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ALFRÉD RÉNYI, 1921–1970

BY P. RÉVÉSZ[1] AND I. VINCZE[2]

Mathematical Institute, Hungarian Academy of Sciences

The death of Alfréd Rényi on February 1, 1970, was a great loss for mathematicians all over the world. His contributions to probability, mathematics, and mathematical statistics will have a lasting impact on each of these disciplines and on all of the people with whom he came in contact.

Alfréd Rényi was born in Budapest on March 20, 1921. He studied mathematics and physics at the University of Budapest. He earned the doctorate degree in 1945 from the University of Szeged, presenting a result in the theory of Fourier series as his dissertation. This was published in part under the title "On the summability of Cauchy-Fourier series," in the *Publ. Math. Debrecen* **1** 162–164, (1949). After studying in Leningrad as a postgraduate fellow under the guidance of Ju. V. Linnik, his career developed rapidly, with the positions of adjunct in 1947–48, private docent at the University of Budapest in 1948, and professor at the University of Debrecen from 1948 to 1950. These were years of planning and organizing. In 1950 Rényi was appointed Director of the new Hungarian Academy of Sciences and Head of the Department of Probability Theory of its Mathematical Institute. He also held the chair of probability theory at the University of Budapest. He was secretary general of the Bolyai János Mathematical Society and secretary of the Class of Mathematics and Physics of the Hungarian Academy of Sciences. He was also the secretary of the newly organized board for granting national postgraduate degrees. His capacity for work, his energy and his initiative were incomparable.

The first published paper by Rényi was on a topic in real analysis (1946: 1). His scientific interest in number theory was stimulated by the number theoretical work of Ju. V. Linnik, of whom he considered himself a pupil. Some early important results are connected with Goldbach's famous problem in number theory (1947: 4, 5; 1948: 7). Based on Linnik's large sieve method, Rényi proved that every integer can be represented as the sum of a prime and an almost prime, that is, an integer whose number of prime factors is less than an absolute constant.

This interest was later expanded to include ergodic properties of representations of real numbers (e.g. 1957: 99, 100; 1959: 115), and applications of probability theory to number theoretic problems (e.g. 1957: 96; 1958: 110; 1960: 128, 131; 1963: 160; 1964: 179; 1968: 220). His interest in the large sieve method continued simultaneously (1948: 11; 1958: 114; 1959: 119), culminating in a purely probabilistic, and much more general, form presented in the paper of 1959. The foundation of the theorem and its proof were simplified using the following two

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[1] Presently at Indiana University, Bloomington.

[2] Presently at Catholic University of America, Washington, D.C. The preparation of this paper was supported in part by the National Science Foundation grant GP-9396.

(known) concepts:

(i) Let ξ and η be two random variables and define the maximum correlation $\rho(\cdots)$ of the random variables ξ and η, by

$$\rho(\xi, \eta) = \sup_{f,g} R(f(\xi), g(\eta)).$$

Here f and g run over the Borel-measurable functions for which $D(f(\xi))$ and $D(g(\eta))$ are finite, and $R(\cdots)$ is the correlation coefficient.

(ii) *The correlation ratio* Q of the random variable η with respect to the random variable ξ, is given by

$$Q_\xi(\eta) = \frac{D(M(\eta \mid \xi))}{D(\eta)}.$$

Rényi's formulation of the large sieve theorem was then as follows.

THEOREM. *Let* $\xi_1, \xi_2, \cdots$ *be a sequence of random variables with finite variance, for which the relation*

$$\left| \sum_n \sum_m \rho(\xi_n, \xi_m) x_n x_m \right| \leqq C \sum_{i=1}^{\infty} x_i^2$$

is valid, where $x_1, x_2, \cdots$ *is an arbitrary sequence of real numbers with* $\sum_1^\infty x_i^2 < \infty$.
Then

$$\sum_{n=1}^{\infty} Q_{\xi_n}(\eta) \leqq C,$$

for any random variable η *which has a finite variance.*

This study of the large sieve method led to a consideration of the interesting and important concept of "mixing." A sequence $A_1, A_2, \cdots, A_n, \cdots$ of events is called mixing if

$$\lim_{n\to\infty} P(A_n \mid B) = \lim_{n\to\infty} P(A_n),$$

for any event B with positive probability. His basic theorem (1958: 107) gives the following simple condition for the sequence $A_1, A_2, \cdots$, to be mixing:

$$\lim_{n\to\infty} P(A_n \mid A_j) = \lim_{n\to\infty} P(A_n) \qquad \text{for any } j.$$

This concept was generalized in collaboration with Révész (1958: 112). Applications of this and similar results in the area of limit theorems and ergodic theory were also studied.

Previously neglected in Hungary, except for the important but isolated work of Charles Jordan, probability theory started to flourish as a consequence of Rényi's activity. The introduction of conditional probability spaces (1954: 69; 1955: 77) provided an interesting and natural foundation for the theory of probability. It may be considered remarkable that the fundamental idea of conditional probability spaces originated from his wish to apply probabilistic methods in number theory. The difficulty in such applications is that (in Kolmogorov's theory) a uniform distribution does not exist on the integers. For instance, we cannot ask for the probability that a *randomly* chosen number is prime. The introduction of conditional probability spaces partly solved this problem.

Rényi's first contributions in mathematical statistics belong to the theory of order statistics. He developed a method by which the elements of an ordered sample could be studied as the sums of independent random variables. With this notion, he was able to derive a simple and unified method for the proofs of many of the known limit theorems in the theory of order statistics.

At the same time, he developed new tests of the goodness-of-fit between empirical distribution functions and the theoretical distribution function (1953: 61, 62; 1954: 61; 1965: 185; 1967: 212; 1968: 214). In particular, he investigated the limit distribution of the random variable

$$\sup_{a<x<\infty} \frac{F_n(x) - F(x)}{F(x)}$$

where $F(x)$ and $F_n(x)$ are the theoretical and the empirical distribution functions, respectively.

Other branches of mathematical statistics also attracted attention. Since a sufficient statistic embodies all of the information contained in the sample, he was able to develop a new foundation of mathematical statistics based on the concepts of information theory (1964: 174, 178; 1966: 196, 197; 1967: 205–208; 1968: 218; 1969: 232).

A study of the theory of stochastic processes resulted in contributions concerning Poisson processes and their generalizations. He was probably the first person to prove that a point process with independent increments is a generalized Poisson process. Rényi later found a connection between Poisson processes and recurrent processes. For a given recurrent process, suppose that each of its points x is omitted with probability p $(0 < p < 1)$ and replaced by px (i.e., the new process is compressed). Rényi proved that the limiting process, when this procedure is iterated, is a Poisson process. A series of investigations resulted from this property, as, for example, in papers [2], [3], [6], [7], [8], [9].

Rényi obtained many important results relating to different aspects of limit theorems. His theorem which states that the limit distribution of a sum of a random number of independent random variables is normally distributed under very general conditions is a major contribution. These results stimulated interesting investigations related to sequential analysis, as in [1], [4], [5].

His textbook on probability theory is indeed an important contribution to the field. It was first published in Hungarian (1954: B1) and thoroughly revised later (1966: B4). Editions have also been published in German (1962: B2), in French (1966: B5), in English (1970: B13) and in Czech (1970: B16). These are not simple translations of his first edition; each subsequent edition contains substantial new modifications and additions.

Rényi's interest in applications of probability theory was not limited to number theoretic problems, as for example, 1969: 235; 1970: 243. Other areas of mathematics attracting his interest in probabilistic applications are analysis (1952: 45; 1957: 96; 1958: 110; 1961: 142; 1967: 203; 1969: 227) and graph theory (1959: 116; 1960: 130; 1962: 156; 1969: 234; 1970: 242). Unfortunately, he was unable to complete the books begun on both these subjects.

He also made significant contributions in applications of mathematical theory to practical problems in a variety of fields, for example, chemistry (1953: 63; 1954: 71; 1957: 102; 1959: 123), biology (1956, 86; 1968: 226; 1969: 235; 1970, 237), economics (1952: 49, 50, 51; 1953: 65; 1956: 88; 1960: 135), engineering (1950: 27; 1952: 50, 51; 1957: 102).

Rényi was a very enthusiastic mathematician; his love for the subject was the secret of his success. In "Dialogue on the applications of mathematics" (1967: B7), Rényi has Archimedes as saying to Hieron:

> "Mathematics is like your daughter, Helena, who suspects every time a suitor appears that he is not really in love with her, but is interested in her only because he wants to be the son-in-law of the king. She wants a husband who loves her for her own beauty, her wit and charm, and not for the wealth and power which he can get by marrying her. Similarly, mathematics reveals its secrets only to those who approach it with pure love, for its own beauty. Those who do this are, of course, also rewarded with results of practical importance. But if somebody asks at each step 'What can I profit by this?' he will not get far."

In his quest for sharing his own appreciation for the beauty of mathematics, Rényi wrote a number of popularizing books and papers (1962: 154; 1964: 170, 175, 176, 177, 181; 1965: B3, 190, 191; 1966: 198, 199: 1967; B6, B7, B8, B9, 209, 210; 1968: 211, 213, 222, 223, 224; 1969: B10, B11, 230, 231; 1970: 136, B17, 244).

The importance of Rényi's work can be measured by the influence which his deep investigations had on the field, as attested to by the large number of books and papers which quote his results and by the contributions of his students and collaborators.

Great recognition was accorded him and his accomplishments in all parts of the world. As an ordinary member of the Hungarian Academy of Sciences, he was twice awarded the Kossuth prize (a prize awarded by the Hungarian Government to the most distinguished scholars and artists). He was the vice-president of the International Statistical Institute, a Fellow of the Institute of Mathematical Statistics, and a member of the editorial board of eight mathematical journals. He accepted invitations to be a visiting professor at several universities in the United States, England, and Germany.

The present brief summary has been limited to Rényi's major activities and papers concerning probability and statistics and their applications in mathematics, but his research in other areas was also considerable. The broad scope of his work is indicated by the accompanying list of publications prepared by our colleague, P. Medgyessy. Besides his contributions to number theory, he obtained significant results in information theory, graph theory, combinatorial mathematics, real and complex analysis, applied mathematics and didactical problems. The contributions which were left uncompleted or almost completed are as yet unknown. It is impossible to say what he could have accomplished if his career had not been terminated at the early and untimely age of 49 years. Rényi was without doubt one of the most outstanding members of the contemporary scientific community, both in Hungary and in the world. His early death was not only a great loss for our Institute, for our University and for our country, but also for the fields of mathematics and mathematical statistics. His colleagues all over the world share in our feeling of supreme loss.

In spite of his recognized stature, Professor Rényi remained ready and willing to discuss mathematics with anyone, whether young or old, apprentice or accomplished. Ever approachable by his associates, his warm and friendly manner encouraged many to take advantage of his deep knowledge in so many branches of mathematics. He spoke with great respect of and indebtedness to his teachers, L. Fejér, F. Riesz, A. N. Kolmogorov, Ju. V. Linnik. His frequent participation in symposia and seminars in all parts of the world was always a stimulating experience for his fellow mathematicians.

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THE PUBLICATIONS OF ALFRÉD RÉNYI

(*Based on a list compiled by P. Medgyessy*)

BOOKS

1954

[1] *Theory of Probability.* Tankönyvkiadó, Budapest (in Hungarian).

1962

[2] *Wahrscheinlichkeitsrechnung, mit einem Anhang über Informationstheorie.* VEB Deutscher Verlag der Wissenschaften, Berlin.

1965

[3] *Dialogues on Mathematics.* Akadémiai Kiadó, Budapest (in Hungarian).

1966

[4] *Theory of Probability.* Tankönyvkiadó, Budapest (in Hungarian).
[5] *Calcul des Probabilités.* Avec un appendice sur la théorie de l'information. Dunod, Paris.

1967

[6] *Letters on Probability.* Akadémiai Kiadó, Budapest (in Hungarian).
[7] *Dialogues on Mathematics.* Holden-Day, San Francisco.
[8] *Dialoge über Mathematik.* VEB Deutscher Verlag der Wissenschaften, Berlin, and Birkhauser, Basel.
[9] *Dialoguri despre matematica.* Editura Stiintifica, Bucuresti.

1969

[10] *Dialogues on Mathematics.* Mir, Moszkva (in Russian).
[11] *Briefe über die Wahrscheinlichkeit.* Akademiai Kiadó, Budapest—VEB Deutscher Verlag der Wissenschaften, Berlin, and Birkhauser, Basel.
[12] *Exercises in Probability Theory* (with co-authors). Tankönyvkiadó, Budapest.
[13] *Probability Theory.* North-Holland, Amsterdam.
[14] *Foundations of Probability Theory.* Holden-Day, San Francisco.
[15] *On the Mathematical Theory of Trees.* North-Holland, Amsterdam.
[16] *Theory of Probability* (in Czech.). In print.
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1946

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1947

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1948

[7] On the representation of an even number as a sum of a prime and an almost prime number. *Izv. Akad. Nauk SSSR* **12** 57–78 (in Russian).
[8] Play with chances. *Mat. Lapok* **1** Part I, 101–111. Part II, 144–157 (in Hungarian).
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[11] Generalization of the "large sieve" of Ju. V. Linnik. *Math. Centrum Amsterdam.* 5 pages.
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1949

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[15] On the measure of equidistribution of point sets. *Acta Sci. Math.* (*Szeged*) **13** 77–92.

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