EXACT OPERATING CHARACTERISTIC FOR TRUNCATED SEQUENTIAL LIFE TESTS IN THE EXPONENTIAL CASE

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1. Introduction. Non-truncated sequential life tests involving the exponential distribution, $f(t) = \exp(-t/\theta)/\theta$, $t \geqq 0$, $\theta \geqq 0$, have been treated extensively by Epstein and Sobel. In [3], the details are given for

(i) computing the approximate OC curve associated with the familiar sequential probability ratio (A, B) rule, where $A = (1 - \beta)/\alpha$, $B = \beta/(1 - \alpha)$, and where α and β are the nominal errors of first and second kind, respectively;

(ii) computing the exact strength (α', β') of the (A, B) rule where $\alpha' \leqq \alpha$, $\beta \leqq \beta' \leqq \beta/(1 - \alpha)$; and

(iii) determining an (A^*, B) rule with strength exactly (α, β) (based on the solution by Dvoretzky, Kiefer, and Wolfowitz, [1], for the exact OC curve for the non-truncated case).

The above results constitute essentially a complete solution for the OC curve in the non-truncated case. Many times, however, it is desirable to truncate the sequential test after some pre-selected V_0 time units or i_0 failures have been observed. Then if a decision has not been reached earlier, accept H_0: $\theta = \theta_0$ if V_0 is observed before i_0 failures, otherwise accept H_1: $\theta = \theta_1$. Upper bounds on the strength of truncated tests have been given by Epstein in [2].

The main purpose of this paper is to determine the exact OC curve for the truncated test. This result is conveniently obtained as the sum of a finite series whose terms are defined recursively (in Sections 3 and 4) by modifying the Dvoretzky, Kiefer, and Wolfowitz solution [1] for the non-truncated test.

Only the case of sequential testing with replacement is considered. For convenience it is assumed that the sample units are tested one at a time. Extension to the case of testing n units simultaneously (with replacement of failures as they occur) is straightforward (Epstein [2]).

2. Preliminaries and notation. Application of Wald's sequential probability ratio test [3, 5] to the exponential distribution yields the following (A, B) rule:

$$\text{Accept } H_0\colon \theta = \theta_0 \quad \text{if} \quad V(t) \geqq a_i ,$$

$$\text{Accept } H_1\colon \theta = \theta_1 \quad \text{if} \quad V(t) \leqq r_i ,$$

Continue test otherwise,

where

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$$\begin{aligned}
&\theta_1 < \theta_0 \\
&a_i = h_0 + is \qquad \text{for } i = 0, 1, 2, \cdots, \\
(1)\quad &r_i = -h_1 + is \qquad \text{for those positive integers for which } (-h_1 + is) > 0, \\
&r_i = 0 \qquad \text{otherwise}, \\
&h_0 = -\ln B/(\theta_1^{-1} - \theta_0^{-1}),\ h_1 = \ln A/(\theta_1^{-1} - \theta_0^{-1}),\ s = \ln(\theta_0/\theta_1)/(\theta_1^{-1} - \theta_0^{-1}), \\
&A = (1 - \beta)/\alpha,\ B = \beta/(1 - \alpha).
\end{aligned}$$

$V(t)$ denotes the accumulated life time of all items tested up to time t and i denotes the observed number of failures up to time t. Clearly [3] the decision to reject can be reached only at the instant a failure occurs while the decision to accept may be made at any time between failures.

In accordance with the notation used in [1], let the integer $n(0)$ denote the largest index i for which $r_i = 0$. Then $n(0)$ is fixed by the (A, B) rule and is the integer satisfying the relation $(h_1/s) - 1 \leqq n(0) < h_1/s$. Further let m be the integer defined by the relation $r_m \leqq h_0 < r_{m+1}$. Then since $h_0 > 0$, it follows that $m + 1 > n(0)$ or $n(0) \leqq m$. (See Fig. 1.)

Let stage i be defined as the time interval from (and including) the ith failure up to (and not including) the $(i + 1)$th failure, and let $E_a^{(i)}$ and $E_c^{(i)}$ denote the events $V(t) \geqq a_i$, and $r_j < V(t) < a_j$ for each $j = 1, 2, \cdots, i$, respectively. Let $p[E_a^{(i)}, E_c^{(i)}]$ be the probability of the joint occurrence of $E_a^{(i)}$ and $E_c^{(i)}$, i.e., the probability of accepting H_0 at the ith stage.

Then the OC curve, which is the probability of accepting material with true mean-time-between-failures (MTBF) θ, can be expressed in the infinite series form

$$(2)\qquad P_A(\theta) = \Pr[\text{accept} \mid \theta] = \sum_{i=0}^{\infty} p(E_a^{(i)}, E_c^{(i)}).$$

3. *OC* curve for the non-truncated case. In the non-truncated case, $P_A(\theta)$ is given by Dvoretzky, Kiefer, and Wolfowitz [1] in the form

$$(3)\qquad P_A(\theta) = \exp(-h_0/\theta)\left\{\sum_{i=0}^{n(0)} b_i z^i \Big/ \sum_{i=0}^{m} c_i z^i\right\},$$

where

$$(4)\quad b_i = (-1)^i[(h_1 - is)/\theta]^i/i!, \qquad c_i = (-1)^i[(h_0 + h_1 - is)/\theta]^i/i!, \qquad z = \exp(-s/\theta).$$

The aim now is to rearrange (3) into the form of (2). Then, as shown in the next section, it will be a simple matter to obtain the OC curve for the particular truncation rule considered herein.

The quotient of the polynomials in (3) forms an infinite series,

$$(5)\qquad \sum_{i=0}^{n(0)} b_i z^i \Big/ \sum_{i=0}^{m} c_i z^i = \sum_{i=0}^{\infty} S_i z^i.$$

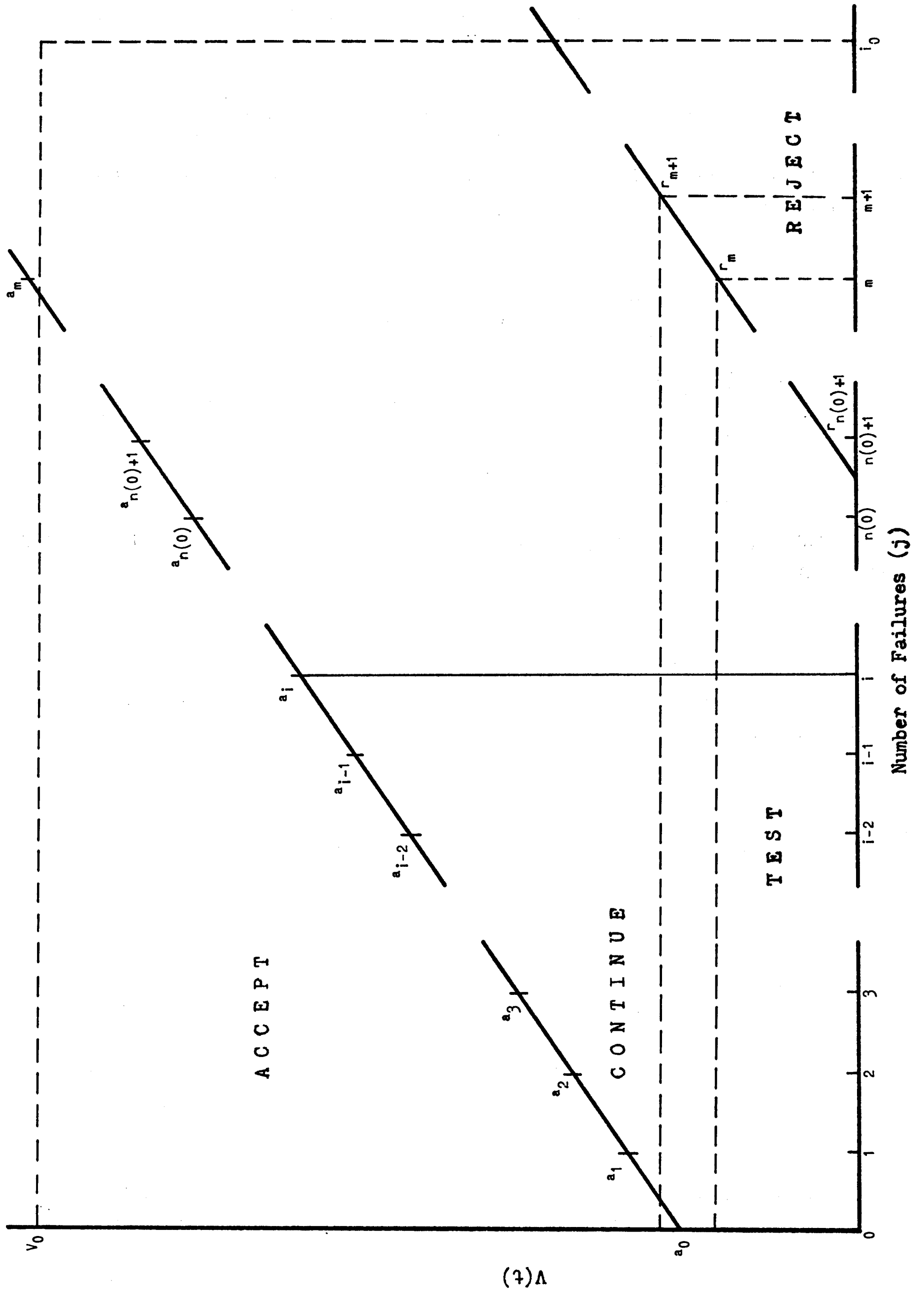
ACCEPT
CONTINUE TEST
REJECT
Number of Failures (j)
V(t)
V_0
a_0
a_1
a_2
a_3
a_{i-2}
a_{i-1}
a_i
$a_{n(0)}$
$a_{n(0)+1}$
a_m
$r_{n(0)+1}$
r_m
r_{m+1}
0
1
2
3
i-2
i-1
i
n(0)
n(0)+1
m
m+1
i_0

FIG. 1

To express the coefficients, S_i, in terms of the known quantities, b_i and c_i, clear fractions in (5), getting

$$(6)\quad \sum_{i=0}^{n(0)} b_i z^i = c_0\left(\sum_{i=0}^{\infty} S_i z^i\right) + c_1\left(\sum_{i=0}^{\infty} S_i z^{i+1}\right) + \cdots + c_m\left(\sum_{i=0}^{\infty} S_i z^{i+m}\right).$$

Equating coefficients of like powers of z (keeping in mind that $n(0) \leqq m$), there results, for fixed integers $n(0)$ and m,

$$(7)\qquad S_i = \begin{cases} 1 & \text{for } i = 0 \\ b_i - \sum_{j=1}^{i} c_j S_{i-j} & \text{for } i = 1, 2, \cdots, n(0) \\ -\sum_{j=1}^{i} c_j S_{i-j} & \text{for } i = n(0) + 1, \cdots, m \\ -\sum_{j=1}^{m} c_j S_{i-j} & \text{for } i = m + 1, m + 2, \cdots. \end{cases}$$

The relations (7) can be expressed in terms of the acceptance and rejection numbers using the identity

$$(8)\quad h_0 + h_1 - js = a_{k-j} - r_k \quad \text{for any} \quad k \geqq n(0) + 1, \quad \text{and} \quad j = 1, 2, \cdots, k,$$

which follows directly from the definitions of a_i and r_i in (1).

Then using (1), (4), and (8), the relations in (7) can be written, after some algebraic manipulations, as

$$(9a)\quad S_i = 1 \qquad \text{for } i = 0$$

$$(9b)\quad = \sum_{j=1}^{i} [(-1)^{j+1}/j!][a_{i-j}/\theta]^j S_{i-j} \qquad \text{for } i = 1, 2, \cdots, n(0)$$

$$(9c)\quad = \sum_{j=1}^{i} [(-1)^{j+1}/j!][(a_{i-j} - r_i)/\theta]^j S_{i-j} \quad \text{for } i = n(0) + 1, \cdots, m$$

$$(9d)\quad = \sum_{j=1}^{m} [(-1)^{j+1}/j!][(a_{i-j} - r_i)/\theta]^j S_{i-j} \quad \text{for } i = m + 1, m + 2, \cdots.$$

Using (1) and (5), (3) can be written as

$$(10)\qquad P_A(\theta) = \sum_{i=0}^{\infty} S_i \exp(-a_i/\theta),$$

where S_i is calculated from the recurrence relations (9a) through (9d).

Now it can be further shown that (2) and (10) are equivalent term-by-term, that is

$$(11)\qquad p[E_a^{(i)}, E_c^{(i)}] = S_i \exp(-a_i/\theta).$$

The proof of (11) is delayed until Section 5 in order to retain continuity at this point.

4. Exact *OC* curve for (*A*, *B*) rule with truncation. The exact *OC* curve for any truncation rule (V_0, i_0) can now be obtained from (9a) through (9d) and (10) by merely replacing a_i by V_0 for those values of i such that $a_i > V_0$ and by terminating the summation at the $(i_0 - 1)$th term. Then (10) reduces to

$$(12) \quad P_A(\theta) = \begin{cases} \sum_{i=0}^{J} S_i \exp(-a_i/\theta) + \sum_{i=J+1}^{i_0-1} S_i^* \exp(-V_0/\theta) & \text{for} \quad V_0 \geqq h_0 \\ \sum_{i=0}^{i_0-1} S_i^* \exp(-V_0/\theta) & \text{for} \quad V_0 < h_0 \end{cases}$$

where J is the largest integer less than $(V_0 - h_0)/s$, and where S_i^* is the same as S_i (as defined in (9a) through (9d)) except that a_i is replaced by V_0 whenever $a_i > V_0$.

As pointed out by Epstein [2], a reasonable truncation procedure is to select V_0 (or i_0) and determine i_0 (or V_0) from the relation $V_0 = i_0 s$.

5. Proof of formula (11). The proof of (11) requires treatment of the distinct cases indicated in equations (9a) through (9d). Consider first the probability of accepting H_0 at any stage i, where $n(0)$ and m are arbitrary integers previously defined and displayed in Fig. 1. Let t_j represent the failure time of the jth item and u_j denote the sum of the failure times for the first j units. For case (9a), the probability of accepting H_0 at stage $i = 0$ is the probability that the first item survives a_0 time units. That is,

$$(13) \quad p[E_a^{(0)}, E_c^{(0)}] = 1 - \int_0^{a_0} [1/\theta] \exp(-t_1/\theta)\, dt_1 = S_0 \exp(-a_0/\theta),$$

which proves (11) for the case (9a).

For the case (9b), it is clear from the continue test region of Fig. 1 that for acceptance at any stage i $(i = 1, 2, \cdots, n(0))$ the event $[E_a^{(i)}, E_c^{(i)}]$ implies the joint occurrence of the following set of events: the first failure time, $t_1 = u_1$, must occur in the interval $(0, a_0)$; the sum of the first two failure times, $u_2 = t_1 + t_2$, must lie in (u_1, a_1); $\cdots$; the sum of the first i failure times, u_i, must lie in (u_{i-1}, a_{i-1}) while the $(i + 1)$th unit must survive the next $(a_i - u_i)$ time units. The probability of this compound event is

$$(14) \quad \begin{aligned} p[E_a^{(i)}, E_c^{(i)}] = \int_{t_1=0}^{a_0} dt_1 \int_{t_2=0}^{a_1-u_1} dt_2 \\ \cdots \int_{t_i=0}^{a_{i-1}-u_{i-1}} dt_i\, (1/\theta)^i \exp\{[-u_i - (a_i - u_i)]/\theta\}. \end{aligned}$$

Under the transformation $u_1 = t_1$, $u_2 = t_1 + t_2$, $\cdots$, $u_i = \sum_{k=1}^{i} t_k$, (14) becomes

$$(15) \quad \begin{aligned} p[E_a^{(i)}, E_c^{(i)}] = (1/\theta)^i \exp(-a_i/\theta) \int_0^{a_0} du_1 \int_{u_1}^{a_1} du_2 \\ \cdots \int_{u_{i-1}}^{a_{i-1}} du_i, \qquad i = 1, 2, \cdots, n(0). \end{aligned}$$

It is easy to show by induction that the multiple integral on the RHS of (15) can

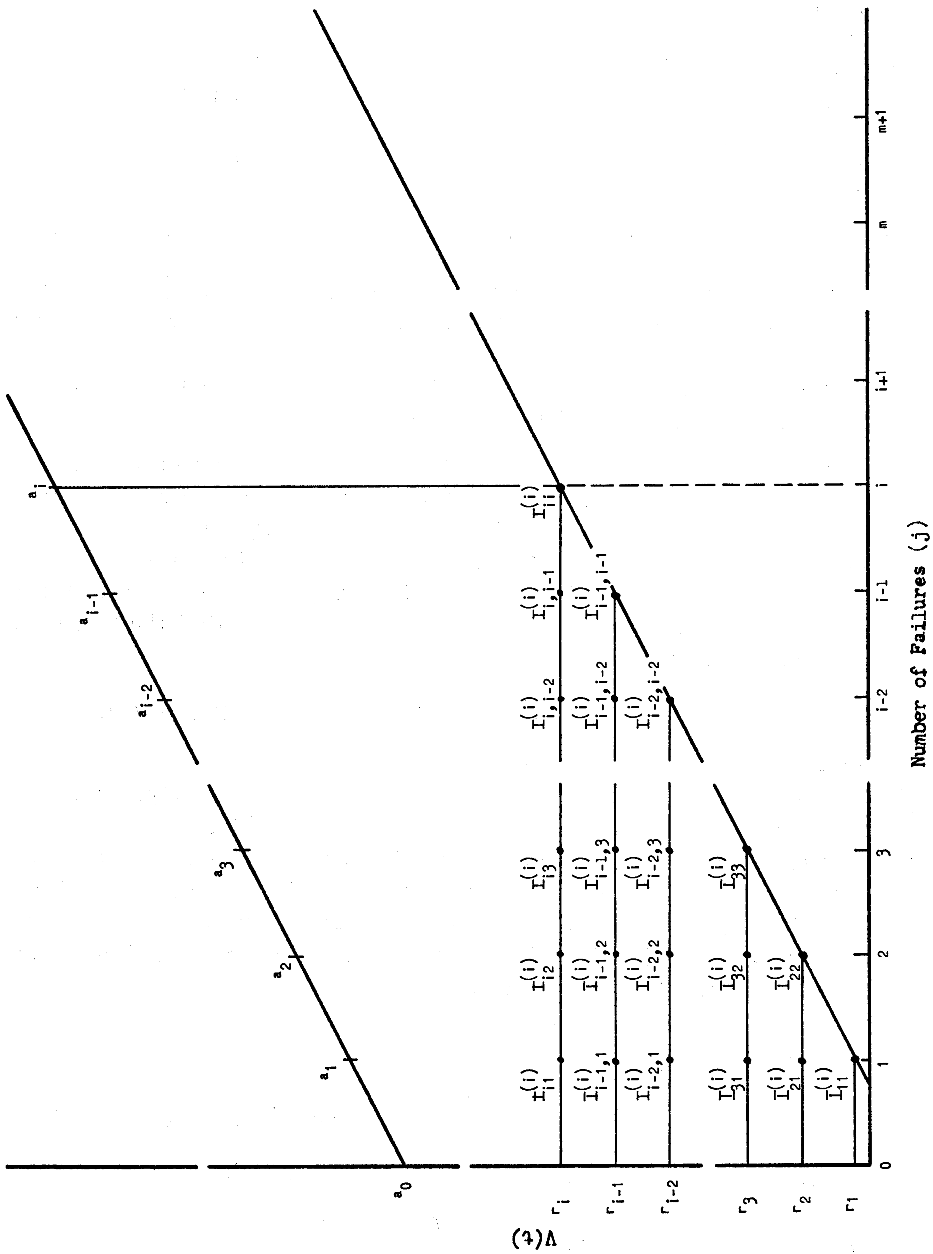

FIG. 2

be expressed as

$$(16)\qquad \int_0^{a_0} du_1 \int_{u_1}^{a_1} du_2 \cdots \int_{u_i}^{a_{i-1}} du_i = (a_0/i!) \sum_{j=1}^{i} (-1)^{j+1} \binom{i}{j} a_{i-j}^{i-1}.$$

Furthermore, using the linear relation between the acceptance numbers (1) it can be shown (by expanding each a_{i-j}^{i-1} in a binomial series and collecting coefficients of $(s/a_0)^k$, $k = 0, 1, 2, \cdots, i-1$), that

$$(17)\qquad \sum_{j=1}^{i} (-1)^{j+1} \binom{i}{j} a_{i-j}^{i-1} = a_i^{i-1}.$$

Putting (16), (17) in (15)

$$(18)\qquad p[E_a^{(i)}, E_c^{(i)}] = [(a_0 a_i^{i-1})/\theta^i i!] \exp(-a_i/\theta).$$

Finally using (17), it is easy to show by induction that

$$(19)\qquad S_i = \sum_{j=1}^{i} [(-1)^{j+1}/j!][a_{i-j}/\theta]^j S_{i-j} = [a_0 a_i^{i-1}]/i!\theta^i.$$

Putting (19) in (18) we get the desired result (11) for the case (9b).

To prove (11) for the case (9c), it is convenient to deal with the special case $n(0) = 0$ and arbitrary m. (Extension to the general case of arbitrary $n(0)$ and m merely involves replacing with zero each r_i for all indices $i \leqq n(0)$, wherever it appears in treating the special case.) For the case $n(0) < i \leqq m$, it is more difficult to determine the event $E_c^{(i)}$ because now one must exclude values of u_j lying in the reject region as well as those in the accept region for all $j = 1, 2, \cdots, i$. To determine $E_c^{(i)}$, it is convenient to divide the continue test region into the intervals or zones on the u_j as shown in Fig. 2. The dots in Fig. 2 are used to represent these intervals for each u_j, $j = 1, 2, \cdots, i$, as follows. Denote the dot in the k_jth row from the bottom and jth column from the left by $I_{k_j j}^{(i)}$, $k_j = j, j+1, \cdots, i; j = 1, 2, \cdots, i$. Since $u_j \geqq u_{j-1}$ for each $j = 1, 2, \cdots, i$, the dots represent the intervals on u_j, given in (20) below, depending on whether the value of u_{j-1} lay in the same or a lower zone. Using this array of dots one can construct each admissible "path" (hence each distinct element of $E_c^{(i)}$) leading to acceptance of H_0 at stage i by connecting i dots, one from each column starting with the first column and terminating with the one dot in the ith column. Since $u_j \geqq u_{j-1}$ for all j, obviously each path must be monotonically nondecreasing. Hence each path can be represented as a vector of i intervals on the u_j ($j = 1, 2, \cdots, i$), say

$$I^{(i)} = [I_{k_1 1}^{(i)}; I_{k_2 2}^{(i)}; \cdots; I_{ii}^{(i)}], \quad \text{where, for} \quad i = 1, 2, \cdots, m,$$

$$(20)\qquad I_{k_j j}^{(i)} = \begin{cases} (r_{k_1}, r_{k_1+1}) & \text{for } j = 1 \text{ and } k_1 = 1, 2, \cdots, i-1 \\ (r_i, a_0) & \text{for } j = 1 \text{ and } k_1 = i \\ (r_{k_j}, r_{k_j+1}) & \text{if } k_{j-1} < k_j \text{ for } \left\{\begin{array}{l} j = 2, 3, \cdots, i-1 \text{ and} \\ k_j = j, j+1, \cdots, i-1 \end{array}\right. \\ (u_{j-1}, r_{k_j+1}) & \text{if } k_{j-1} = k_j \\ (r_i, a_{j-1}) & \text{if } k_{j-1} < i \text{ for } \left\{\begin{array}{l} j = 2, 3, \cdots, i \text{ and} \\ k_j = i \end{array}\right. \\ (u_{j-1}, a_{j-1}) & \text{if } k_{j-1} = i \end{cases}$$

Note that we can pass from the i-stage array to the $(i - 1)$ stage array by deleting the dot labeled $I_{ii}^{(i)}$ in Fig. 2 and adding the intervals in the jth column of the ith row to the intervals in the jth column of the $(i - 1)$st row, $(j = 1, 2, \cdots, i - 1)$, all other intervals remaining unchanged. That is

$$(21) \qquad \begin{aligned} I_{i-1,j}^{(i-1)} &= I_{ij}^{(i)} + I_{i-1,j}^{(i)}, & j &= 1, 2, \cdots, i-1 \\ I_{k_j j}^{(i-1)} &= I_{k_j j}^{(i)}, & j &= 1, 2, \cdots, i-2, \text{ and} \\ & & k_j &= j, j+1, \cdots, i-2. \end{aligned}$$

To avoid counting all the paths leading to acceptance at stage i, we define, consistently with (16), $R(u_i \varepsilon I_{ii}^{(i)})$ to be the definite integral over any path terminating at $I_{ii}^{(i)}$, that is

$$(22) \qquad R(u_i \varepsilon I_{ii}^{(i)}) = \int \underset{I^{(i)}}{\cdots} \int du_1\, du_2 \cdots du_i .$$

Also define $\sum R(u_i \varepsilon I_{ii}^{(i)})$ to be the sum of all such integrals over distinct paths $I^{(i)}$ each terminating at the position $I_{ii}^{(i)}$. For example, for $i = 2$ there would be two paths $I_1^{(2)} = [I_{11}^{(2)}; I_{22}^{(2)}] \equiv [(r_1, r_2); (r_2, a_1)]$ and $I_2^{(2)} = [I_{21}^{(2)}; I_{22}^{(2)}] \equiv [(r_2, a_0); (u_1, a_1)]$, and then

$$\sum R(u_2 \varepsilon I_{22}^{(2)}) = \int_{u_1=r_1}^{r_2} \int_{u_2=r_2}^{a_1} du_1\, du_2 + \int_{u_1=r_2}^{a_0} \int_{u_2=u_1}^{a_1} du_1\, du_2 .$$

Then, by the same considerations leading to (15), we can write for the probability of acceptance at stage j

$$(23) \qquad p[E_a^{(j)}, E_c^{(j)}] = (1/\theta^j) \exp(-a_j/\theta) \sum R(u_j \varepsilon I_{jj}^{(j)}).$$

It is easy to show that the RHS of (23) reduces to (11) for $j = 1$ and $j = 2$. Assume now that (23) reduces to (11) for all j up to $j = i - 1$, or equivalently, that

$$(24) \qquad \sum R(u_j \varepsilon I_{jj}^{(j)}) = \theta^j S_j \quad \text{for all } j = 1, 2, \cdots, i-1.$$

We show now that (24), hence (11), is true at stage i for the case $i \leqq m$. Since the position $I_{ii}^{(i)}$ can be reached from only the positions $I_{i-1,i-1}^{(i)}$ and $I_{i,i-1}^{(i)}$ we can write (using (20)),

$$(25) \qquad \begin{aligned} \sum R(u_i \varepsilon I_{ii}^{(i)}) = {} & [\sum R(u_{i-1} \varepsilon I_{i-1,i-1}^{(i)})] \int_{r_i}^{a_{i-1}} du_i \\ & + [\sum R(u_{i-1} \varepsilon I_{i,i-1}^{(i)})] \int_{u_{i-1}}^{a_{i-1}} du_i . \end{aligned}$$

Substituting $\int_{u_{i-1}}^{a_{i-1}} du_i = \int_{r_i}^{a_{i-1}} du_i - \int_{r_i}^{u_{i-1}} du_i$ in (25) and rearranging, we get

$$(26) \qquad \begin{aligned} \sum R(u_i \varepsilon I_{ii}^{(i)}) = {} & [\sum R(u_{i-1} \varepsilon I_{i-1,i-1}^{(i)}) + \sum R(u_{i-1} \varepsilon I_{i,i-1}^{(i)})] \int_{r_i}^{a_{i-1}} du_i \\ & - [\sum R(u_{i-1} \varepsilon I_{i,i-1}^{(i)})] \int_{r_i}^{u_{i-1}} du_i . \end{aligned}$$

But, as a consequence of the definition of R and $\sum R$, we can write

$$(27)\quad \sum R(u_{i-1}\,\varepsilon\, I^{(i)}_{i-1,i-1}) + \sum R(u_{i-1}\,\varepsilon\, I^{(i)}_{i,i-1}) = \sum R(u_{i-1}\,\varepsilon\, I^{(i)}_{i-1,i-1} + I^{(i)}_{i,i-1}).$$

Using (21) and (24) in (27) and putting that result into (26), we get

$$(28)\quad \sum R(u_i\,\varepsilon\, I^{(i)}_{ii}) = (a_{i-1} - r_i)\theta^{i-1}S_{i-1} - [\sum R(u_{i-1}\,\varepsilon\, I^{(i)}_{i,i-1})]\int_{r_i}^{u_{i-1}} du_i\,.$$

Similarly, the $\sum R$-expression in RHS of (28) can be broken up into,

$$(29)\quad \begin{aligned} \sum R(u_{i-1}\,\varepsilon\, I^{(i)}_{i,i-1}) &= [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i-2,i-2})]\int_{r_i}^{a_{i-2}} du_{i-1} \\ &\quad + [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i-1,i-2})]\int_{r_i}^{a_{i-2}} du_{i-1} + [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i,i-2})]\int_{u_{i-2}}^{a_{i-2}} du_{i-1} \\ &= [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i-2,i-2} + I^{(i)}_{i-1,i-2})]\int_{r_i}^{a_{i-2}} du_{i-1} \\ &\quad + [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i,i-2})]\int_{u_{i-2}}^{a_{i-2}} du_{i-1}. \end{aligned}$$

Substituting $\int_{u_{i-2}}^{a_{i-2}} du_{i-1} = \int_{r_i}^{a_{i-2}} du_{i-1} - \int_{r_i}^{u_{i-2}} du_{i-1}$ in (29) and making repeated application of (21), (28) reduces to

$$(30)\quad \begin{aligned} \sum R(u_i\,\varepsilon\, I^{(i)}_{ii}) &= (a_{i-1} - r_i)\theta^{i-1}S_{i-1} - [(a_{i-2} - r_i)^2/2!]\theta^{i-2}S_{i-2} \\ &\quad + [\sum R(u_{i-2}\,\varepsilon\, I^{(i)}_{i,i-2})]\int_{r_i}^{u_{i-2}} du_{i-1}\int_{r_i}^{u_{i-1}} du_i\,. \end{aligned}$$

Continuing this process of breaking up the $\sum R$-integral forms (a total of i times), and using, at each step, the well-known result ([4], p. 218)

$$(31)\quad \int_{u_1=r_i}^{a_{i-j}}\int_{u_2=r_i}^{u_1}\cdots\int_{u_j=r_i}^{u_{j-1}} du_1\,du_2\cdots du_j = (a_{i-j} - r_i)^j/j!$$

for $j = 1, 2, \cdots, i$, (30) reduces to

$$(32)\quad \sum R(u_i\,\varepsilon\, I^{(i)}_{ii}) = \sum_{j=1}^{i} (-1)^{j+1}[(a_{i-j} - r_i)^j/j!]\theta^{i-j}S_{i-j} = \theta^i S_i\,.$$

Multiplying through by $(1/\theta^i)\exp(-a_i/\theta)$ we get the desired result, (11), for the case of (9c).

The proof for the case of (9d), i.e., for $i > m$, follows an identical argument but terminates at the term involving the factor $(a_{i-m} - r_m)^m/m!$ because it follows directly from the definition of m that $(a_{i-(m+k)} - r_i)$ is negative for all $k = 1, 2, \cdots$, and hence can't represent an admissible interval (r_i, a_{i-m-k}) on any u_j.

6. Remarks. Before this work was started, the solution of a particular truncated sequential life test with parameters $\theta_0 = 328$ min., $\theta_1 = 95$ min., $\alpha = 0.05$,

$\beta = 0.10$, $V_0 = 1140$ min., $i_0 = 7$ failures, $n(0) = 2$, and $m = 4$, was obtained by a Monte Carlo technique by Holt and Pettit of Collins Radio Company. In that experiment, 2,000 random observations were generated by machine methods from each of the exponential distributions with MTBF of $\theta = 95$, 162, 250, and 328 min. The (A, B) rule with truncation at 1140 min., or 7 failures, whichever occurred first, was applied to the observations for each θ with the following results:

θ(min.)	$P_A(\theta)$	
	Monte Carlo	Exact
95	.105	.107
162	.532	.524
250	.853	.848
328	.950	.942

The exact values listed above were computed from (12) with $n(0) = 2$, and $m = 4$.

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