near $u = 1$, this resemblance may be exploited to give (after elementary but tedious calculations)

$$\pi \Sigma p_m^2 = B(n + 1, \tfrac{1}{2}) + e \qquad (10)$$

with

$$0 < e < 2e^{-n\delta} + (\tfrac{2}{3})[\delta/(1 - a)]^{3/2}$$

whenever $n > a/(1 - a)$. Here δ is any number $< pq$. Picking $\delta = n^{-\theta}$, $\theta < 1$, shows that the error goes to zero almost as fast as $n^{-3/2}$. A similar result may be obtained by the methods of Uspensky.

From (10) we have easily

$$\Sigma p_m^2 \sim 1/(2\sqrt{\pi npq}) \qquad (n \to \infty), \qquad (11)$$

which is correct even for $p = q$.

It was pointed out by the referee that (9) and (11) are special cases of the relation

$$\Sigma p_m^2 \sim (\tfrac{1}{2}) \sqrt{\text{variance}}$$

which generally holds whenever the shape of the distribution curve approaches a limit.

REFERENCES

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APPROXIMATION TO THE POINT BINOMIAL

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The following approximation to the sum of the first $(t + 1)$ terms of the point binomial appears to be useful. Let this sum be denoted by S_{t+1}, and let the point binomial be the expansion of $(p + q)^N$; *i.e.*, let

$$S_{t+1} = p^N + Np^{N-1}q + \cdots + \binom{N}{t} p^{N-t}q^t. \qquad (1)$$

Then S_{t+1} is approximately equal to the probability that a unit normal deviate will exceed x, where

$$(2) \qquad x = \frac{\frac{1}{3}\left[\left(\frac{9s-1}{s}\right)\left(\frac{s}{t+1}\frac{q}{b}\right)^{1/3} - \frac{9t+8}{t+1}\right]}{\left[\frac{1}{s}\left(\frac{s}{t+1}\frac{q}{p}\right)^{2/3} + \frac{1}{t+1}\right]^{1/2}}, \qquad s = N - t.$$

This approximation is a corollary to an approximation given by Paulson [1] to the table of the integral of Snedecor's F (Fisher and Yates' $w = e^{2z}$), and the known facts that this integral is an incomplete Beta-function [2] of a simple transform of F, and that S_{t+1} is also an incomplete Beta function of suitable arguments. Paulson's approximation appeared to be quite close. Since it was essentially an approximation to the incomplete Beta function we must now have a similarly close approximation to the point binomial. Therefore two illustrations will suffice.

Example 1. $(.8 + .2)^8$

t	S_{t+1}		Error
	Approx.	True	
0	.166	.168	−.002
1	.505	.503	.002
2	.801	.797	.004
3	.943	.944	−.001
5	.999	.999	.000

Example 2. $(.9 + .1)^{50}$

t	S_{t+1}		Error
	Approx.	True	
0	.005	.005	.000
1	.033	.034	−.001
3	.250	.250	.000
5	.617	.616	.001
10	.992	.991	.001

Both these examples involve strongly skewed distributions, one with a small value of N and the other with a fairly large value of N. Considering the amount of computation involved this approximation is much more satisfactory than any other in this author's experience.

REFERENCES

[1] E. Paulson, "An approximate normalization of the analysis of variance distribution," *Annals of Math. Stat.*, Vol. 13 (1942), p. 233.

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