for two periods 1994–95 and 2000–01 is given in Section 4. Finally, a Lorenz curve, a GLC and a COMIC are drawn to visualize the inequality measures for the empirical data used in this study.

2 The generalised lambda distribution family

The GLD family was originally suggested by Hasting et al. (1947) and was generalised by Ramberg and Schmeiser (1972, 1974) to provide an algorithm for generating values for symmetric and asymmetric random variables. Unlike other families of distributions, the members of the GLD family are represented in terms of their quantile function given by

$$x_p = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}, \qquad 0 \le p \le 1, \lambda_2 \ne 0, \tag{2.1}$$

where $p = P(X \le x)$, λ_1 and λ_2 are location and scale parameters, λ_3 and λ_4 are the shape parameters. The probability density function (PDF) of the GLD is defined explicitly by the density quantile function in terms of p.

$$f(x_p) = \lambda_2[\lambda_3 p^{\lambda_3 - 1} + \lambda_4(1-p)^{\lambda_4 - 1}]^{-1} \tag{2.2}$$

GLD($\lambda_1, \lambda_2, \lambda_3, \lambda_4$) specifies a valid distribution if and only if

$$\frac{\lambda_2}{\lambda_3 p^{\lambda_3 - 1} + \lambda_4 (1-p)^{\lambda_4 - 1}} \geq 0.$$

The support of the random variable is $[\lambda_1 - \frac{1}{\lambda_2}, \lambda_1 + \frac{1}{\lambda_2}]$ when $\lambda_3 > 0, \lambda_4 > 0$ and $(-\infty, \infty)$ when $\lambda_3 < 0, \lambda_4 < 0$. When $\lambda_3 < 0, \lambda_4 = 0$, the support is $(-\infty, \lambda_1 + 1/\lambda_2)$ and $(\lambda_1 - 1/\lambda_2, \infty)$ if $\lambda_3 = 0, \lambda_4 < 0$. If $\lambda_3 = \lambda_4$ then (2.1) is symmetric about the pole $X = \lambda_1$.

When scale and location are changed such as through the tranformation $Y = a + bX$, the transformed distribution is another member of the GLD family with parameters λ_1 and λ_2 replaced by $a + b\lambda_1$ and $b\lambda_2$, respectively. The fact that the GLD is not invertible is not a serious drawback because the same property holds for many popular distribution models such as a normal, a lognormal, a gamma and a beta distributions, respectively. The use of distinctive values of p and $(1 - p)$ is an attractive feature of the GLD.

The k^{th} order moment of X about λ_1 denoted by μ_k is given by

$$\begin{aligned}\mu_k &= E(X - \lambda_1)^k \\ &= \frac{1}{\lambda_2^k} \sum_{i=0}^{k} \binom{k}{i} (-1)^i \beta\big(\lambda_3(k-i) + 1, \lambda_4 i + 1\big),\end{aligned}$$

where $\beta(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$ is a well-known beta function and $k = 1, 2, 3, \ldots$. The k^{th} order moment exists if and only if $\min\{\lambda_3, \lambda_4\} > -\frac{1}{k}$.

The mean μ of the distribution is

$$\mu = \lambda_1 + \frac{1}{\lambda_2(1+\lambda_3)} - \frac{1}{\lambda_2(1+\lambda_4)}. \tag{2.3}$$

Different methods of estimation of the parameters of GLD are suggested by different authors: method of moments by Ramberg and et al. (1979); method of least squares by Ozturk and Dale (1985); the starship estimation method by King and MacGillivray (1999); and the controlled randomization approach by Lakhany and Mausser (2000) and the discretized approach by Su (2005). Extensive tables are provided in Karian and Dudewicz (2000), which give the values of the parameters of a GLD corresponding to assumed pairs of values of skewness and kurtosis or based on selected percentiles of the distribution.

Karian and Dudewicz (2000) have demonstrated the closeness of almost all the traditional distributions with their corresponding GLD representations. Numerical results reveal that GLD approximation for any known continuous distribution almost agrees with the true distribution, both in terms of their PDF and cumulative distribution function (CDF). The greatest advantage of the GLD family is the availability of a single quantile function for all the members of the family including the traditional one and two parameter distributions. If the parent distribution is unknown, then based on the first four moments or the selected four quantiles of sample data drawn from the population, the appropriate GLD representation of the distribution can be made. Once the GLD member is identified, we can easily use it for modeling and decision making.

3 Measures of inequality

In this section different measures of inequality are discussed and explicit expressions for these measures are derived based on the GLD family.

3.1 Lorenz curve, Lorenz dominance and Gini index

The Lorenz curve and Gini index, powerful tools for a variety of scientific problems, are very popular in the study and quantification of inequality, its measurement and interpretation, particularly in economics. Let X be a nonnegative random variable with CDF $F(x)$, a continuous derivative $f(x)$ and finite mean μ. The general definition of the Lorenz curve $L_F(p)$ (Gastwirth (1971)) corresponding to the distribution function $F(x)$ is

$$L_F(p) = \mu^{-1} \int_0^p F^{-1}(t)\,dt, \qquad 0 \le p \le 1, \tag{3.1}$$

where $F^{-1}(t) = \inf\{x : F(x) \ge t\}$ is the inverse of $F(x) = P[X \le x]$ and is left continuous inverse of the CDF $F(x)$ (also known as quantile function). Twice the area between Lorenz curve and the line of equal distribution $F(x) = p$, is called the Gini index (also known as Lorenz ratio or concentration ratio) and it is given by

$$G = 1 - 2\int_0^1 L_F(p)\,dp. \tag{3.2}$$

Suppose we have two income distributions $F(x)$ and $G(x)$ with associated Lorenz curves, then we say that the distribution $F(x)$ Lorenz dominates distribution $G(x)$ if $L_F(p) \ge L_G(p)$ for $p \in (0, 1]$. In other words, a distribution function $F(x)$ is said to have less inequality in the Lorenz curve sense than a distribution function $G(x)$, if their Lorenz curves $L_F(p)$ and $L_G(p)$ satisfy the condition $L_F(p) \ge L_G(p)$ for all p, where $>$ applies for at least one $p \in (0, 1)$. From the definition of Lorenz curve, it is evident that the Lorenz partial order is invariant with respect to scale transformation.

Proposition 1. *The Lorenz curve denoted as $L_F(p)$ of the GLD is*

$$L_F(p) = \mu^{-1}\left\{\lambda_1 p + \frac{p^{\lambda_3+1}}{\lambda_2(\lambda_3+1)} + \frac{(1-p)^{\lambda_4+1} - 1}{\lambda_2(\lambda_4+1)}\right\} \tag{3.3}$$

provided $\lambda_2\lambda_3\lambda_4 \ge 0$ and μ is the mean of the GLD as defined in (2.3).

Proof. By (2.1) and (3.1)

$$\begin{aligned} L_F(p) &= \mu^{-1}\int_0^p F^{-1}(t)\,dt \\ &= \mu^{-1}\int_0^p \left[\lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}\right] dp \\ &= \mu^{-1}\left\{\lambda_1 p + \frac{p^{\lambda_3+1}}{\lambda_2(\lambda_3+1)} + \frac{(1-p)^{\lambda_4+1} - 1}{\lambda_2(\lambda_4+1)}\right\}. \end{aligned}$$

The condition $\lambda_2\lambda_3\lambda_4 \geq 0$ suffices to ensure the convexity of the Lorenz curve as long as the mean exists and $f(x_p)$ the PDF of GLD is a valid density function. □

Proposition 2. *The Gini index denoted as G of the GLD is*

$$G = \mu^{-1}\left\{\mu - \lambda_1 - \frac{2\lambda_2}{(\lambda_3+1)(\lambda_3+2)} + \frac{2\lambda_2}{(\lambda_4+1)(\lambda_4+2)} - \frac{2\lambda_2}{(\lambda_4+1)}\right\}. \tag{3.4}$$

Proof. By (3.2) and (3.3)

$$\begin{aligned} G &= 1 - 2\int_0^1 L_F(p)\,dp \\ &= \mu^{-1}\left\{\mu - \lambda_1 - \frac{2\lambda_2}{(\lambda_3+1)(\lambda_3+2)} + \frac{2\lambda_2}{(\lambda_4+1)(\lambda_4+2)} - \frac{2\lambda_2}{(\lambda_4+1)}\right\}. \end{aligned}$$ □

3.1.1 *Moments of the Lorenz curve.* As the Lorenz curve itself can be considered as a CDF on the unit interval, having bounded support, it can be characterized by the sequence of its moments. Furthermore, these Lorenz curve moments characterize the underlying distribution up to a scale parameter. Let X_L be a random variable supported on [0, 1] with a well-defined CDF as the Lorenz curve $L_F(p)$. Then, the k^{th} order raw moment of the Lorenz curve is

$$E(X_L^k) = k\int_0^1 p^{k-1}\big(1 - L_F(p)\big)\,dp. \tag{3.5}$$

Proposition 3. *The k^{th} order raw moment μ'_k of the Lorenz curve of the GLD is*

$$\begin{aligned} E(X_L^k) = 1 - \frac{k}{\mu}\bigg\{&\frac{\lambda_1}{k+1} + \frac{\beta(\lambda_3+k+1,1)}{\lambda_2(\lambda_3+1)} \\ &+ \frac{\beta(k,\lambda_4+2)}{\lambda_2(\lambda_4+1)} - \frac{\beta(k,1)}{\lambda_2(\lambda_4+1)}\bigg\}, \end{aligned} \tag{3.6}$$

where $\beta(p,q) = \Gamma(p)\Gamma(q)/\Gamma(p+q)$ and $k = 1, 2, 3, \ldots$.

Table 1 *Characteristics of the Lorenz curve of Pareto and lognormal distributions using their GLD representations*

Distribution	μ'_1	μ'_2	μ'_3	μ'_4	Variance	Skewness	Kurtosis
Pareto (1, 2)	0.67	0.5333	0.4571	0.4064	0.0889	0.6389	−0.8571
Pareto (1, 5)	0.56	0.3968	0.3133	0.2611	0.0882	0.1969	−1.2040
Lognormal (0, 1/3)	0.59	0.4308	0.3416	0.2847	0.0790	0.3427	−1.0502

Proof. By definition (3.5)

$$E(X_L^k) = k\int_0^1 p^{k-1}(1 - L_F(p))\,dp \qquad \text{using (3.3) for } L_F(p) \text{ and integrating}$$

$$= 1 - \frac{k}{\mu}\left\{\frac{\lambda_1}{k+1} + \frac{\beta(\lambda_3+k+1,1)}{\lambda_2(\lambda_3+1)} + \frac{\beta(k,\lambda_4+2)}{\lambda_2(\lambda_4+1)} - \frac{\beta(k,1)}{\lambda_2(\lambda_4+1)}\right\},$$

$$E(X_L) = \frac{G}{2} + \frac{1}{2}, \qquad \text{when } k = 1 \text{ and } G \text{ is as defined in (3.2).}$$

As can be seen in Table 1, mean, variance, skewness and kurtosis of the respective Lorenz curve of Pareto (1, 2), Pareto (1, 5) and lognormal (0, 1/3) distributions are presented. The Gini indices of the distributions are 0.33, 0.11 and 0.19, respectively. □

3.2 GLC and COMIC

Note that the GLC is defined by

$$GL_F(p) = \int_0^p F^{-1}(t)\,dt. \tag{3.7}$$

The Lorenz curve and GLC are related and $GL_F(p) = \mu L_F(p)$. Whenever the ordinary Lorenz curves cross and the Lorenz dominating distribution has a lower income, then the ordinary Lorenz curves fail to rank income distribution on welfare grounds. GLC is introduced to overcome this limitation and it can be used to identify the best income distribution on welfare grounds within a set of alternative income distributions, generated by different policy options. If the GLC of the distributions are $GL_F(p)$ and $GL_G(p)$, then the distribution $F(x)$ exhibits generalised Lorenz dominance over the distribution $G(x)$ if $GL_F(p) \geq GL_G(p)$ for $p \in (0, 1]$. When the GLC of one distribution dominates the other and if the decision maker is a profit maximizer and inequality averse, then social welfare is higher for the dominant distribution. Thus, we have to conclude that if there is generalised Lorenz dominance, then the dominating distribution has higher welfare associated with it.

COMIC is another analytical device for measuring inequality. COMIC, denoted as $C_F(p)$ with respect to a CDF $F(x)$ having mean μ, is defined as the conditional mean of incomes less than or equal to ζ_p (p^{th} income quantile) where $F(\zeta_p) = p$. This means that COMIC is the mean income received by the poorest $100p$ percent of the income recipients where the incomes are arranged in an ascending order. Mathematically,

$$C_F(p) = GL_F(p)/p, \qquad p \in (0, 1]. \tag{3.8}$$

The relationship between COMIC and a Lorenz curve is

$$C_F(p) = \frac{\mu L_F(p)}{p}, \qquad 0 < p \leq 1.$$

COMIC may not always start from the origin because as $p \to 0$, $C_F(p)$ takes $(0/0)$ form and egalitarian line connects the point $(0, \mu)$ to $(1, \mu)$. The graph of $C_F(p)$ is strictly increasing since $C'_F(p) \geq 0$. The sign of $C''_F(p)$ can be either positive or negative and hence COMIC can be convex in some parts and concave in some other parts. Since $GL_F(p) = pC_F(p) \leq C_F(p)$ for $0 < p \leq 1$, COMIC always lies above the GLC. $F(x)$ is said to have less income inequality than $G(x)$ in the COMIC sense, that is, $F \leq_C G$ if $C_F(p) \geq C_G(p)$. COMIC ordering will be useful to depict a particular dominance relationship which may not be obvious using Lorenz ordering. It gives a reverse relationship as compared to a Lorenz ordering and can be used to interpret dominance of certain distributions in terms of their welfare interpretations. The COMICs drawn for the two distributions enable us to make an immediate comparison. Thus, if $C_F(p)$ dominates $C_G(p)$ in the neighbourhood of $p = 0$, then $F(x)$ has more income among the poorest quantile than the poorest quantile of distribution $G(x)$; and $F(x)$ is said to rank higher than $G(x)$.

3.3 Social welfare function

The social welfare function (SWF) for the distribution $F(x)$ is defined as

$$W_F = \int U(x) f(x)\, dx$$

for every $U(x)$ such that $U'(x) > 0$ and $U''(x) < 0$. The function $U(x)$ is the utility function and social welfare is evaluated as the average utility. Suppose $F(x)$ and $G(x)$ are two income distributions with equal means $\mu_F = \mu_G$, then $L_F(p) \geq L_G(p)$ for all $p \in [0, 1]$ for every function $U(x)$. If a distribution F Lorenz dominates another distribution G, then social welfare under F will be higher than under G, provided average incomes are the same. Thus, in this case, Lorenz dominance is equivalent to social welfare dominance. In the case of GLCs, the relationship between dominance and social welfare is then given by

$$\int U(x) f(x)\, dx \geq \int U(x) g(x)\, dx$$

if and only if $GL_F(p) \geq GL_G(p)$ for all $p \in [0, 1]$. Thus, if generalised Lorenz dominance holds, welfare dominance can be inferred for all increasing strictly concave social welfare functions. If generalised Lorenz dominance does not hold and the GLCs cross, then it is always possible to find two increasing and concave social welfare functions which will rank the two income distributions differently. As both the Lorenz dominance and the generalised Lorenz dominance provide only partial ordering of the social welfare of a society, for complete ordering, we need a cardinal SWF that provides numerical values to all possible social states. Sen (1974), assuming social marginal utility to be inversely related to income rank, introduced axiomatically a SWF as

$$W = \mu(1 - G), \tag{3.9}$$

where μ is the mean income of the society and G is the Gini coefficient of the income distribution. The Sen index is twice the area below the GLC,

$$2 \int_0^1 GL_F(p) = \mu(1 - G).$$

As the Gini index is some area and can be represented by a single number, the Sen SWF can rank distributions according to both mean income and inequality in cases even where generalised Lorenz curves intersect.

Proposition 4. *By definition* (3.7), *the GLC denoted as* $GL_F(p)$ *of the GLD is*

$$GL_F(p) = \lambda_1 p + \frac{p^{\lambda_3+1}}{\lambda_2(\lambda_3+1)} + \frac{(1-p)^{\lambda_4+1} - 1}{\lambda_2(\lambda_4+1)}, \qquad p \in (0, 1]. \tag{3.10}$$

Proposition 5. *By definition* (3.8), *the COMIC denoted* $C_F(p)$ *of the GLD is*

$$C_F(p) = \frac{1}{p}\left\{\lambda_1 p + \frac{p^{\lambda_3+1}}{\lambda_2(\lambda_3+1)} + \frac{(1-p)^{\lambda_4+1} - 1}{\lambda_2(\lambda_4+1)}\right\}, \qquad p \in (0, 1]. \tag{3.11}$$

It may be noted that numerous forms of income data arise frequently. The pattern of these data are highly unpredictable. However, only very few distributions, namely lognormal and Pareto, are used to model and analyse income data. As the GLD contains unlimited choice of distributions with nonnegative support with varying characteristics and arbitrary shapes, it is ideal for realistic modeling of income data. Moreover, as demonstrated above, using GLD, it is possible to get explicit expressions for all most all inequality measures. Once the GLD parameters of the income distribution are identified, these measures can be computed easily.

Table 2 *The Lorenz curve, Gini index, GLC and COMIC of Pareto* $(1, 5)$ *and lognormal* $(0, 1/3)$ *functions encompassed by the GLD family*

	Pareto			Lognormal		
p	LC	GLC	COMIC	LC	GLC	COMIC
0.1	0.05132	0.10263	1.02633	0.05157	0.05432	0.54321
0.2	0.10557	0.21115	1.05573	0.11881	0.12514	0.62571
0.3	0.16334	0.32668	1.08893	0.19476	0.20515	0.68382
0.4	0.22540	0.45081	1.12702	0.27833	0.29317	0.73291
0.5	0.29289	0.58579	1.17157	0.36940	0.38909	0.77818
0.6	0.36754	0.73509	1.22515	0.46845	0.49342	0.82236
0.7	0.45228	0.90456	1.29222	1.38197	0.60728	0.86754
0.8	0.55279	1.10557	1.38197	1.38197	0.73284	0.91605
0.9	0.68377	1.36754	1.51949	0.83051	0.87479	0.97199
1	1	2	2	1	1.05331	1.05331
G	0.33333			0.18618		

As a numerical illustration of the results derived, we have computed Gini index values and cumulative decile shares of Lorenz curves, GLC and COMIC for Pareto $(1, 5)$ and lognormal $(0, 1/3)$ distributions using their GLD approximation and given in Table 2.

4 Application

Data on per capita domestic product at current prices for various states/union territories of India for two periods 1994–95 and 2000–01 as given in Table 3 are considered to illustrate the various measures of inequality. (Source: Central Statistical Organization based on Directorate of Economics and Statistics of respective State Governments as on 30–11–2004.)

We have tabulated the GLC and the COMIC of the data using our derived results. We make an unambiguous statement regarding the change in welfare between 1994–95 and 2000–01 from the numerical values of Table 4. As the GLC for 2000–01 is everywhere above that for 1994–95, all social welfare functions which are increasing show an increase in social welfare over the period. The per capita domestic product at current prices for 2000–01 dominates the year 1994–95. Similarly, there is COMIC dominance in the year 2000–01 as compared to the year 1994–95. This means that the income inequality in 2000–01 is less than the income inequality in 1994–95.

The Lorenz curve, GLC and COMIC are drawn to visualize inequality in 1994–95 and 2000–01 inherent in the empirical income graduation for these various Indian states. From Figures 1–3, it is observed that $C_1(p)$, the COMIC for 2000–01 dominates everywhere $C(p)$, the COMIC for 1994–95. So one can conclude

Table 3 *Per capita net domestic product at current prices*

Sl. no.	States UT	1994–95	2000–01
1	Andhra Pradesh	8732	16708
2	Arunachal Pradesh	9148	14699
3	Assam	6493	10718
4	Bihar	3372	5157
5	Jharkhand	6455	8749
6	Goa	19317	49693
7	Gujarat	12640	17938
8	Haryana	12879	23194
9	Himachal Pradesh	9451	19925
10	Jammu & Kashmir	6915	12781
11	Karnataka	8960	17816
12	Kerala	9632	20107
13	Madhya Pradesh	7099	10777
14	Chattisgarh	6983	9922
15	Maharashtra	13654	21883
16	Manipur	9105	11047
17	Meghalaya	7347	14632
18	Mizoram	8793	18491
19	Nagaland	10175	17629
20	Orissa	5795	9281
21	Punjab	14068	24183
22	Rajasthan	7647	12514
23	Sikkim	7696	16658
24	Tamil Nadu	10503	20346
25	Tripura	5656	15253
26	Uttar Pradesh	5767	9162
27	Uttaranchal	8119	12687
28	West Bengal	7711	16146
29	Andaman & Nicobar Islands	17688	24418
30	Chandigargh	22824	44476
31	Delhi	21568	42508
32	Pondicherry	10997	35190

there appears to be less income inequality in the year 2000–01 as compared to the year 1994–95 in the various states of India.

Conclusion

The generalised lambda distribution family is a flexible form that encompasses a complete system of continuous distributions. As the distribution of income and similar variables are arbitrary, one can identify the distribution as a member of the GLD family either based on the first four moments or the selected four quantiles of sample data drawn from the population. One can then apply the derived results

Table 4 *GLC and COMIC values of per capita net domestic product*

Decile	1994–95 $GL(p)$	1994–95 $C(p)$	2000–01 $GL_1(p)$	2000–01 $C_1(p)$	% change in GLC
0.1	513.68	5136.78	807.98	8079.80	57.29
0.2	1094.73	5473.63	1759.16	8795.80	60.69
0.3	1749.91	5833.03	2865.91	9553.04	63.77
0.4	2488.00	6220.01	4147.27	10368.20	66.69
0.5	3320.51	6641.02	5628.85	11257.70	69.52
0.6	4263.19	7105.31	7346.33	12243.90	72.32
0.7	5339.10	7627.28	9352.69	13361.00	75.17
0.8	6585.71	8232.14	11735.5	14669.40	78.19
0.9	8077.05	8974.50	14671.90	16302.10	81.16
1	10102.40	10102.40	18900.80	18900.80	87.09

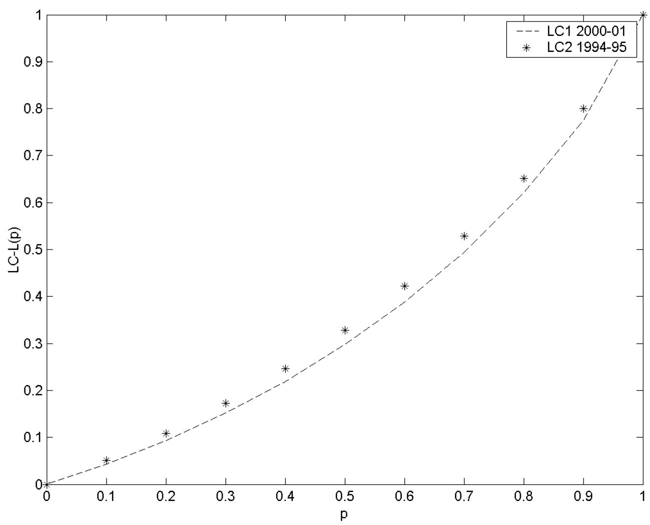

Figure 1 *Lorenz curves of per capita domestic product for 1994–95 and 2000–01.*

to compute measures of inequality and social welfare. The results are true for all distributions that can be represented as a member of the GLD family.

Acknowledgment

The authors are thankful to both referees for their valuable suggestions and comments which helped in improving the earlier draft of this paper.

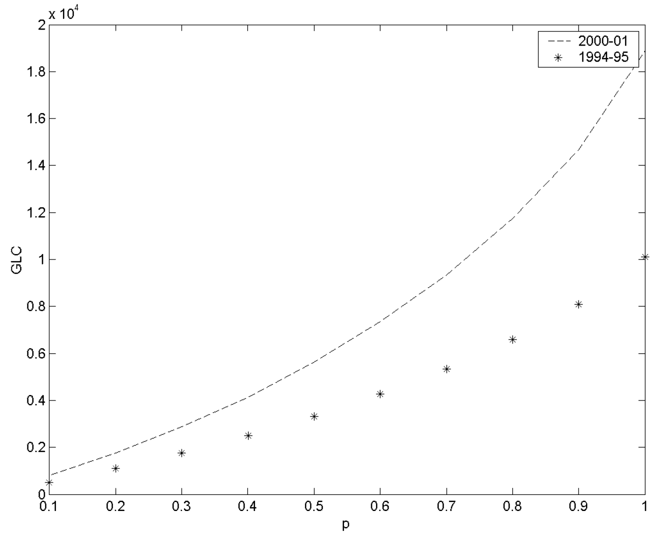

Figure 2 *GLC of per capita domestic product for 1994–95 and 2000–01.*

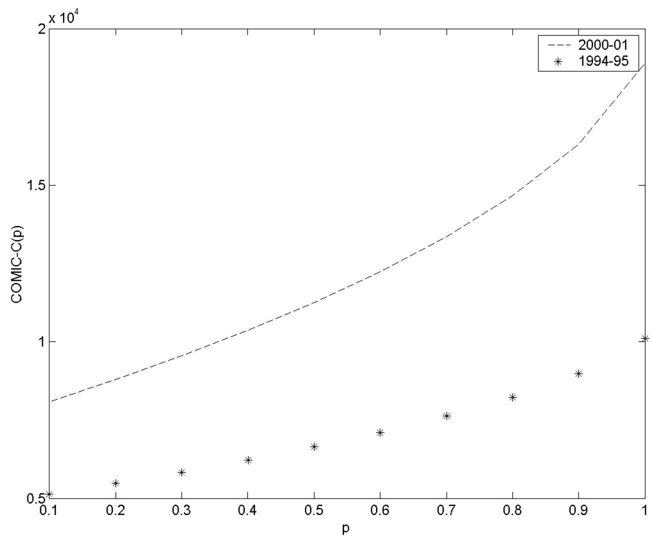

Figure 3 *COMIC of per capita domestic product for 1994–95 and 2000–01.*

References

Arnold, B. C. and Nagaraja, H. N. (1991). Lorenz ordering of exponential order statistics. *Statistics and Probability Letters* **11** 485–490. MR1116741

Arora, S., Jain, K. and Pudir, S. (2005/06). On cumulated income curve. *Model Assisted Statistics and Applications* **1** 107–114. MR2339090

Gastwirth, J. L. (1971). A general definition of the Lorenz curve. *Econometrica* **39** 1037–1039.

Gastwirth, J. L. (1974). Large sample theory of some measures of income inequality. *Econometrica* **42** 191–196. MR0411100

Hastings, Jr., C., Mosteller, F., Tukey, J. W. and Winsor, C. P. (1947). Low moments for small samples: A comparative study of statistics. *Annals of Mathematical Statistics* **18** 413–426. MR0022335

Kakwani, N. (1977). Applications of Lorenz curves in economic analysis. *Econometrica* **45** 719–727. MR0436932

Kakwani, N. (1980). On a class of poverty measures. *Econometrica* **48** 437–446. MR0560520

Karian, Z. A. and Dudewicz, E. J. (2000). *Fitting Statistical Distributions: The Generalized Lambda Distribution and Generalized Bootstrap Methods*. CRC Press, Boca Raton, FL. MR1850548

Kleiber, C. (1999). On the Lorenz order within parametric families of income distribution. *Sankhya: The Indian Journal of Statistics, Series B* **61** 514–517. MR1745750

King, R. and MacGillivray, H. L. (1999). A starship estimation method for the generalised lambda distributions. *Australian and New-Zealand Journal of Statistics* **41** 353–374. MR1718029

Lakhany, A. and Mausser, H. (2000). Estimating the parameters of the generalized lambda distribution. *Algo Research Quarterly* **3** 47–58.

Moothathu, T. S. K. (1981). On Lorenz curves of lognormal and Pareto distributions. *Journal of the Indian Statistical Association* **19** 103–108. MR0673873

Moothathu, T. S. K. (1983). Properties of Gastwirth's Lorenz curve and bounds for general Gini index. *Journal of the Indian Statistical Association* **21** 149–154. MR0773234

Moothathu, T. S. K. (1991). On a sufficient condition for two non-intersecting Lorenz curves. *Sankhya: The Indian Journal of Statistics, Series B* **53** 268–274. MR1180067

Ozturk, A. and Dale, R. (1985). Least squares estimation of the parameters of the generalized lambda distribution. *Technometrics* **27** 81–84.

Ramberg, J. S. and Schmeiser, B. W. (1972). An approximate method for generating symmetric random variables. *Communications of the ACM* **15** 987–990.

Ramberg, J. S. and Schmeiser, B. W. (1974). An approximate method for generating asymmetric random variables. *Communications of the ACM* **17** 78–82. MR0331711

Ramberg, J. S., Tadikamalla, P. R., Dudewicz, E. J. and Mykytka, E. F. (1979). A probability distribution and its uses in fitting data. *Technometrics* **21** 201–214.

Sen, A. K. (1974). Informational bases of alternative welfare approaches: Aggregation of income distribution. *Journal of Public Economics* **3** 387–403.

Shorrocks, A. F. (1983). Ranking income distributions. *Economica* **50** 3–17.

Su, S. (2005). A discretized approach to flexibly fit generalized lambda distributions to data. *Journal of Modern Applied Statistical Methods* **4** 408–424.

Govt. College for Women
Thiruvananthapuram 695014
Kerala
India
E-mail: beenamadhu@rediffmail.com

Nehru Arts and Science College
Kanhangad 671328
Kerala
India