Correction to

On the Diophantine equation $1^k+2^k+\ldots+x^k+R(x)=y^z$

by

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In the above article the authors claim that a polynomial P with rational integer coefficients which is congruent (mod 4) to

$$3x^8+2x^6+x^4+2x^2$$

has at least three simple roots. Their argumentation is incorrect. In this corrigendum, they wish to repair this defect by proving claim (i) in case B of Lemma 4 in a correct way.

Suppose P can be written as

$$P(x)\equiv Q(x)\,T^2(x), \tag{*}$$

with $\deg Q\leqslant 2$.

If $\deg Q=0$, then clearly Q is an odd constant, so $T^2(x)\equiv x^8+x^4 \pmod 2$, hence $T(x)\equiv x^4+x^2 \pmod 2$ and $T^2(x)\equiv x^8+2x^6+x^4 \pmod 4$, which is clearly not the case. If $\deg Q=1$, then either $Q(x)\equiv x$ or $Q(x)\equiv x+1 \pmod 2$. In both cases, the quotient of P and Q cannot be written as a square (mod 2). If $\deg Q=2$, then either

$$Q(x)\equiv x^2 \quad\text{or}\quad Q(x)\equiv x^2+x \quad\text{or}\quad Q(x)\equiv x^2+1 \pmod 2,$$

since x^2+x+1 does not divide P (mod 2). In the first case $T(x)\equiv x^3+x \pmod 2$, hence $T^2(x)\equiv x^6+2x^4+x^2 \pmod 4$ which does not divide P (mod 4). In the second case, the quotient of P and Q is not even a square (mod 2). In the third case $T(x) \equiv x^3 + x^2 \pmod 2$, hence $T^2(x) \equiv x^6 + 2x^5 + x^4 \pmod 4$ which does not divide $P \pmod 4$. We conclude that P cannot be written in the form (*) with $\deg Q < 3$, proving our claim.

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