TABLES OF RANGE AND STUDENTIZED RANGE[1]

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0. Summary. A description is given of the computation of tables of percentage points of the range, moments of the range, and percentage points of the studentized range for samples from a normal population. Percentage points of the (standardized) range $W = w/\sigma$ corresponding to cumulative probability $P = 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.025, 0.05, 0.1\ (0.1)\ 0.9, 0.95, 0.975, 0.99, 0.995, 0.999, 0.9995$ and 0.9999 are given to six decimal places for samples of size $n = 2\ (1)\ 20\ (2)\ 40\ (10)\ 100$. Moments (mean, variance, skewness, and elongation) of the range W are given to eight or more significant figures for samples of size $n = 2\ (1)\ 100$. Percentage points of the studentized range $Q = w/s$ corresponding to cumulative probability $P = 0.9, 0.95, 0.975, 0.99, 0.995$, and 0.999 are given to four significant figures or four decimal places, whichever is less accurate, for samples of size $n = 2\ (1)\ 20\ (2)\ 40\ (10)\ 100$, with degrees of freedom $\nu = 1\ (1)\ 20, 24, 30, 40, 60, 120$, and ∞ for the independent estimate s^2 of the population variance. All tabular values are accurate to within a unit in the last place.

1. History and Introduction.

1.1. *Theory and Tables.* One of the earliest writers to give serious consideration to the use of the sample range as a measure of dispersion was Tippett [26]. Tippett tabulated (to five decimal places) the mean range (in terms of the population standard deviation) of samples of size n, taken from a normal population, for $n = 2\ (1)\ 1000$. He also calculated a few values of σ_W, $\beta_{1:W}$, and $\beta_{2:W}$. This work was extended by E. S. Pearson [16], who computed modified values of β_1 and β_2 for $n = 60$ and 100. "Student" [24] fitted Pearson curves to the first four theoretical moments of the range for several values of n. Using "Student's" results with modifications, E. S. Pearson [17] computed lower and upper 0.5%, 1%, 5%, and 10% points (to two decimal places) for $n = 2\ (1)\ 30\ (5)\ 100$. In collaboration with McKay [13], he found the exact distribution of the range for samples of 3 from a normal population, and gave certain new results regarding the form of the range curve at its terminals.

The idea of using the ratio of the range to an independent estimate s of the population standard deviation (studentized range) was first proposed by "Student" in a letter written to and later published by E. S. Pearson [19]. Newman [14] developed this notion further, and tabulated a number of 5% and 1% points of $Q = w/s$, which he obtained by quadrature from the approximate probability law of $W = w/\sigma$ given by E. S. Pearson [17], except for $n = 3$, where the exact distribution of W obtained by McKay and Pearson [13] was used.

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The probability integral of the range was first tabulated by Pearson and Hartley [20], for $n = 2\ (1)\ 20$, $W = 0.00\ (0.05)\ 7.25$. Values were computed to five decimal places, but only four decimal places were published. Included in the same paper was a table of lower and upper 0.1%, 0.5%, 1.0%, 2.5%, 5%, and 10% points for $n = 2\ (1)\ 12$. The theoretical basis for the computations was outlined by Hartley [8] in another paper.

Pearson and Hartley [21] also tabulated the probability integral of the studentized range, which they wrote in the form

$$_{\nu}P_n(Q) = P_n(Q) + \nu^{-1}a_n(Q) + \nu^{-2}b_n(Q),$$

where ν is the number of degrees of freedom for the independent estimate s of the population standard deviation and $P_n(Q)$ is the probability integral of the range. The tables gave values (to four, two and one decimal places, respectively) of $P_n(Q)$, $a_n(Q)$, and $b_n(Q)$ for $Q = 0.00\ (0.25)\ 6.50$ and $n = 3\ (1)\ 20$, with the observation that the results were somewhat inaccurate for small values of $\nu(<10)$ and for large values of $Q(>6)$. Lower and upper 5% and 1% points were given for $n = 2\ (1)\ 20$, $\nu = 10\ (1)\ 20$, 24, 30, 40, 60, 120, and ∞, the results for $n = 2$ being obtained by multiplying by $(2)^{\frac{1}{2}}$ the corresponding percentage points of the Student t distribution. Upper 5% and 1% points for $n = 3\ (1)\ 20$ and $\nu = 10\ (1)\ 19$ were tabulated to only one decimal place, with two decimal places for the other tabular values. May [12] published extended and corrected tables of the upper percentage points. Values were given to two decimal places down through $\nu = 2$ and to one decimal place for $\nu = 1$. Values were based on exact quadrature for $\nu = 1\ (1)\ 4$, 6, 8, 12, and 24 and for 6 to 8 equidistant values of $1/Q$, with other values obtained by Lagrangian and/or harmonic interpolation. Hartley [9] published further corrections for $\nu = 5$ and $\nu = 7$.

1.2. *Applications.* One of the earliest applications of the sample range was in statistical quality control. Shewhart [23], the father of statistical quality control, considered and tentatively rejected the use of the range as a measure of dispersion, but E. S. Pearson [18] justified its use and tabulated factors for the calculation of control limits for the range. Due to the greater ease in calculating the range, it soon replaced the standard deviation in most applications of quality control.

E. S. Pearson [17] showed that if the population standard deviation is to be estimated from the mean range of a number of subgroups, the optimum subgroup size is 8, but it remained for Grubbs and Weaver [3] to make an extensive study of the use of this method.

Newman [14], following a suggestion by "Student", proposed a systematic method of applying the studentized range to multiple comparisons of treatment means in an experiment, but it was a number of years before any further work on this subject was published.

The use of the range in estimating all the variances involved in the analysis of variance was explored by Rodgers [22], who credited W. J. Jennett with suggesting this procedure. Further work on the use of the range in the analysis of variance was done by Patnaik [15], Hartley [10], and David [1], but the use of the range in the overall analysis (as contrasted with the use of the studentized range in the multiple comparisons procedures which may be used to determine which means differ significantly when overall significance has been established) has never met with wide acceptance.

One of the notable occurrences of the past ten years has been the development of a number of multiple comparisons procedures, several of which are based on the studentized range. Among these are the Newman-Keuls test (a modification by Keuls [11] of a test first proposed by Newman [14]), two tests proposed by Tukey [27], [28], and the new multiple range test proposed by Duncan [2]. The author [4] made a study of the error rates and sample sizes for a number of multiple comparisons tests based on the studentized range.

2. Computation of the Tables.

2.1. *Percentage Points of the Range.* For samples of size n from $N(\mu, 1)$, the values of the range corresponding to certain cumulative probabilities P were computed by inverse interpolation, using Aitken's method with a tolerance of 5×10^{-7}, in a new table of the probability integral of the range. The probability integral was computed at an interval of 2^{-6} in W for $n = 2\ (1)\ 20\ (2)\ 40\ (10)\ 100$, accurate to within a unit in the eighth decimal place. The method of computation of the probability integral, and a table at an interval of 0.01 in W, obtained by subtabulation, have been given by Harter and Clemm [6], and will not be included here. The percentage points were computed for the same values of n as the probability integral. The cumulative probabilities P involved were 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.025, 0.05, 0.1 (0.1) 0.9, 0.95, 0.975, 0.99, 0.995, 0.999, 0.9995, and 0.9999. The interpolation was performed on the Univac Scientific (ERA 1103A) computer. In some cases, it was necessary to divide each of three intervals in the relevant portion of the table into eight subintervals and subtabulate the probability integral, in order to meet the tolerance on the inverse interpolation. This happened especially for small values of P and for values of n near 4. The resulting percentage points were rounded to six decimal places, with error not exceeding a unit in the last place, then punched on cards and printed on the IBM 407 tabulator. The results are shown in Table 1.

2.2. *Moments of the Range.* The expected value of the kth power of the range W, for samples of size n from $N(\mu, 1)$ is given by the equation

$$E(W^k) = n(n-1) \int_{-\infty}^{\infty} \left\{ \int_0^{\infty} W^k \, [\Phi(X+W) - \Phi(X)]^{n-2} \phi(X+W) \, dW \right\} \phi(X) \, dx, \tag{1}$$

with

$$\phi(X) = (2\pi)^{-\frac{1}{2}} e^{-X^2/2}; \qquad \Phi(X) = \int_0^X \phi(X) \, dX.$$

TABLE 1

Percentage points of the range for samples of n from N (μ,1)
(Values for which the cumulative probability is P)

P\n	2	3	4	5	6	7
.0001	0.000177	0.019046	0.092394	0.205489	0.334168	0.464515
.0005	0.000886	0.042594	0.158155	0.308222	0.463700	0.612589
.0010	0.001772	0.060245	0.199446	0.367392	0.534736	0.691347
.0050	0.008862	0.134847	0.342702	0.554904	0.748983	0.921825
.0100	0.017725	0.190945	0.433676	0.665015	0.869515	1.048144
.0250	0.044319	0.303071	0.594643	0.849672	1.065951	1.250500
.0500	0.088681	0.431402	0.759533	1.029940	1.252885	1.440141
.1000	0.177712	0.618352	0.979366	1.261398	1.488195	1.676051
.2000	0.358287	0.900092	1.285672	1.573441	1.799995	1.985445
.3000	0.544925	1.138259	1.531485	1.818447	2.042028	2.223993
.4000	0.741614	1.362597	1.756529	2.040097	2.259641	2.437704
.5000	0.953873	1.587788	1.978320	2.256882	2.471652	2.645452
.6000	1.190232	1.826320	2.210281	2.482427	2.691658	2.860733
.7000	1.465738	2.094590	2.468799	2.732888	2.935559	3.099199
.8000	1.812388	2.423529	2.783758	3.037317	3.231739	3.388684
.9000	2.326174	2.902380	3.240446	3.478281	3.660721	3.808098
.9500	2.771808	3.314493	3.633160	3.857656	4.030092	4.169554
.9750	3.169822	3.682268	3.984015	4.197026	4.360906	4.493624
.9900	3.642773	4.120303	4.402801	4.602821	4.757047	4.882166
.9950	3.969745	4.424235	4.694087	4.885585	5.033479	5.153613
.9990	4.653508	5.063453	5.308804	5.483754	5.619333	5.729754
.9995	4.922533	5.316400	5.552855	5.721773	5.852849	5.959710
.9999	5.502128	5.864157	6.082864	6.239691	6.361710	6.461392

P\n	8	9	10	11	12	13
.0001	0.590186	0.708709	0.819433	0.922514	1.018443	1.107820
.0005	0.751013	0.878357	0.995220	1.102585	1.201493	1.292918
.0010	0.834826	0.965508	1.084583	1.193404	1.293250	1.385252
.0050	1.075281	1.212115	1.334927	1.445920	1.546898	1.639327
.0100	1.204819	1.343385	1.467033	1.578303	1.679205	1.771331
.0250	1.410019	1.549720	1.673517	1.784355	1.884474	1.975611
.0500	1.600414	1.739853	1.862843	1.972582	2.071455	2.161277
.1000	1.835449	1.973327	2.094446	2.202195	2.299057	2.386902
.2000	2.141656	2.276121	2.393844	2.498317	2.592064	2.676969
.3000	2.376728	2.507898	2.622556	2.724195	2.815329	2.897818
.4000	2.586852	2.714772	2.826491	2.925467	3.014177	3.094450
.5000	2.790841	2.915438	3.024202	3.120531	3.206853	3.284960
.6000	3.002059	3.123122	3.228778	3.322347	3.406194	3.482065
.7000	3.235931	3.353046	3.455258	3.545785	3.626919	3.700346
.8000	3.519834	3.632192	3.730280	3.817183	3.895093	3.965627
.9000	3.931349	4.037023	4.129346	4.211200	4.284635	4.351158
.9500	4.286309	4.386509	4.474124	4.551864	4.621655	4.684920
.9750	4.604857	4.700411	4.784033	4.858286	4.924993	4.985497
.9900	4.987183	5.077506	5.156635	5.226963	5.290196	5.347592
.9950	5.254550	5.341439	5.417616	5.485364	5.546312	5.601663
.9990	5.822728	5.902906	5.973307	6.036000	6.092468	6.143802
.9995	6.049760	6.127468	6.195739	6.256568	6.311378	6.361227
.9999	6.545530	6.618237	6.682189	6.739227	6.790668	6.837491

TABLE 1 (*Continued*)

P \ n	14	15	16	17	18	19
.0001	1.191258	1.269336	1.342579	1.411460	1.476396	1.537757
.0005	1.377729	1.456682	1.530432	1.599544	1.664501	1.725726
.0010	1.470384	1.549474	1.623228	1.692245	1.757038	1.818045
.0050	1.724403	1.803104	1.876236	1.944471	2.008372	2.068413
.0100	1.855958	1.934115	2.006645	2.074243	2.137488	2.196866
.0250	2.059129	2.136113	2.207442	2.273835	2.335884	2.394087
.0500	2.243459	2.319117	2.389145	2.454272	2.515097	2.572117
.1000	2.467168	2.540983	2.609248	2.672690	2.731907	2.787396
.2000	2.754467	2.825680	2.891495	2.952632	3.009675	3.063109
.3000	2.973079	3.042215	3.106097	3.165428	3.220781	3.272628
.4000	3.167678	3.234938	3.297083	3.354799	3.408646	3.459083
.5000	3.356208	3.421650	3.482118	3.538280	3.590680	3.639767
.6000	3.551278	3.614858	3.673612	3.728188	3.779115	3.826829
.7000	3.767343	3.828898	3.885792	3.938651	3.987986	4.034216
.8000	4.030005	4.089172	4.143877	4.194716	4.242179	4.286668
.9000	4.411913	4.467782	4.519464	4.567519	4.612403	4.654494
.9500	4.742732	4.795924	4.845154	4.890951	4.933745	4.973892
.9750	5.040817	5.091743	5.138897	5.182782	5.223806	5.262307
.9900	5.400105	5.448476	5.493291	5.535020	5.574047	5.610690
.9950	5.652328	5.699017	5.742289	5.782597	5.820307	5.855724
.9990	6.190836	6.234215	6.274452	6.311958	6.347071	6.380070
.9995	6.406916	6.449068	6.488177	6.524643	6.558791	6.590890
.9999	6.880436	6.920084	6.956892	6.991231	7.023403	7.053659

P \ n	20	22	24	26	28	30
.0001	1.595869	1.703469	1.801126	1.890368	1.972413	2.048246
.0005	1.783582	1.890415	1.987092	2.075236	2.156122	2.230775
.0010	1.875646	1.981895	2.077933	2.165415	2.245638	2.319635
.0050	2.125000	2.229147	2.323063	2.408454	2.486648	2.558691
.0100	2.252791	2.355636	2.448296	2.532489	2.609546	2.680513
.0250	2.448862	2.549497	2.640077	2.722319	2.797547	2.866799
.0500	2.625752	2.724238	2.812828	2.893229	2.966750	3.034415
.1000	2.839570	2.935330	3.021432	3.099551	3.170970	3.236692
.2000	3.113340	3.205511	3.288367	3.363534	3.432252	3.495491
.3000	3.321365	3.410792	3.491185	3.564123	3.630811	3.692189
.4000	3.506497	3.593503	3.671730	3.742714	3.807625	3.867378
.5000	3.685915	3.770611	3.846776	3.915902	3.979129	4.037342
.6000	3.871692	3.954046	4.028125	4.095376	4.156902	4.213563
.7000	4.077692	4.157522	4.229355	4.294588	4.354286	4.409280
.8000	4.328517	4.405388	4.474592	4.537464	4.595024	4.648069
.9000	4.694104	4.766906	4.832494	4.892122	4.946746	4.997113
.9500	5.011689	5.081193	5.143852	5.200850	5.253094	5.301290
.9750	5.298566	5.365277	5.425452	5.480220	5.530445	5.576798
.9900	5.645215	5.708769	5.766138	5.818385	5.866325	5.910592
.9950	5.889103	5.950573	6.006087	6.056666	6.103092	6.145978
.9990	6.411188	6.468538	6.520381	6.567657	6.611084	6.651229
.9995	6.621165	6.676981	6.727454	6.773495	6.815802	6.854920
.9999	7.082210	7.134877	7.182537	7.226043	7.266043	7.303048

TABLE 1 (*Continued*)

P\n	32	34	36	38	40	50
.0001	2.118672	2.184354	2.245845	2.303612	2.358049	2.590592
.0005	2.300019	2.364536	2.424886	2.481541	2.534898	2.762552
.0010	2.388240	2.452137	2.511888	2.567966	2.620768	2.845954
.0050	2.625423	2.687528	2.745568	2.800013	2.851256	3.069615
.0100	2.746227	2.807369	2.864498	2.918079	2.968502	3.183321
.0250	2.930905	2.990534	3.046239	3.098477	3.147630	3.357004
.0500	3.097040	3.155285	3.209693	3.260711	3.308715	3.513199
.1000	3.297515	3.354083	3.406923	3.456472	3.503096	3.701734
.2000	3.554019	3.608458	3.659314	3.707009	3.751895	3.943206
.3000	3.749002	3.801853	3.851234	3.897553	3.941149	4.127044
.4000	3.922697	3.974166	4.022264	4.067386	4.109863	4.291064
.5000	4.091246	4.141409	4.188294	4.232287	4.273707	4.450481
.6000	4.266041	4.314888	4.360553	4.403408	4.443766	4.616090
.7000	4.460228	4.507662	4.552018	4.593654	4.632872	4.800425
.8000	4.697229	4.743013	4.785838	4.826050	4.863937	5.025917
.9000	5.043816	5.087333	5.128057	5.166312	5.202369	5.356687
.9500	5.346000	5.387678	5.426697	5.463363	5.497935	5.646026
.9750	5.619817	5.659933	5.697502	5.732818	5.766127	5.908917
.9900	5.951695	5.990041	6.025967	6.059751	6.091626	6.228393
.9950	6.185810	6.222982	6.257818	6.290585	6.321508	6.454269
.9990	6.688538	6.723377	6.756044	6.786787	6.815815	6.940586
.9995	6.891285	6.925249	6.957102	6.987086	7.015401	7.137165
.9999	7.337465	7.369626	7.399801	7.428215	7.455058	7.570602

P\n	60	70	80	90	100	
.0001	2.775321	2.927787	3.057130	3.169149	3.267739	
.0005	2.943168	3.092146	3.218493	3.327905	3.424199	
.0010	3.024532	3.171801	3.296690	3.404839	3.500025	
.0050	3.242654	3.385323	3.506312	3.611097	3.703340	
.0100	3.353527	3.493863	3.612885	3.715979	3.806747	
.0250	3.522895	3.659695	3.775746	3.876293	3.964843	
.0500	3.675244	3.808908	3.922332	4.020633	4.107227	
.1000	3.859209	3.989161	4.099483	4.195134	4.279427	
.2000	4.094985	4.220324	4.326798	4.419167	4.500610	
.3000	4.274626	4.396574	4.500222	4.590182	4.669535	
.4000	4.435016	4.554032	4.655239	4.743118	4.820666	
.5000	4.591012	4.707268	4.806177	4.892097	4.967946	
.6000	4.753185	4.866667	4.963266	5.047218	5.121358	
.7000	4.933836	5.044345	5.138468	5.220308	5.292614	
.8000	5.155024	5.262057	5.353283	5.432651	5.502810	
.9000	5.479870	5.582115	5.669346	5.745304	5.812496	
.9500	5.764388	5.862730	5.946701	6.019871	6.084638	
.9750	6.023168	6.118178	6.199362	6.270146	6.332834	
.9900	6.337964	6.429173	6.507175	6.575233	6.635542	
.9950	6.560722	6.649397	6.725273	6.791507	6.850224	
.9990	7.040805	7.124401	7.196012	7.258582	7.314095	
.9995	7.235034	7.316710	7.386706	7.447888	7.502186	
.9999	7.663595	7.741285	7.807923	7.866213	7.917977	

For $n = 2$, the double integration on the right-hand side of equation (1) can be performed in closed form, giving the results $E(W) = 2/(\pi)^{\frac{1}{2}}$, $E(W^2) = 2$, $E(W^3) = 8/(\pi)^{\frac{1}{2}}$, $E(W^4) = 12$, etc. For $n \geqq 3$, it is necessary to resort to numerical integration; this was done for $n = 3$ (1) 100, $k = 1$ (1) 4, using double precision arithmetic on the ERA 1103A computer, employing the seven-point Lagrangian integration formula for the inner integral and the trapezoidal rule for the outer integral (since the integrand tends strongly to zero at both limits). From the values of $E(W^k)$, values of the variance σ_W^2, the standard deviation σ_W, and the third and fourth moments about the mean, $\mu_{3:W}$ and $\mu_{4:W}$, were computed. From these in turn the standard third and fourth moments, $\alpha_{3:W}$ and $\alpha_{4:W}$, were computed. For $n = 3$ (1) 15, intervals $h = 0.05$ and $2h = 0.10$ were used for the numerical integration; for $n = 16$ (1) 100, the intervals used were $h = 0.10$ and $2h = 0.20$. The agreement between the results for h and $2h$ is such as to indicate that if one retains 10 decimal places (11 significant figures) for $E(W)$, 10 decimal places (10 significant figures) for σ_W^2, 8 decimal places (8 significant figures) for $\alpha_{3:W}$, and 7 decimal places (8 significant figures) for $\alpha_{4:W}$, the error will not exceed a unit in the last place, except possibly in the case of σ_W^2 for $n = 5$ (1) 11. A few corrections of one in the last place of the mean and of up to three in the last place of the variance for sample sizes up through 20 were made in order to make the results agree with values computed from an unpublished 20-decimal-place version (ten decimal places were published) of tables of the expected values of order statistics and products of order statistics computed by Teichroew [25]. The resulting Table 2 was printed on the IBM 407 tabulator.

2.3. *Percentage Points of the Studentized Range.* The values of the studentized range corresponding to certain cumulative probabilities P were computed, for samples of size n from a normal distribution with ν degrees of freedom for the independent estimate s of the population standard deviation, by inverse interpolation in a new table of the probability integral of the studentized range. The probability integral was computed at varying intervals, small enough to make the table interpolable, for $n = 2$ (1) 20 (2) 40 (10) 100 and $\nu = 1$ (1) 20, 24, 30, 40, 60, and 120, accurate to within a unit in the sixth decimal place, except for values greater than 0.999995, which are given as 1.00000. The method of computation of the probability integral and the table itself have been given by Harter, Clemm, and Guthrie [7], and will not be included here. The percentage points were computed for the same values of n and ν as the probability integral. Values for $\nu = \infty$, obtained by rounding values from Table 1, are included for convenience in interpolation. Results have been tabulated by Harter, Clemm, and Guthrie [7] for cumulative probability 0.001, 0.005, 0.01, 0.025, 0.05, 0.1 (0.1) 0.9, 0.95, 0.975, 0.99, 0.995, and 0.999, but values will be given here only for the highest six of these values of P. The interpolation necessary to obtain the percentage points was performed on the ERA 1103A computer, using an iterative procedure involving the following steps:

1. In the table of the probability integral of the studentized range for the

desired values of n and ν, find the two successive probabilities, y_0 and y_1, between which the required cumulative probability P lies. Call the two corresponding arguments (studentized ranges) x_0 and x_1, respectively. The required studentized range Q corresponding to cumulative probability P will lie between x_0 and x_1.

2. Compute the tolerance T for P corresponding to a tolerance $5 \times 10^{u-5}$ for Q by means of the equation $T = (\Delta P/\Delta Q) \times 5 \times 10^{u-5}$, where $\Delta P = y_1 - y_0$, $\Delta Q = x_1 - x_0$, and u is the number of digits before the decimal point in numbers between x_0 and x_1.

3. Perform linear inverse interpolation to find an approximation x to the required Q, using the relation

$$x = \frac{(x_1 - x_0)(P - y_0)}{(y_1 - y_0)} + x_0 .$$

4. Perform direct interpolation, using Aitken's method with a tolerance of 5×10^{-7} and with provision for up to 16-point interpolation if the tolerance is not met for fewer points, to find the cumulative probability y corresponding to the value x of the studentized range.

5. Compare the result y of step (4) with the required cumulative probability P, using the tolerance T computed in step (2):

a. If $| y - P | \leqq T$, stop and set $Q = x$.

b. If $(y - P) > T$, replace y_1 by y and x_1 by x, then repeat the procedure, starting with step (3).

c. If $(y - P) < -T$, replace y_0 by y and x_0 by x, then repeat the procedure, starting with step (3).

The tolerance for the direct interpolation was set at 5×10^{-7}, so that the interpolation error would not add appreciably to the error already present in the new table of the probability integral of the studentized range, which is accurate to within a unit in the sixth decimal place, and hence the interpolated values are substantially as accurate as the values in the input table. Inverse interpolation is, of course, not as accurate as direct interpolation, the error being $\Delta Q/\Delta P$ times as great for inverse interpolation as for direct interpolation. Thus the tolerance for P was found by multiplying the tolerance for $Q(5 \times 10^{u-5})$ by $1/(\Delta Q/\Delta P) = \Delta P/\Delta Q$. Since u is defined as the number of digits before the decimal point in the studentized range interval under consideration, this would guarantee that the error in Q would not exceed 5 units in the fifth significant digit for $Q \geqq 0.1$, or in the fifth decimal place for $Q < 1$, if the ratio of the change in P to the change in Q were constant throughout the interval under consideration. This condition (P piecewise linear in Q) is obviously not satisfied in practice, but as long as the weaker condition

$$\max [\Delta P_0/\Delta Q_0 , \Delta P_1/\Delta Q_1] \leqq 2\Delta P/\Delta Q, \tag{2}$$

where $\Delta P_i = | y - y_i |$ and $\Delta Q_i = | x - x_i |$ $(i = 0, 1)$, is satisfied, the error in Q will not exceed a unit in the fourth significant digit for $Q \geqq 0.1$, or in the

TABLE 2

Moments of the range for samples of n from N(μ, 1)

n	Mean, E(W)	Variance, σ^2_W	Skewness, $\alpha_{3:W}$	Elongation, $\alpha_{4:W}$
2	1.12837 91671	.72676 04553	.9952 7175	3.869 1773
3	1.69256 87506	.78919 77107	.6460 7471	3.286 4021
4	2.05875 07460	.77406 24738	.5229 5734	3.188 3555
5	2.32592 89473	.74663 76009	.4655 1383	3.169 1478
6	2.53441 27212	.71917 13092	.4349 7555	3.169 6424
7	2.70435 67512	.69423 11313	.4175 5240	3.175 9064
8	2.84720 06121	.67212 36717	.4072 5611	3.183 7977
9	2.97002 63244	.65259 62151	.4011 3245	3.191 9210
10	3.07750 54617	.63528 97762	.3975 8678	3.199 7848
11	3.17287 27038	.61986 43117	.3956 9856	3.207 2265
12	3.25845 52798	.60602 85277	.3949 0804	3.214 2105
13	3.33598 03541	.59354 11244	.3948 6050	3.220 7503
14	3.40676 31082	.58220 42445	.3953 2354	3.226 8773
15	3.47182 68899	.57185 57265	.3961 4066	3.232 6279
16	3.53198 27861	.56236 21426	.3972 0409	3.238 0376
17	3.58788 39618	.55361 30572	.3984 3819	3.243 1394
18	3.64006 37579	.54551 64487	.3997 8908	3.247 9628
19	3.68896 30232	.53799 51043	.4012 1787	3.252 5340
20	3.73495 01196	.53098 37904	.4026 9623	3.256 8763
21	3.77833 58298	.52442 70274	.4042 0335	3.261 0099
22	3.81938 46434	.51827 73314	.4057 2387	3.264 9529
23	3.85832 34233	.51249 38181	.4072 4641	3.268 7211
24	3.89534 81485	.50704 10861	.4087 6255	3.272 3285
25	3.93062 92195	.50188 83188	.4102 6606	3.275 7877
26	3.96431 56795	.49700 85564	.4117 5237	3.279 1097
27	3.99653 86040	.49237 81028	.4132 1818	3.282 3044
28	4.02741 38482	.48797 60384	.4146 6112	3.285 3810
29	4.05704 42921	.48378 38184	.4160 7955	3.288 3472
30	4.08552 16883	.47978 49392	.4174 7239	3.291 2106
31	4.11292 81953	.47596 46599	.4188 3901	3.293 9775
32	4.13933 76559	.47230 97683	.4201 7907	3.296 6540
33	4.16481 66719	.46880 83838	.4214 9251	3.299 2456
34	4.18942 55115	.46544 97900	.4227 7944	3.301 7572
35	4.21321 88792	.46222 42928	.4240 4013	3.304 1933
36	4.23624 65735	.45912 30982	.4252 7496	3.306 5582
37	4.25855 40507	.45613 82080	.4264 8438	3.308 8557
38	4.28018 29105	.45326 23296	.4276 6890	3.311 0893
39	4.30117 13155	.45048 87982	.4288 2909	3.313 2624
40	4.32155 43564	.44781 15084	.4299 6552	3.315 3779
41	4.34136 43695	.44522 48562	.4310 7881	3.317 4387
42	4.36063 12150	.44272 36864	.4321 6955	3.319 4474
43	4.37938 25208	.44030 32479	.4332 3836	3.321 4065
44	4.39764 38975	.43795 91537	.4342 8586	3.323 3181
45	4.41543 91280	.43568 73457	.4353 1265	3.325 1845
46	4.43279 03360	.43348 40636	.4363 1932	3.327 0076
47	4.44971 81351	.43134 58177	.4373 0645	3.328 7892
48	4.46624 17617	.42926 93638	.4382 7462	3.330 5311
49	4.48237 91942	.42725 16817	.4392 2438	3.332 2350
50	4.49814 72588	.42528 99556	.4401 5628	3.333 9024

TABLE 2 (*Continued*)

n	Mean, E(W)	Variance, σ_W^2	Skewness, $\alpha_{3:W}$	Elongation, $\alpha_{4:W}$
51	4.51356 17253	.42338 15568	.4410 7083	3.335 5347
52	4.52863 73928	.42152 40275	.4419 6854	3.337 1333
53	4.54338 81668	.41971 50671	.4428 4992	3.338 6994
54	4.55782 71291	.41795 25197	.4437 1543	3.340 2344
55	4.57196 66012	.41623 43619	.4445 6553	3.341 7393
56	4.58581 82013	.41455 86931	.4454 0068	3.343 2152
57	4.59939 28961	.41292 37256	.4462 2129	3.344 6632
58	4.61270 10487	.41132 77766	.4470 2779	3.346 0843
59	4.62575 24613	.40976 92598	.4478 2058	3.347 4793
60	4.63855 64145	.40824 66789	.4486 0004	3.348 8491
61	4.65112 17036	.40675 86207	.4493 6654	3.350 1947
62	4.66345 66714	.40530 37495	.4501 2045	3.351 5168
63	4.67556 92383	.40388 08018	.4508 6211	3.352 8161
64	4.68746 69302	.40248 85808	.4515 9186	3.354 0933
65	4.69915 69040	.40112 59526	.4523 1001	3.355 3493
66	4.71064 59712	.39979 18416	.4530 1689	3.356 5846
67	4.72194 06193	.39848 52266	.4537 1278	3.357 7998
68	4.73304 70321	.39720 51377	.4543 9799	3.358 9955
69	4.74397 11082	.39595 06527	.4550 7278	3.360 1724
70	4.75471 84777	.39472 08940	.4557 3743	3.361 3310
71	4.76529 45186	.39351 50265	.4563 9220	3.362 4718
72	4.77570 43714	.39233 22541	.4570 3735	3.363 5953
73	4.78595 29522	.39117 18179	.4576 7310	3.364 7021
74	4.79604 49664	.39003 29940	.4582 9971	3.365 7924
75	4.80598 49196	.38891 50914	.4589 1739	3.366 8669
76	4.81577 71291	.38781 74497	.4595 2637	3.367 9259
77	4.82542 57340	.38673 94380	.4601 2686	3.368 9699
78	4.83493 47049	.38568 04527	.4607 1905	3.369 9992
79	4.84430 78526	.38463 99165	.4613 0316	3.371 0143
80	4.85354 88369	.38361 72763	.4618 7936	3.372 0154
81	4.86266 11740	.38261 20024	.4624 4786	3.373 0030
82	4.87164 82439	.38162 35871	.4630 0882	3.373 9773
83	4.88051 32976	.38065 15435	.4635 6241	3.374 9388
84	4.88925 94630	.37969 54044	.4641 0882	3.375 8877
85	4.89788 97515	.37875 47214	.4646 4820	3.376 8244
86	4.90640 70634	.37782 90636	.4651 8070	3.377 7490
87	4.91481 41933	.37691 80172	.4657 0649	3.378 6620
88	4.92311 38348	.37602 11844	.4662 2570	3.379 5635
89	4.93130 85861	.37513 81825	.4667 3849	3.380 4538
90	4.93940 09535	.37426 86432	.4672 4499	3.381 3334
91	4.94739 33562	.37341 22122	.4677 4534	3.382 2022
92	4.95528 81299	.37256 85481	.4682 3966	3.383 0607
93	4.96308 75311	.37173 73222	.4687 2809	3.383 9089
94	4.97079 37398	.37091 82175	.4692 1075	3.384 7472
95	4.97840 88637	.37011 09284	.4696 8776	3.385 5757
96	4.98593 49408	.36931 51602	.4701 5923	3.386 3948
97	4.99337 39426	.36853 06286	.4706 2527	3.387 2044
98	5.00072 77771	.36775 70590	.4710 8601	3.388 0050
99	5.00799 82910	.36699 41863	.4715 4154	3.388 7966
100	5.01518 72729	.36624 17546	.4719 9196	3.389 5794

TABLE 3

Percentage points of the studentized range for samples of n from $N(\mu, \sigma^2)$ with ν degrees of freedom for independent estimate s^2 of σ^2
(Values of the studentized range Q corresponding to cumulative probability P)

P = .90

ν \ n	2	3	4	5	6	7	8	9	10
1	8.929	13.44	16.36	18.49	20.15	21.51	22.64	23.62	24.48
2	4.130	5.733	6.773	7.538	8.139	8.633	9.049	9.409	9.725
3	3.328	4.467	5.199	5.738	6.162	6.511	6.806	7.062	7.287
4	3.015	3.976	4.586	5.035	5.388	5.679	5.926	6.139	6.327
5	2.850	3.717	4.264	4.664	4.979	5.238	5.458	5.648	5.816
6	2.748	3.559	4.065	4.435	4.726	4.966	5.168	5.344	5.499
7	2.680	3.451	3.931	4.280	4.555	4.780	4.972	5.137	5.283
8	2.630	3.374	3.834	4.169	4.431	4.646	4.829	4.987	5.126
9	2.592	3.316	3.761	4.084	4.337	4.545	4.721	4.873	5.007
10	2.563	3.270	3.704	4.018	4.264	4.465	4.636	4.783	4.913
11	2.540	3.234	3.658	3.965	4.205	4.401	4.568	4.711	4.838
12	2.521	3.204	3.621	3.922	4.156	4.349	4.511	4.652	4.776
13	2.505	3.179	3.589	3.885	4.116	4.305	4.464	4.602	4.724
14	2.491	3.158	3.563	3.854	4.081	4.267	4.424	4.560	4.680
15	2.479	3.140	3.540	3.828	4.052	4.235	4.390	4.524	4.641
16	2.469	3.124	3.520	3.804	4.026	4.207	4.360	4.492	4.608
17	2.460	3.110	3.503	3.784	4.004	4.183	4.334	4.464	4.579
18	2.452	3.098	3.488	3.767	3.984	4.161	4.311	4.440	4.554
19	2.445	3.087	3.474	3.751	3.966	4.142	4.290	4.418	4.531
20	2.439	3.078	3.462	3.736	3.950	4.124	4.271	4.398	4.510
24	2.420	3.047	3.423	3.692	3.900	4.070	4.213	4.336	4.445
30	2.400	3.017	3.386	3.648	3.851	4.016	4.155	4.275	4.381
40	2.381	2.988	3.349	3.605	3.803	3.963	4.099	4.215	4.317
60	2.363	2.959	3.312	3.562	3.755	3.911	4.042	4.155	4.254
120	2.344	2.930	3.276	3.520	3.707	3.859	3.987	4.096	4.191
∞	2.326	2.902	3.240	3.478	3.661	3.808	3.931	4.037	4.129

ν \ n	11	12	13	14	15	16	17	18	19
1	25.24	25.92	26.54	27.10	27.62	28.10	28.54	28.96	29.35
2	10.01	10.26	10.49	10.70	10.89	11.07	11.24	11.39	11.54
3	7.487	7.667	7.832	7.982	8.120	8.249	8.368	8.479	8.584
4	6.495	6.645	6.783	6.909	7.025	7.133	7.233	7.327	7.414
5	5.966	6.101	6.223	6.336	6.440	6.536	6.626	6.710	6.789
6	5.637	5.762	5.875	5.979	6.075	6.164	6.247	6.325	6.398
7	5.413	5.530	5.637	5.735	5.826	5.910	5.988	6.061	6.130
8	5.250	5.362	5.464	5.558	5.644	5.724	5.799	5.869	5.935
9	5.127	5.234	5.333	5.423	5.506	5.583	5.655	5.723	5.786
10	5.029	5.134	5.229	5.317	5.397	5.472	5.542	5.607	5.668
11	4.951	5.053	5.146	5.231	5.309	5.382	5.450	5.514	5.573
12	4.886	4.986	5.077	5.160	5.236	5.308	5.374	5.436	5.495
13	4.832	4.930	5.019	5.100	5.176	5.245	5.311	5.372	5.429
14	4.786	4.882	4.970	5.050	5.124	5.192	5.256	5.316	5.373
15	4.746	4.841	4.927	5.006	5.079	5.147	5.209	5.269	5.324
16	4.712	4.805	4.890	4.968	5.040	5.107	5.169	5.227	5.282
17	4.682	4.774	4.858	4.935	5.005	5.071	5.133	5.190	5.244
18	4.655	4.746	4.829	4.905	4.975	5.040	5.101	5.158	5.211
19	4.631	4.721	4.803	4.879	4.948	5.012	5.073	5.129	5.182
20	4.609	4.699	4.780	4.855	4.924	4.987	5.047	5.103	5.155
24	4.541	4.628	4.708	4.780	4.847	4.909	4.966	5.021	5.071
30	4.474	4.559	4.635	4.706	4.770	4.830	4.886	4.939	4.988
40	4.408	4.490	4.564	4.632	4.695	4.752	4.807	4.857	4.905
60	4.342	4.421	4.493	4.558	4.619	4.675	4.727	4.775	4.821
120	4.276	4.353	4.422	4.485	4.543	4.597	4.647	4.694	4.738
∞	4.211	4.285	4.351	4.412	4.468	4.519	4.568	4.612	4.654

TABLE 3 (*Continued*)

P = .90

ν \ n	20	22	24	26	28	30	32	34	36
1	29.71	30.39	30.99	31.54	32.04	32.50	32.93	33.33	33.71
2	11.68	11.93	12.16	12.36	12.55	12.73	12.89	13.04	13.18
3	8.683	8.864	9.029	9.177	9.314	9.440	9.557	9.666	9.768
4	7.497	7.650	7.789	7.914	8.029	8.135	8.234	8.326	8.412
5	6.863	7.000	7.123	7.236	7.340	7.435	7.523	7.606	7.683
6	6.467	6.593	6.708	6.812	6.908	6.996	7.078	7.155	7.227
7	6.195	6.315	6.422	6.521	6.611	6.695	6.773	6.845	6.913
8	5.997	6.111	6.214	6.308	6.395	6.475	6.549	6.618	6.683
9	5.846	5.956	6.055	6.146	6.229	6.306	6.378	6.444	6.507
10	5.726	5.833	5.930	6.017	6.098	6.173	6.242	6.307	6.368
11	5.630	5.734	5.828	5.914	5.992	6.065	6.132	6.196	6.255
12	5.550	5.652	5.744	5.827	5.904	5.976	6.042	6.103	6.161
13	5.483	5.583	5.673	5.755	5.830	5.900	5.965	6.025	6.082
14	5.426	5.524	5.612	5.693	5.767	5.836	5.899	5.959	6.014
15	5.376	5.473	5.560	5.639	5.713	5.780	5.843	5.901	5.956
16	5.333	5.428	5.515	5.593	5.665	5.732	5.793	5.851	5.905
17	5.295	5.389	5.474	5.552	5.623	5.689	5.750	5.806	5.860
18	5.262	5.355	5.439	5.515	5.585	5.650	5.711	5.767	5.820
19	5.232	5.324	5.407	5.483	5.552	5.616	5.676	5.732	5.784
20	5.205	5.296	5.378	5.453	5.522	5.586	5.645	5.700	5.752
24	5.119	5.208	5.287	5.360	5.427	5.489	5.546	5.600	5.650
30	5.034	5.120	5.197	5.267	5.332	5.392	5.447	5.499	5.547
40	4.949	5.032	5.107	5.174	5.236	5.294	5.347	5.397	5.444
60	4.864	4.944	5.015	5.081	5.141	5.196	5.247	5.295	5.340
120	4.779	4.856	4.924	4.987	5.044	5.097	5.146	5.192	5.235
∞	4.694	4.767	4.832	4.892	4.947	4.997	5.044	5.087	5.128

ν \ n	38	40	50	60	70	80	90	100	
1	34.06	34.38	35.79	36.91	37.83	38.62	39.30	39.91	
2	13.31	13.44	13.97	14.40	14.75	15.05	15.31	15.54	
3	9.864	9.954	10.34	10.65	10.91	11.12	11.31	11.48	
4	8.493	8.569	8.896	9.156	9.373	9.557	9.718	9.860	
5	7.756	7.825	8.118	8.353	8.548	8.715	8.859	8.988	
6	7.294	7.358	7.630	7.848	8.029	8.184	8.319	8.438	
7	6.976	7.036	7.294	7.500	7.672	7.818	7.946	8.059	
8	6.744	6.801	7.048	7.245	7.409	7.550	7.672	7.780	
9	6.566	6.621	6.859	7.050	7.208	7.343	7.461	7.566	
10	6.425	6.479	6.709	6.895	7.048	7.180	7.295	7.396	
11	6.310	6.363	6.588	6.768	6.918	7.047	7.158	7.258	
12	6.215	6.267	6.487	6.663	6.810	6.936	7.045	7.142	
13	6.135	6.186	6.402	6.575	6.719	6.842	6.949	7.045	
14	6.067	6.116	6.329	6.499	6.641	6.762	6.868	6.961	
15	6.008	6.057	6.266	6.433	6.573	6.692	6.796	6.888	
16	5.956	6.004	6.210	6.376	6.513	6.631	6.734	6.825	
17	5.910	5.958	6.162	6.325	6.461	6.577	6.679	6.769	
18	5.870	5.917	6.118	6.280	6.414	6.529	6.630	6.719	
19	5.833	5.880	6.079	6.239	6.372	6.486	6.585	6.674	
20	5.801	5.847	6.044	6.203	6.335	6.447	6.546	6.633	
24	5.697	5.741	5.933	6.086	6.214	6.324	6.419	6.503	
30	5.593	5.636	5.821	5.969	6.093	6.198	6.291	6.372	
40	5.488	5.529	5.708	5.850	5.969	6.071	6.160	6.238	
60	5.382	5.422	5.593	5.730	5.844	5.941	6.026	6.102	
120	5.275	5.313	5.476	5.606	5.715	5.808	5.888	5.960	
∞	5.166	5.202	5.357	5.480	5.582	5.669	5.745	5.812	

TABLE 3 (*Continued*)

P = .95

ν \ n	2	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.085	8.331	9.798	10.88	11.74	12.44	13.03	13.54	13.99
3	4.501	5.910	6.825	7.502	8.037	8.478	8.853	9.177	9.462
4	3.927	5.040	5.757	6.287	6.707	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.802	6.995
6	3.461	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.060	5.359	5.606	5.815	5.998	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.597	5.767	5.918
9	3.199	3.949	4.415	4.756	5.024	5.244	5.432	5.595	5.739
10	3.151	3.877	4.327	4.654	4.912	5.124	5.305	5.461	5.599
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.487
12	3.082	3.773	4.199	4.508	4.751	4.950	5.119	5.265	5.395
13	3.055	3.735	4.151	4.453	4.690	4.885	5.049	5.192	5.318
14	3.033	3.702	4.111	4.407	4.639	4.829	4.990	5.131	5.254
15	3.014	3.674	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.897	5.031	5.150
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
18	2.971	3.609	3.997	4.277	4.495	4.673	4.824	4.956	5.071
19	2.960	3.593	3.977	4.253	4.469	4.645	4.794	4.924	5.038
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.896	5.008
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
30	2.888	3.486	3.845	4.102	4.302	4.464	4.602	4.720	4.824
40	2.858	3.442	3.791	4.039	4.232	4.389	4.521	4.635	4.735
60	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560
∞	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474

ν \ n	11	12	13	14	15	16	17	18	19
1	50.59	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83
2	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.37	16.57
3	9.717	9.946	10.15	10.35	10.53	10.69	10.84	10.98	11.11
4	8.027	8.208	8.373	8.525	8.664	8.794	8.914	9.028	9.134
5	7.168	7.324	7.466	7.596	7.717	7.828	7.932	8.030	8.122
6	6.649	6.789	6.917	7.034	7.143	7.244	7.338	7.426	7.508
7	6.302	6.431	6.550	6.658	6.759	6.852	6.939	7.020	7.097
8	6.054	6.175	6.287	6.389	6.483	6.571	6.653	6.729	6.802
9	5.867	5.983	6.089	6.186	6.276	6.359	6.437	6.510	6.579
10	5.722	5.833	5.935	6.028	6.114	6.194	6.269	6.339	6.405
11	5.605	5.713	5.811	5.901	5.984	6.062	6.134	6.202	6.265
12	5.511	5.615	5.710	5.798	5.878	5.953	6.023	6.089	6.151
13	5.431	5.533	5.625	5.711	5.789	5.862	5.931	5.995	6.055
14	5.364	5.463	5.554	5.637	5.714	5.786	5.852	5.915	5.974
15	5.306	5.404	5.493	5.574	5.649	5.720	5.785	5.846	5.904
16	5.256	5.352	5.439	5.520	5.593	5.662	5.727	5.786	5.843
17	5.212	5.307	5.392	5.471	5.544	5.612	5.675	5.734	5.790
18	5.174	5.267	5.352	5.429	5.501	5.568	5.630	5.688	5.743
19	5.140	5.231	5.315	5.391	5.462	5.528	5.589	5.647	5.701
20	5.108	5.199	5.282	5.357	5.427	5.493	5.553	5.610	5.663
24	5.012	5.099	5.179	5.251	5.319	5.381	5.439	5.494	5.545
30	4.917	5.001	5.077	5.147	5.211	5.271	5.327	5.379	5.429
40	4.824	4.904	4.977	5.044	5.106	5.163	5.216	5.266	5.313
60	4.732	4.808	4.878	4.942	5.001	5.056	5.107	5.154	5.199
120	4.641	4.714	4.781	4.842	4.898	4.950	4.998	5.044	5.086
∞	4.552	4.622	4.685	4.743	4.796	4.845	4.891	4.934	4.974

TABLE 3 (*Continued*)

P = .95

ν \ n	20	22	24	26	28	30	32	34	36
1	59.56	60.91	62.12	63.22	64.23	65.15	66.01	66.81	67.56
2	16.77	17.13	17.45	17.75	18.02	18.27	18.50	18.72	18.92
3	11.24	11.47	11.68	11.87	12.05	12.21	12.36	12.50	12.63
4	9.233	9.418	9.584	9.736	9.875	10.00	10.12	10.23	10.34
5	8.208	8.368	8.512	8.643	8.764	8.875	8.979	9.075	9.165
6	7.587	7.730	7.861	7.979	8.088	8.189	8.283	8.370	8.452
7	7.170	7.303	7.423	7.533	7.634	7.728	7.814	7.895	7.972
8	6.870	6.995	7.109	7.212	7.307	7.395	7.477	7.554	7.625
9	6.644	6.763	6.871	6.970	7.061	7.145	7.222	7.295	7.363
10	6.467	6.582	6.686	6.781	6.868	6.948	7.023	7.093	7.159
11	6.326	6.436	6.536	6.628	6.712	6.790	6.863	6.930	6.994
12	6.209	6.317	6.414	6.503	6.585	6.660	6.731	6.796	6.858
13	6.112	6.217	6.312	6.398	6.478	6.551	6.620	6.684	6.744
14	6.029	6.132	6.224	6.309	6.387	6.459	6.526	6.588	6.647
15	5.958	6.059	6.149	6.233	6.309	6.379	6.445	6.506	6.564
16	5.897	5.995	6.084	6.166	6.241	6.310	6.374	6.434	6.491
17	5.842	5.940	6.027	6.107	6.181	6.249	6.313	6.372	6.427
18	5.794	5.890	5.977	6.055	6.128	6.195	6.258	6.316	6.371
19	5.752	5.846	5.932	6.009	6.081	6.147	6.209	6.267	6.321
20	5.714	5.807	5.891	5.968	6.039	6.104	6.165	6.222	6.275
24	5.594	5.683	5.764	5.838	5.906	5.968	6.027	6.081	6.132
30	5.475	5.561	5.638	5.709	5.774	5.833	5.889	5.941	5.990
40	5.358	5.439	5.513	5.581	5.642	5.700	5.753	5.803	5.849
60	5.241	5.319	5.389	5.453	5.512	5.566	5.617	5.664	5.708
120	5.126	5.200	5.266	5.327	5.382	5.434	5.481	5.526	5.568
∞	5.012	5.081	5.144	5.201	5.253	5.301	5.346	5.388	5.427

ν \ n	38	40	50	60	70	80	90	100
1	68.26	68.92	71.73	73.97	75.82	77.40	78.77	79.98
2	19.11	19.28	20.05	20.66	21.16	21.59	21.96	22.29
3	12.75	12.87	13.36	13.76	14.08	14.36	14.61	14.82
4	10.44	10.53	10.93	11.24	11.51	11.73	11.92	12.09
5	9.250	9.330	9.674	9.949	10.18	10.38	10.54	10.69
6	8.529	8.601	8.913	9.163	9.370	9.548	9.702	9.839
7	8.043	8.110	8.400	8.632	8.824	8.989	9.133	9.261
8	7.693	7.756	8.029	8.248	8.430	8.586	8.722	8.843
9	7.428	7.488	7.749	7.958	8.132	8.281	8.410	8.526
10	7.220	7.279	7.529	7.730	7.897	8.041	8.166	8.276
11	7.053	7.110	7.352	7.546	7.708	7.847	7.968	8.075
12	6.916	6.970	7.205	7.394	7.552	7.687	7.804	7.909
13	6.800	6.854	7.083	7.267	7.421	7.552	7.667	7.769
14	6.702	6.754	6.979	7.159	7.309	7.438	7.550	7.650
15	6.618	6.669	6.888	7.065	7.212	7.339	7.449	7.546
16	6.544	6.594	6.810	6.984	7.128	7.252	7.360	7.457
17	6.479	6.529	6.741	6.912	7.054	7.176	7.283	7.377
18	6.422	6.471	6.680	6.848	6.989	7.109	7.213	7.307
19	6.371	6.419	6.626	6.792	6.930	7.048	7.152	7.244
20	6.325	6.373	6.576	6.740	6.877	6.994	7.097	7.187
24	6.181	6.226	6.421	6.579	6.710	6.822	6.920	7.008
30	6.037	6.080	6.267	6.417	6.543	6.650	6.744	6.827
40	5.893	5.934	6.112	6.255	6.375	6.477	6.566	6.645
60	5.750	5.789	5.958	6.093	6.206	6.303	6.387	6.462
120	5.607	5.644	5.802	5.929	6.035	6.126	6.205	6.275
∞	5.463	5.498	5.646	5.764	5.863	5.947	6.020	6.085

TABLE 3 (*Continued*)

P = .975

ν \ n	2	3	4	5	6	7	8	9	10
1	35.99	54.00	65.69	74.22	80.87	86.29	90.85	94.77	98.20
2	8.776	11.94	14.01	15.54	16.75	17.74	18.58	19.31	19.95
3	5.907	7.661	8.808	9.660	10.34	10.89	11.37	11.78	12.14
4	4.943	6.244	7.088	7.716	8.213	8.625	8.976	9.279	9.548
5	4.474	5.558	6.257	6.775	7.186	7.527	7.816	8.068	8.291
6	4.199	5.158	5.772	6.226	6.586	6.884	7.138	7.359	7.554
7	4.018	4.897	5.455	5.868	6.194	6.464	6.695	6.895	7.072
8	3.892	4.714	5.233	5.616	5.919	6.169	6.382	6.568	6.732
9	3.797	4.578	5.069	5.430	5.715	5.950	6.151	6.325	6.479
10	3.725	4.474	4.943	5.287	5.558	5.782	5.972	6.138	6.285
11	3.667	4.391	4.843	5.173	5.433	5.648	5.831	5.989	6.130
12	3.620	4.325	4.762	5.081	5.332	5.540	5.716	5.869	6.004
13	3.582	4.269	4.694	5.004	5.248	5.449	5.620	5.769	5.900
14	3.550	4.222	4.638	4.940	5.178	5.374	5.540	5.684	5.811
15	3.522	4.182	4.589	4.885	5.118	5.309	5.471	5.612	5.737
16	3.498	4.148	4.548	4.838	5.066	5.253	5.412	5.550	5.672
17	3.477	4.118	4.512	4.797	5.020	5.204	5.361	5.496	5.615
18	3.458	4.092	4.480	4.761	4.981	5.162	5.315	5.448	5.565
19	3.442	4.068	4.451	4.728	4.945	5.123	5.275	5.405	5.521
20	3.427	4.047	4.426	4.700	4.914	5.089	5.238	5.368	5.481
24	3.381	3.983	4.347	4.610	4.816	4.984	5.126	5.250	5.358
30	3.337	3.919	4.271	4.523	4.720	4.881	5.017	5.134	5.238
40	3.294	3.858	4.197	4.439	4.627	4.780	4.910	5.022	5.120
60	3.251	3.798	4.124	4.356	4.536	4.682	4.806	4.912	5.006
120	3.210	3.739	4.053	4.276	4.447	4.587	4.704	4.805	4.894
∞	3.170	3.682	3.984	4.197	4.361	4.494	4.605	4.700	4.784

ν \ n	11	12	13	14	15	16	17	18	19
1	101.3	104.0	106.5	108.8	110.8	112.7	114.5	116.2	117.7
2	20.52	21.03	21.49	21.91	22.30	22.67	23.01	23.32	23.62
3	12.46	12.75	13.01	13.26	13.48	13.69	13.88	14.06	14.23
4	9.788	10.01	10.20	10.39	10.55	10.71	10.85	10.99	11.11
5	8.490	8.670	8.834	8.984	9.124	9.253	9.374	9.486	9.593
6	7.729	7.887	8.031	8.163	8.286	8.399	8.506	8.605	8.698
7	7.230	7.373	7.504	7.624	7.735	7.839	7.935	8.025	8.111
8	6.879	7.011	7.132	7.244	7.347	7.443	7.532	7.616	7.695
9	6.617	6.742	6.856	6.961	7.058	7.148	7.232	7.311	7.385
10	6.416	6.534	6.643	6.742	6.834	6.920	7.000	7.075	7.146
11	6.256	6.369	6.473	6.568	6.657	6.739	6.815	6.887	6.955
12	6.125	6.235	6.335	6.427	6.512	6.591	6.665	6.734	6.799
13	6.017	6.123	6.220	6.309	6.392	6.468	6.539	6.607	6.670
14	5.926	6.029	6.123	6.210	6.290	6.364	6.434	6.499	6.560
15	5.848	5.949	6.041	6.125	6.203	6.276	6.344	6.407	6.467
16	5.781	5.879	5.969	6.052	6.128	6.199	6.265	6.328	6.386
17	5.722	5.818	5.907	5.987	6.062	6.132	6.197	6.258	6.315
18	5.670	5.765	5.852	5.931	6.004	6.073	6.137	6.197	6.253
19	5.624	5.718	5.803	5.881	5.954	6.020	6.083	6.142	6.198
20	5.583	5.675	5.759	5.836	5.907	5.974	6.036	6.093	6.148
24	5.455	5.543	5.623	5.697	5.764	5.827	5.886	5.941	5.994
30	5.330	5.414	5.490	5.560	5.624	5.684	5.740	5.792	5.841
40	5.208	5.288	5.360	5.426	5.487	5.544	5.597	5.646	5.693
60	5.089	5.164	5.232	5.295	5.352	5.406	5.456	5.503	5.546
120	4.972	5.043	5.107	5.166	5.221	5.271	5.318	5.362	5.403
∞	4.858	4.925	4.985	5.041	5.092	5.139	5.183	5.224	5.262

TABLE 3 (*Continued*)

P = .975

ν \ n	20	22	24	26	28	30	32	34	36
1	119.2	121.9	124.3	126.5	128.6	130.4	132.1	133.7	135.2
2	23.89	24.41	24.87	25.29	25.67	26.03	26.35	26.66	26.95
3	14.39	14.69	14.95	15.19	15.41	15.62	15.81	15.99	16.15
4	11.23	11.46	11.66	11.84	12.00	12.16	12.30	12.44	12.56
5	9.693	9.878	10.04	10.20	10.34	10.47	10.59	10.70	10.80
6	8.787	8.949	9.097	9.231	9.355	9.469	9.575	9.674	9.767
7	8.191	8.339	8.473	8.595	8.708	8.812	8.909	8.999	9.084
8	7.769	7.907	8.031	8.145	8.250	8.346	8.436	8.520	8.599
9	7.455	7.585	7.702	7.809	7.908	7.999	8.084	8.163	8.237
10	7.212	7.335	7.447	7.549	7.643	7.729	7.810	7.885	7.956
11	7.019	7.137	7.244	7.341	7.431	7.514	7.592	7.664	7.732
12	6.861	6.974	7.078	7.172	7.258	7.338	7.413	7.483	7.548
13	6.730	6.840	6.939	7.031	7.115	7.192	7.265	7.332	7.396
14	6.619	6.726	6.823	6.911	6.993	7.069	7.139	7.204	7.266
15	6.523	6.628	6.723	6.809	6.889	6.962	7.031	7.095	7.155
16	6.441	6.543	6.636	6.721	6.799	6.870	6.938	7.000	7.059
17	6.370	6.469	6.560	6.644	6.720	6.790	6.856	6.917	6.975
18	6.306	6.404	6.493	6.575	6.650	6.720	6.784	6.844	6.900
19	6.250	6.347	6.434	6.514	6.588	6.656	6.719	6.779	6.835
20	6.200	6.295	6.381	6.460	6.532	6.600	6.662	6.720	6.775
24	6.043	6.133	6.215	6.290	6.359	6.423	6.482	6.538	6.589
30	5.888	5.974	6.052	6.123	6.188	6.248	6.305	6.357	6.406
40	5.737	5.818	5.891	5.958	6.020	6.077	6.130	6.179	6.226
60	5.588	5.664	5.733	5.797	5.854	5.908	5.958	6.004	6.048
120	5.442	5.513	5.578	5.637	5.691	5.741	5.788	5.831	5.872
∞	5.299	5.365	5.425	5.480	5.530	5.577	5.620	5.660	5.698

ν \ n	38	40	50	60	70	80	90	100
1	136.6	137.9	143.6	148.1	151.8	154.9	157.7	160.0
2	27.22	27.47	28.55	29.42	30.13	30.74	31.27	31.74
3	16.31	16.46	17.08	17.59	18.00	18.36	18.67	18.95
4	12.68	12.79	13.27	13.65	13.96	14.23	14.47	14.68
5	10.91	11.00	11.40	11.72	11.99	12.21	12.41	12.59
6	9.855	9.938	10.30	10.58	10.81	11.02	11.19	11.35
7	9.164	9.239	9.563	9.822	10.04	10.23	10.38	10.53
8	8.673	8.743	9.044	9.286	9.487	9.660	9.810	9.944
9	8.307	8.373	8.657	8.885	9.076	9.238	9.381	9.507
10	8.023	8.086	8.356	8.574	8.755	8.911	9.046	9.167
11	7.796	7.856	8.116	8.325	8.499	8.648	8.779	8.894
12	7.610	7.668	7.919	8.120	8.289	8.433	8.559	8.671
13	7.455	7.512	7.755	7.950	8.113	8.253	8.375	8.484
14	7.324	7.379	7.615	7.806	7.965	8.101	8.220	8.325
15	7.212	7.265	7.496	7.682	7.837	7.970	8.086	8.189
16	7.115	7.167	7.393	7.574	7.726	7.856	7.969	8.070
17	7.030	7.081	7.302	7.480	7.628	7.756	7.868	7.966
18	6.954	7.005	7.221	7.396	7.543	7.667	7.777	7.874
19	6.887	6.936	7.150	7.322	7.465	7.589	7.696	7.792
20	6.827	6.876	7.086	7.255	7.397	7.518	7.624	7.718
24	6.639	6.685	6.885	7.046	7.180	7.296	7.397	7.486
30	6.453	6.497	6.686	6.839	6.965	7.075	7.171	7.255
40	6.270	6.311	6.489	6.633	6.753	6.855	6.945	7.025
60	6.089	6.127	6.295	6.429	6.540	6.636	6.720	6.795
120	5.910	5.946	6.101	6.225	6.329	6.418	6.495	6.564
∞	5.733	5.766	5.909	6.023	6.118	6.199	6.270	6.333

TABLE 3 (*Continued*)

P = .99

ν \ n	2	3	4	5	6	7	8	9	10
1	90.03	135.0	164.3	185.6	202.2	215.8	227.2	237.0	245.6
2	14.04	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69
3	8.261	10.62	12.17	13.33	14.24	15.00	15.64	16.20	16.69
4	6.512	8.120	9.173	9.958	10.58	11.10	11.55	11.93	12.27
5	5.702	6.976	7.804	8.421	8.913	9.321	9.669	9.972	10.24
6	5.243	6.331	7.033	7.556	7.973	8.318	8.613	8.869	9.097
7	4.949	5.919	6.543	7.005	7.373	7.679	7.939	8.166	8.368
8	4.746	5.635	6.204	6.625	6.960	7.237	7.474	7.681	7.863
9	4.596	5.428	5.957	6.348	6.658	6.915	7.134	7.325	7.495
10	4.482	5.270	5.769	6.136	6.428	6.669	6.875	7.055	7.213
11	4.392	5.146	5.621	5.970	6.247	6.476	6.672	6.842	6.992
12	4.320	5.046	5.502	5.836	6.101	6.321	6.507	6.670	6.814
13	4.260	4.964	5.404	5.727	5.981	6.192	6.372	6.528	6.667
14	4.210	4.895	5.322	5.634	5.881	6.085	6.258	6.409	6.543
15	4.168	4.836	5.252	5.556	5.796	5.994	6.162	6.309	6.439
16	4.131	4.786	5.192	5.489	5.722	5.915	6.079	6.222	6.349
17	4.099	4.742	5.140	5.430	5.659	5.847	6.007	6.147	6.270
18	4.071	4.703	5.094	5.379	5.603	5.788	5.944	6.081	6.201
19	4.046	4.670	5.054	5.334	5.554	5.735	5.889	6.022	6.141
20	4.024	4.639	5.018	5.294	5.510	5.688	5.839	5.970	6.087
24	3.956	4.546	4.907	5.168	5.374	5.542	5.685	5.809	5.919
30	3.889	4.455	4.799	5.048	5.242	5.401	5.536	5.653	5.756
40	3.825	4.367	4.696	4.931	5.114	5.265	5.392	5.502	5.599
60	3.762	4.282	4.595	4.818	4.991	5.133	5.253	5.356	5.447
120	3.702	4.200	4.497	4.709	4.872	5.005	5.118	5.214	5.299
∞	3.643	4.120	4.403	4.603	4.757	4.882	4.987	5.078	5.157

ν \ n	11	12	13	14	15	16	17	18	19
1	253.2	260.0	266.2	271.8	277.0	281.8	286.3	290.4	294.3
2	32.59	33.40	34.13	34.81	35.43	36.00	36.53	37.03	37.50
3	17.13	17.53	17.89	18.22	18.52	18.81	19.07	19.32	19.55
4	12.57	12.84	13.09	13.32	13.53	13.73	13.91	14.08	14.24
5	10.48	10.70	10.89	11.08	11.24	11.40	11.55	11.68	11.81
6	9.301	9.485	9.653	9.808	9.951	10.08	10.21	10.32	10.43
7	8.548	8.711	8.860	8.997	9.124	9.242	9.353	9.456	9.554
8	8.027	8.176	8.312	8.436	8.552	8.659	8.760	8.854	8.943
9	7.647	7.784	7.910	8.025	8.132	8.232	8.325	8.412	8.495
10	7.356	7.485	7.603	7.712	7.812	7.906	7.993	8.076	8.153
11	7.128	7.250	7.362	7.465	7.560	7.649	7.732	7.809	7.883
12	6.943	7.060	7.167	7.265	7.356	7.441	7.520	7.594	7.665
13	6.791	6.903	7.006	7.101	7.188	7.269	7.345	7.417	7.485
14	6.664	6.772	6.871	6.962	7.047	7.126	7.199	7.268	7.333
15	6.555	6.660	6.757	6.845	6.927	7.003	7.074	7.142	7.204
16	6.462	6.564	6.658	6.744	6.823	6.898	6.967	7.032	7.093
17	6.381	6.480	6.572	6.656	6.734	6.806	6.873	6.937	6.997
18	6.310	6.407	6.497	6.579	6.655	6.725	6.792	6.854	6.912
19	6.247	6.342	6.430	6.510	6.585	6.654	6.719	6.780	6.837
20	6.191	6.285	6.371	6.450	6.523	6.591	6.654	6.714	6.771
24	6.017	6.106	6.186	6.261	6.330	6.394	6.453	6.510	6.563
30	5.849	5.932	6.008	6.078	6.143	6.203	6.259	6.311	6.361
40	5.686	5.764	5.835	5.900	5.961	6.017	6.069	6.119	6.165
60	5.528	5.601	5.667	5.728	5.785	5.837	5.886	5.931	5.974
120	5.375	5.443	5.505	5.562	5.614	5.662	5.708	5.750	5.790
∞	5.227	5.290	5.348	5.400	5.448	5.493	5.535	5.574	5.611

TABLE 3 (*Continued*)

P = .99

ν \ n	20	22	24	26	28	30	32	34	36
1	298.0	304.7	310.8	316.3	321.3	326.0	330.3	334.3	338.0
2	37.95	38.76	39.49	40.15	40.76	41.32	41.84	42.33	42.78
3	19.77	20.17	20.53	20.86	21.16	21.44	21.70	21.95	22.17
4	14.40	14.68	14.93	15.16	15.37	15.57	15.75	15.92	16.08
5	11.93	12.16	12.36	12.54	12.71	12.87	13.02	13.15	13.28
6	10.54	10.73	10.91	11.06	11.21	11.34	11.47	11.58	11.69
7	9.646	9.815	9.970	10.11	10.24	10.36	10.47	10.58	10.67
8	9.027	9.182	9.322	9.450	9.569	9.678	9.779	9.874	9.964
9	8.573	8.717	8.847	8.966	9.075	9.177	9.271	9.360	9.443
10	8.226	8.361	8.483	8.595	8.698	8.794	8.883	8.966	9.044
11	7.952	8.080	8.196	8.303	8.400	8.491	8.575	8.654	8.728
12	7.731	7.853	7.964	8.066	8.159	8.246	8.327	8.402	8.473
13	7.548	7.665	7.772	7.870	7.960	8.043	8.121	8.193	8.262
14	7.395	7.508	7.611	7.705	7.792	7.873	7.948	8.018	8.084
15	7.264	7.374	7.474	7.566	7.650	7.728	7.800	7.869	7.932
16	7.152	7.258	7.356	7.445	7.527	7.602	7.673	7.739	7.802
17	7.053	7.158	7.253	7.340	7.420	7.493	7.563	7.627	7.687
18	6.968	7.070	7.163	7.247	7.325	7.398	7.465	7.528	7.587
19	6.891	6.992	7.082	7.166	7.242	7.313	7.379	7.440	7.498
20	6.823	6.922	7.011	7.092	7.168	7.237	7.302	7.362	7.419
24	6.612	6.705	6.789	6.865	6.936	7.001	7.062	7.119	7.173
30	6.407	6.494	6.572	6.644	6.710	6.772	6.828	6.881	6.932
40	6.209	6.289	6.362	6.429	6.490	6.547	6.600	6.650	6.697
60	6.015	6.090	6.158	6.220	6.277	6.330	6.378	6.424	6.467
120	5.827	5.897	5.959	6.016	6.069	6.117	6.162	6.204	6.244
∞	5.645	5.709	5.766	5.818	5.866	5.911	5.952	5.990	6.026

ν \ n	38	40	50	60	70	80	90	100
1	341.5	344.8	358.9	370.1	379.4	387.3	394.1	400.1
2	43.21	43.61	45.33	46.70	47.83	48.80	49.64	50.38
3	22.39	22.59	23.45	24.13	24.71	25.19	25.62	25.99
4	16.23	16.37	16.98	17.46	17.86	18.20	18.50	18.77
5	13.40	13.52	14.00	14.39	14.72	14.99	15.23	15.45
6	11.80	11.90	12.31	12.65	12.92	13.16	13.37	13.55
7	10.77	10.85	11.23	11.52	11.77	11.99	12.17	12.34
8	10.05	10.13	10.47	10.75	10.97	11.17	11.34	11.49
9	9.521	9.594	9.912	10.17	10.38	10.57	10.73	10.87
10	9.117	9.187	9.486	9.726	9.927	10.10	10.25	10.39
11	8.798	8.864	9.148	9.377	9.568	9.732	9.875	10.00
12	8.539	8.603	8.875	9.094	9.277	9.434	9.571	9.693
13	8.326	8.387	8.648	8.859	9.035	9.187	9.318	9.436
14	8.146	8.204	8.457	8.661	8.832	8.978	9.106	9.219
15	7.992	8.049	8.295	8.492	8.658	8.800	8.924	9.035
16	7.860	7.916	8.154	8.347	8.507	8.646	8.767	8.874
17	7.745	7.799	8.031	8.219	8.377	8.511	8.630	8.735
18	7.643	7.696	7.924	8.107	8.261	8.393	8.508	8.611
19	7.553	7.605	7.828	8.008	8.159	8.288	8.401	8.502
20	7.473	7.523	7.742	7.919	8.067	8.194	8.305	8.404
24	7.223	7.270	7.476	7.642	7.780	7.900	8.004	8.097
30	6.978	7.023	7.215	7.370	7.500	7.611	7.709	7.796
40	6.740	6.782	6.960	7.104	7.225	7.328	7.419	7.500
60	6.507	6.546	6.710	6.843	6.954	7.050	7.133	7.207
120	6.281	6.316	6.467	6.588	6.689	6.776	6.852	6.919
∞	6.060	6.092	6.228	6.338	6.429	6.507	6.575	6.636

TABLE 3 (*Continued*)

P = .995

ν \ n	2	3	4	5	6	7	8	9	10
1	180.1	270.1	328.5	371.2	404.4	431.6	454.4	474.0	491.1
2	19.93	26.97	31.60	35.02	37.73	39.95	41.83	43.46	44.89
3	10.55	13.50	15.45	16.91	18.06	19.01	19.83	20.53	21.15
4	7.916	9.814	11.06	11.99	12.74	13.35	13.88	14.33	14.74
5	6.751	8.196	9.141	9.847	10.41	10.88	11.28	11.63	11.93
6	6.105	7.306	8.088	8.670	9.135	9.522	9.852	10.14	10.40
7	5.699	6.750	7.429	7.935	8.339	8.674	8.961	9.211	9.433
8	5.420	6.370	6.981	7.435	7.797	8.097	8.354	8.578	8.777
9	5.218	6.096	6.657	7.074	7.405	7.680	7.915	8.120	8.303
10	5.065	5.888	6.412	6.800	7.109	7.365	7.584	7.775	7.944
11	4.945	5.727	6.222	6.588	6.878	7.119	7.325	7.505	7.664
12	4.849	5.597	6.068	6.416	6.693	6.922	7.118	7.288	7.439
13	4.770	5.490	5.943	6.277	6.541	6.760	6.947	7.111	7.255
14	4.704	5.401	5.838	6.160	6.414	6.626	6.805	6.962	7.101
15	4.647	5.325	5.750	6.061	6.308	6.511	6.685	6.837	6.971
16	4.599	5.261	5.674	5.977	6.216	6.413	6.582	6.729	6.859
17	4.557	5.205	5.608	5.903	6.136	6.329	6.493	6.636	6.763
18	4.521	5.156	5.550	5.839	6.067	6.255	6.415	6.554	6.678
19	4.488	5.113	5.500	5.783	6.005	6.189	6.346	6.482	6.603
20	4.460	5.074	5.455	5.732	5.951	6.131	6.285	6.418	6.537
24	4.371	4.955	5.315	5.577	5.783	5.952	6.096	6.221	6.332
30	4.285	4.841	5.181	5.428	5.621	5.780	5.914	6.031	6.135
40	4.202	4.731	5.053	5.284	5.465	5.614	5.739	5.848	5.944
60	4.122	4.625	4.928	5.146	5.316	5.454	5.571	5.673	5.762
120	4.045	4.523	4.809	5.013	5.172	5.301	5.410	5.504	5.586
∞	3.970	4.424	4.694	4.886	5.033	5.154	5.255	5.341	5.418

ν \ n	11	12	13	14	15	16	17	18	19
1	506.3	520.0	532.4	543.6	554.0	563.6	572.5	580.9	588.7
2	46.16	47.31	48.35	49.30	50.17	50.99	51.74	52.45	53.12
3	21.70	22.20	22.66	23.08	23.46	23.82	24.15	24.46	24.76
4	15.10	15.42	15.72	15.99	16.24	16.48	16.70	16.90	17.09
5	12.21	12.46	12.69	12.90	13.09	13.27	13.44	13.60	13.75
6	10.63	10.83	11.02	11.20	11.36	11.51	11.65	11.78	11.90
7	9.632	9.812	9.977	10.13	10.27	10.40	10.52	10.64	10.75
8	8.955	9.117	9.265	9.401	9.527	9.644	9.754	9.857	9.953
9	8.466	8.614	8.749	8.874	8.990	9.097	9.198	9.292	9.381
10	8.096	8.234	8.360	8.476	8.583	8.683	8.777	8.865	8.947
11	7.807	7.937	8.055	8.164	8.265	8.359	8.447	8.530	8.608
12	7.575	7.697	7.810	7.914	8.009	8.099	8.183	8.261	8.335
13	7.384	7.502	7.609	7.708	7.800	7.886	7.965	8.040	8.111
14	7.225	7.338	7.442	7.537	7.625	7.707	7.784	7.856	7.924
15	7.091	7.200	7.300	7.392	7.477	7.556	7.630	7.699	7.765
16	6.976	7.081	7.178	7.267	7.349	7.426	7.498	7.566	7.629
17	6.876	6.979	7.072	7.159	7.239	7.314	7.384	7.449	7.511
18	6.788	6.888	6.980	7.064	7.142	7.215	7.283	7.347	7.407
19	6.711	6.809	6.898	6.981	7.057	7.128	7.195	7.257	7.316
20	6.642	6.738	6.826	6.907	6.981	7.051	7.116	7.177	7.235
24	6.431	6.520	6.602	6.677	6.747	6.812	6.872	6.930	6.983
30	6.227	6.310	6.387	6.456	6.521	6.581	6.638	6.691	6.741
40	6.030	6.108	6.179	6.244	6.304	6.360	6.412	6.461	6.507
60	5.841	5.913	5.979	6.039	6.094	6.146	6.194	6.239	6.281
120	5.660	5.726	5.786	5.842	5.893	5.940	5.984	6.025	6.064
∞	5.485	5.546	5.602	5.652	5.699	5.742	5.783	5.820	5.856

TABLE 3 (*Continued*)

P = .995

ν \ n	20	22	24	26	28	30	32	34	36
1	596.0	609.5	621.7	632.6	642.7	652.0	660.6	668.5	676.0
2	53.74	54.89	55.92	56.86	57.73	58.52	59.26	59.95	60.59
3	25.03	25.54	26.00	26.42	26.80	27.15	27.48	27.79	28.07
4	17.28	17.61	17.91	18.19	18.44	18.68	18.89	19.09	19.28
5	13.89	14.14	14.38	14.59	14.79	14.96	15.13	15.29	15.44
6	12.02	12.23	12.43	12.61	12.77	12.92	13.06	13.19	13.32
7	10.85	11.03	11.21	11.36	11.50	11.64	11.76	11.88	11.99
8	10.04	10.22	10.37	10.51	10.64	10.76	10.87	10.97	11.07
9	9.465	9.620	9.761	9.890	10.01	10.12	10.22	10.32	10.41
10	9.026	9.170	9.302	9.422	9.532	9.635	9.730	9.820	9.904
11	8.682	8.818	8.941	9.055	9.159	9.256	9.345	9.430	9.509
12	8.405	8.534	8.652	8.759	8.858	8.950	9.036	9.116	9.191
13	8.178	8.302	8.414	8.516	8.611	8.699	8.781	8.857	8.929
14	7.988	8.107	8.215	8.314	8.404	8.489	8.568	8.641	8.710
15	7.827	7.942	8.046	8.141	8.229	8.311	8.387	8.458	8.524
16	7.689	7.800	7.901	7.994	8.078	8.158	8.231	8.300	8.365
17	7.569	7.677	7.775	7.865	7.948	8.024	8.096	8.163	8.226
18	7.464	7.570	7.665	7.753	7.833	7.908	7.978	8.043	8.104
19	7.372	7.474	7.568	7.653	7.732	7.805	7.873	7.937	7.996
20	7.289	7.390	7.481	7.565	7.642	7.713	7.780	7.842	7.901
24	7.034	7.128	7.213	7.291	7.362	7.429	7.491	7.549	7.603
30	6.788	6.875	6.954	7.026	7.093	7.154	7.212	7.265	7.316
40	6.550	6.631	6.704	6.770	6.832	6.889	6.942	6.991	7.038
60	6.321	6.396	6.462	6.523	6.580	6.632	6.681	6.726	6.769
120	6.101	6.169	6.230	6.286	6.337	6.385	6.428	6.470	6.508
∞	5.889	5.951	6.006	6.057	6.103	6.146	6.186	6.223	6.258

ν \ n	38	40	50	60	70	80	90	100
1	683.0	689.6	717.8	740.2	758.8	774.5	788.2	800.3
2	61.19	61.76	64.19	66.13	67.74	69.10	70.29	71.35
3	28.34	28.60	29.68	30.55	31.27	31.88	32.42	32.90
4	19.46	19.63	20.36	20.93	21.42	21.83	22.18	22.50
5	15.58	15.71	16.27	16.72	17.09	17.41	17.69	17.94
6	13.43	13.54	14.02	14.40	14.71	14.98	15.21	15.43
7	12.09	12.18	12.60	12.93	13.21	13.44	13.65	13.84
8	11.16	11.25	11.63	11.93	12.18	12.39	12.58	12.75
9	10.49	10.58	10.92	11.20	11.43	11.63	11.80	11.96
10	9.983	10.06	10.38	10.64	10.86	11.04	11.20	11.35
11	9.583	9.654	9.957	10.20	10.41	10.59	10.74	10.88
12	9.262	9.328	9.617	9.850	10.04	10.21	10.36	10.49
13	8.997	9.061	9.337	9.560	9.747	9.907	10.05	10.17
14	8.775	8.837	9.103	9.317	9.497	9.652	9.787	9.907
15	8.587	8.647	8.904	9.111	9.285	9.434	9.565	9.680
16	8.425	8.483	8.733	8.933	9.102	9.247	9.373	9.486
17	8.285	8.341	8.583	8.779	8.943	9.084	9.206	9.316
18	8.162	8.217	8.452	8.643	8.803	8.940	9.061	9.167
19	8.053	8.106	8.337	8.523	8.679	8.813	8.931	9.036
20	7.956	8.008	8.234	8.416	8.569	8.700	8.815	8.917
24	7.655	7.704	7.914	8.083	8.226	8.348	8.455	8.551
30	7.364	7.409	7.603	7.760	7.893	8.006	8.105	8.193
40	7.082	7.123	7.302	7.447	7.568	7.672	7.763	7.845
60	6.808	6.846	7.010	7.143	7.252	7.347	7.431	7.504
120	6.545	6.580	6.728	6.846	6.946	7.032	7.107	7.173
∞	6.291	6.322	6.454	6.561	6.649	6.725	6.792	6.850

TABLE 3 (*Continued*)

P = .999

ν \ n	2	3	4	5	6	7	8	9	10
1	900.3	1351.	1643.	1856.	2022.	2158.	2272.	2370.	2455.
2	44.69	60.42	70.77	78.43	84.49	89.46	93.67	97.30	100.5
3	18.28	23.32	26.65	29.13	31.11	32.74	34.12	35.33	36.39
4	12.18	14.99	16.84	18.23	19.34	20.26	21.04	21.73	22.33
5	9.714	11.67	12.96	13.93	14.71	15.35	15.90	16.38	16.81
6	8.427	9.960	10.97	11.72	12.32	12.83	13.26	13.63	13.97
7	7.648	8.930	9.768	10.40	10.90	11.32	11.68	11.99	12.27
8	7.130	8.250	8.978	9.522	9.958	10.32	10.64	10.91	11.15
9	6.762	7.768	8.419	8.906	9.295	9.619	9.897	10.14	10.36
10	6.487	7.411	8.006	8.450	8.804	9.099	9.352	9.573	9.769
11	6.275	7.136	7.687	8.098	8.426	8.699	8.933	9.138	9.319
12	6.106	6.917	7.436	7.821	8.127	8.383	8.601	8.793	8.962
13	5.970	6.740	7.231	7.595	7.885	8.126	8.333	8.513	8.673
14	5.856	6.594	7.062	7.409	7.685	7.915	8.110	8.282	8.434
15	5.760	6.470	6.920	7.252	7.517	7.736	7.925	8.088	8.234
16	5.678	6.365	6.799	7.119	7.374	7.585	7.766	7.923	8.063
17	5.608	6.275	6.695	7.005	7.250	7.454	7.629	7.781	7.916
18	5.546	6.196	6.604	6.905	7.143	7.341	7.510	7.657	7.788
19	5.492	6.127	6.525	6.817	7.049	7.242	7.405	7.549	7.676
20	5.444	6.065	6.454	6.740	6.966	7.154	7.313	7.453	7.577
24	5.297	5.877	6.238	6.503	6.712	6.884	7.031	7.159	7.272
30	5.156	5.698	6.033	6.278	6.470	6.628	6.763	6.880	6.984
40	5.022	5.528	5.838	6.063	6.240	6.386	6.509	6.616	6.711
60	4.894	5.365	5.653	5.860	6.022	6.155	6.268	6.366	6.451
120	4.771	5.211	5.476	5.667	5.815	5.937	6.039	6.128	6.206
∞	4.654	5.063	5.309	5.484	5.619	5.730	5.823	5.903	5.973

ν \ n	11	12	13	14	15	16	17	18	19
1	2532.	2600.	2662.	2718.	2770.	2818.	2863.	2904.	2943.
2	103.3	105.9	108.2	110.4	112.3	114.2	115.9	117.4	118.9
3	37.34	38.20	38.98	39.69	40.35	40.97	41.54	42.07	42.58
4	22.87	23.36	23.81	24.21	24.59	24.94	25.27	25.58	25.87
5	17.18	17.53	17.85	18.13	18.41	18.66	18.89	19.10	19.31
6	14.27	14.54	14.79	15.01	15.22	15.42	15.60	15.78	15.94
7	12.52	12.74	12.95	13.14	13.32	13.48	13.64	13.78	13.92
8	11.36	11.56	11.74	11.91	12.06	12.21	12.34	12.47	12.59
9	10.55	10.73	10.89	11.03	11.18	11.30	11.42	11.54	11.64
10	9.946	10.11	10.25	10.39	10.52	10.64	10.75	10.85	10.95
11	9.482	9.630	9.766	9.892	10.01	10.12	10.22	10.31	10.41
12	9.115	9.254	9.381	9.498	9.606	9.707	9.802	9.891	9.975
13	8.817	8.948	9.068	9.178	9.281	9.376	9.466	9.550	9.629
14	8.571	8.696	8.809	8.914	9.012	9.103	9.188	9.267	9.343
15	8.365	8.483	8.592	8.693	8.786	8.872	8.954	9.030	9.102
16	8.189	8.303	8.407	8.504	8.593	8.676	8.755	8.828	8.897
17	8.037	8.148	8.248	8.342	8.427	8.508	8.583	8.654	8.720
18	7.906	8.012	8.110	8.199	8.283	8.361	8.434	8.502	8.567
19	7.790	7.893	7.988	8.075	8.156	8.232	8.303	8.369	8.432
20	7.688	7.788	7.880	7.966	8.044	8.118	8.186	8.251	8.312
24	7.374	7.467	7.551	7.629	7.701	7.768	7.831	7.890	7.946
30	7.077	7.162	7.239	7.310	7.375	7.437	7.494	7.548	7.599
40	6.796	6.872	6.942	7.007	7.067	7.122	7.174	7.223	7.269
60	6.528	6.598	6.661	6.720	6.774	6.824	6.871	6.914	6.956
120	6.276	6.339	6.396	6.448	6.496	6.542	6.583	6.623	6.660
∞	6.036	6.092	6.144	6.191	6.234	6.274	6.312	6.347	6.380

TABLE 3 (*Continued*)

P = .999

ν \ n	20	22	24	26	28	30	32	34	36
1	2980.	3047.	3108.	3163.	3213.	3260.	3303.	3343.	3380.
2	120.3	122.9	125.2	127.3	129.3	131.0	132.7	134.2	135.7
3	43.05	43.92	44.70	45.42	46.07	46.68	47.24	47.77	48.26
4	26.14	26.65	27.10	27.51	27.89	28.24	28.57	28.88	29.16
5	19.51	19.86	20.19	20.48	20.75	21.01	21.24	21.46	21.66
6	16.09	16.38	16.64	16.87	17.08	17.28	17.47	17.64	17.81
7	14.04	14.29	14.50	14.70	14.88	15.05	15.20	15.35	15.49
8	12.70	12.91	13.09	13.26	13.42	13.57	13.71	13.84	13.96
9	11.75	11.93	12.10	12.25	12.39	12.53	12.65	12.77	12.87
10	11.03	11.20	11.36	11.50	11.63	11.75	11.87	11.97	12.07
11	10.49	10.65	10.79	10.92	11.04	11.16	11.26	11.35	11.45
12	10.06	10.20	10.34	10.46	10.57	10.68	10.78	10.87	10.96
13	9.704	9.843	9.969	10.09	10.19	10.29	10.39	10.47	10.55
14	9.414	9.546	9.666	9.776	9.878	9.972	10.06	10.14	10.22
15	9.170	9.296	9.411	9.517	9.613	9.703	9.788	9.867	9.940
16	8.963	9.084	9.194	9.295	9.388	9.475	9.556	9.631	9.702
17	8.784	8.900	9.007	9.104	9.194	9.277	9.355	9.429	9.497
18	8.628	8.741	8.844	8.938	9.025	9.106	9.181	9.251	9.318
19	8.491	8.601	8.701	8.792	8.876	8.955	9.028	9.096	9.161
20	8.370	8.477	8.574	8.663	8.745	8.821	8.892	8.959	9.021
24	7.999	8.097	8.185	8.267	8.342	8.411	8.476	8.537	8.594
30	7.647	7.735	7.816	7.890	7.958	8.021	8.080	8.135	8.188
40	7.312	7.393	7.466	7.533	7.594	7.651	7.704	7.754	7.801
60	6.995	7.067	7.133	7.193	7.248	7.299	7.347	7.392	7.433
120	6.695	6.760	6.818	6.872	6.921	6.966	7.008	7.048	7.085
∞	6.411	6.469	6.520	6.568	6.611	6.651	6.689	6.723	6.756

ν \ n	38	40	50	60	70	80	90	100
1	3415.	3448.	3589.	3701.	3794.	3873.	3941.	4002.
2	137.0	138.3	143.7	148.0	151.6	154.7	157.4	159.7
3	48.72	49.16	51.02	52.51	53.75	54.81	55.72	56.53
4	29.43	29.68	30.78	31.65	32.37	32.98	33.52	34.00
5	21.86	22.03	22.82	23.45	23.97	24.41	24.80	25.15
6	17.96	18.10	18.73	19.22	19.64	20.00	20.31	20.58
7	15.62	15.74	16.27	16.69	17.04	17.35	17.61	17.85
8	14.07	14.18	14.64	15.01	15.32	15.59	15.82	16.02
9	12.97	13.07	13.49	13.82	14.10	14.34	14.55	14.74
10	12.16	12.25	12.63	12.94	13.20	13.42	13.61	13.78
11	11.53	11.62	11.97	12.25	12.49	12.70	12.88	13.04
12	11.03	11.11	11.44	11.71	11.94	12.13	12.29	12.45
13	10.63	10.70	11.01	11.27	11.48	11.66	11.82	11.97
14	10.30	10.37	10.66	10.91	11.11	11.28	11.43	11.57
15	10.01	10.08	10.37	10.59	10.79	10.96	11.10	11.23
16	9.769	9.833	10.11	10.34	10.52	10.68	10.82	10.95
17	9.562	9.623	9.888	10.10	10.29	10.44	10.58	10.70
18	9.381	9.440	9.696	9.904	10.08	10.23	10.36	10.48
19	9.221	9.279	9.528	9.730	9.899	10.04	10.17	10.29
20	9.081	9.137	9.379	9.575	9.740	9.881	10.01	10.12
24	8.648	8.700	8.921	9.100	9.250	9.380	9.494	9.596
30	8.237	8.283	8.484	8.647	8.783	8.901	9.004	9.096
40	7.845	7.887	8.067	8.214	8.337	8.442	8.535	8.618
60	7.473	7.510	7.671	7.802	7.911	8.005	8.088	8.161
120	7.121	7.153	7.296	7.411	7.507	7.590	7.662	7.726
∞	6.787	6.816	6.941	7.041	7.124	7.196	7.259	7.314

fourth decimal place for $Q < 1$. The condition (2) is in fact satisfied, and hence it can be stated that the error in the percentage points, when rounded to four significant digits or four decimal places, whichever is less accurate, does not exceed a unit in the last place.

For $\nu = 1$ and $n \geqq 6$, the value of Q corresponding to cumulative probability $P = 0.999$ exceeds 2000. Since the new table of the probability integral of the studentized range extends only to $Q = 2000$, those percentage points which exceed 2000 cannot be found by interpolation. It has been shown, however, that for $\nu = 1$ and $\nu = 2$ and for Q_0 sufficiently large, the cumulative probability $P(Q, \nu, n)$ corresponding to the value Q for the studentized range of a normal sample of size n, with ν degrees of freedom, can be approximated by

$$P(Q, \nu, n) \cong 1 - (Q_0/Q)^\nu[1 - P(Q_0, \nu, n)]. \tag{3}$$

Hence the percentage points in question can be found by setting $Q_0 = 2000$ in equation (3) and solving for Q.

The percentage points of the studentized range are given by Table 3, which was printed on the IBM 407 tabulator.

In addition to the percentage points shown in Table 3, critical values for Duncan's new multiple range test, which are percentage points of the studentized range for special protection levels based upon degrees of freedom, have been computed and will be published elsewhere (see Harter [5]).

3. Interpolation in the Tables.

3.1. *Percentage Points of the Range.* Table 1 gives the percentage points of the range for samples of size $n = 2\ (1)\ 20\ (2)\ 40\ (10)\ 100$. One may wish to interpolate for odd values of n between 20 and 40 and/or for values of n, not multiples of ten, between 40 and 100. Harmonic interpolation in n (interpolation using $1/n$ as the independent variable) is recommended. The maximum errors, in units of the sixth decimal place, are approximately as follows:

Type of Harmonic Interpolation	Maximum Error			
	$20 < n < 40$	$40 < n < 60$	$60 < n < 80$	$80 < n < 100$
Linear	1300	5600	2600	1500
3-point	70	600	350	150
4-point	8	125	60	35
5-point	2	40	20	7
6-point	1	10	7	3
7-point		7	3	2
8-point		4	2	

Interpolation P-wise is not recommended. If percentage points are needed for values of P not included in Table 1, the best procedure is to interpolate, by the method outlined in Section 2.1, in the table of the probability integral of the range (see [6]).

3.2. *Percentage Points of the Studentized Range.* Table 3 gives the percentage points of the studentized range for samples of size $n = 2\ (1)\ 20\ (2)\ 40\ (10)\ 100$, with degrees of freedom $\nu = 1\ (1)\ 20, 24, 30, 40, 60, 120, \infty$. One may wish to interpolate n-wise and/or ν-wise. For n-wise interpolation, the maximum errors, in units of the fourth significant digit, are approximately as follows:

Type of Interpolation	Maximum Errors	
	$20 < n < 40$	$40 < n < 100$
Linear	3	11
3-point	1	3
4-point		1

Linear harmonic ν-wise interpolation[2] (linear interpolation for $1/\nu$) is accurate to within 4 units in the fourth significant digit for $P = .999$, 2 units in the fourth significant digit for $P = .995, .99$, and 1 unit in the fourth significant digit for other values of P. Three-point harmonic ν-wise interpolation is accurate to within 1 unit in the fourth significant digit for all values of P. For convenience in harmonic interpolation, values of ν were chosen for inclusion in the table so as to form a harmonic series (20, 24, 30, 40, 60, 120, ∞). As in the case of the percentage points of the range, P-wise interpolation is not recommended. If percentage points are needed for values of P not included in Table 3, the best procedure is to interpolate, by the method outlined in section 2.3, in the table of the probability integral of the studentized range (see [7]).

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