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REMINISCENCES OF KAROL BORSUK

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Dedicated to the memory of our teacher, Karol Borsuk

The death of Karol Borsuk on January 24, 1982, in Warsaw has brought sorrow and regret to a wide circle of his friends and former students. His creative mathematical work and his profoundly unique personality left us with a feeling that something of inestimable value has departed and yet remains as a treasured remembrance.

Karol Borsuk was born in Warsaw on May 8, 1905. In 1936 he married Zofia Majewska by whom he had two daughters, Elżbieta and Magdalena. On graduation from the Warsaw University in 1927, he taught for three years at a private school. During these years he wrote his doctoral dissertation "Sur les rétractes" which was published in 1931 in Fundamenta Mathematicae. His thesis initiated a steady and remarkable stream of research interrupted only by the outbreak of the Second World War in 1939.

In 1932, at the age of 27, he probably made his most beautiful discovery, the "Antipodensatz", which was presented at the International Mathematical Congress in Zürich in 1932. Around the same time, Borsuk "discovered", at the University of Warsaw, a young student, Samuel Eilenberg, who in 1933, under his influence, published a first paper in topology and two years later wrote his doctoral thesis (at the age of 22). This "discovery" turned out to be a good fortune both for the "student" and for the "teacher": between 1933–1939, the intense activity of each of them provided a stimulus and inspiration for the other. The resulting mutual interaction generated almost 100 publications, some of them of the first order of importance.

In 1939 at the age of 34, Borsuk was already an author of more than 60 papers covering numerous topics in topology, which by their originality marked him as one of the leading topologists of his time. This article is not intended as Borsuk's biography or as an analysis of his mathematical work. What we would like to present here are some rather personal reminiscences of this remarkable man.

In the fall of 1947 we entered the University of Warsaw. As freshmen students of physics, we had to take some courses of mathematics. At that time the attendance of courses was optional; all we had to do was to take examinations at some point. The main mathematical courses were given by K. Borsuk, K. Kuratowski, S. Mazur, A. Mostowski and W. Sierpiński; the students had lively discussions about the differences in style between the various courses. Borsuk at that time was 42; he was tall, dark-haired and always had a kind and friendly smile. The first impression he made upon us could not be easily forgotten. Borsuk's lectures were extremely popular and were inspiring in a very special way: he appreciated deeply the beauty of mathematics and was able to communicate this appreciation to others. At the same time (in contrast to some other lectures, especially in physics) his lectures were models of clarity, systematic presentation and preparation. Under their influence we began to think about switching from physics to mathematics but that decision came for us some years later.

Borsuk's course on Analytic Geometry was our first exposure to the beauty of Geometry and, at the same time, to Borsuk's deep convictions and views of Mathematics as a whole. The way Borsuk taught this first year course reflected his entire philosophy and approach to Mathematics. The course began with a definition of a metric space and then went on to the construction of n-dimensional Euclidean space as an affine space and to a discussion of isometries, similarities and affine transformations. Borsuk was an admirer of Felix Klein's "Erlangen Program" and emphasized the role and importance of various groups of transformations and their invariants. We were especially impressed and even thrilled by his construction of projective spaces and by their topological models: it was like an opening of a new world to us. This was followed by a discussion of complex affine spaces and crowned with a complete classification of quadratic varieties in (real and complex) affine and projective spaces of dimension n. Borsuk even invented a Polish name for these varieties, a name which, as far as we know, had not officially existed in Polish dictionaries; the name was "twór". It has in Polish a certain antique or ancient, almost biblical, connotation, sounding somewhat like "an object of creation"; it decorated these varieties, at least for us, with a certain mysterious beauty. Throughout the entire course not even once was the term "vector space" mentioned (as second year students, we were reading Halmos' "Finite Dimensional Vector Spaces" in a private seminar). Of course, Borsuk was using vectors, he constructed the vector space associated to the Euclidean space, he introduced, used and discussed the standard linear groups of transformations like GL(n), O(n), U(n), PL(n), eigenvalues and eigenvectors; but in his course these concepts appeared mostly in disguise, under different names or camouflaged in geometric costumes. In his beautiful classification of quadratic varieties in projective spaces, the names "quadratic form" or "signature" were not mentioned even once. We were told, however, that non-singular varieties were "manifolds". We also knew what (geometric) orientation was and which of the projective spaces were orientable.

After "Analytic Geometry" there were three other courses we took from Borsuk: "Introductory Topology", "Analytic Functions" and "Differential Geometry". In each of them Borsuk found a fertile ground to show us what he thought was important and crucial in Mathematics. For example in "Analytic Functions" he showed us the beauty of complex analysis, of Moebius transformations, conformal mappings, Riemann surfaces; and, above all, the idea of homotopy and of lifting a function. His "Differential Geometry" was a masterpiece as well. He did not go beyond dimension three; but even for curves he was able to make a strong case for his convictions and for what he felt a right approach to Geometry should be. He was extremely careful to keep a distinction between a "curve" (which was a subset of a Euclidean space), and a "circuit", which was a class of parametrizations of a curve. He was also careful in keeping track of the orientation (of course, the terms "clockwise" or "right hand screws" did not exist for him; he despised them and never mentioned them). The course concluded with Gauss' Theorema Eggregium, with the Gauss-Bonnett theorem and its relation to the Euler characteristic.

We first came into a closer personal contact with Borsuk in 1949, when we started to attend his Topology Seminar in the newly created Mathematical Institute of the Polish Academy of Sciences. The seminar met on Thursday, 2–4 P.M. Besides a few "more advanced" participants, Borsuk had then a small circle of students each of whom received personal attention. Our first mathematical papers originated at that seminar. The opportunity to be with Borsuk and of talking to him about our ideas and results that were not yet quite complete was very important for us. The main papers we were reading in the seminar were Eilenberg's doctoral thesis "Transformations continues en circonférence et la topologie du plan", Fund. Math., 26 (1936), and Borsuk's "Set theoretical approach to disconnection theory in $\mathbb{R}^n$", Fund. Math. (1950). These were beautiful and inspiring papers; for us it was like an opening and a first introduction to homotopy theory, maps into spheres, degree theory, cohomotopy groups and other ideas of algebraic topology. During the seminar Borsuk would suggest problems, some of which were about possible extensions of the results of Eilenberg's Thesis from $\mathbb{R}^2$ to $\mathbb{R}^n$. Some years later many of the ideas we learned at the Borsuk seminar were extended even to Banach spaces but this involved the use of the category of compact vector fields introduced in the late thirties by J. Schauder in Lwów. An elegant illustration of this extensive development is the proof due to K. Gęba of the bifurcation theorem of Eberhard Hopf which uses the well-known theorem of Heinz Hopf concerning maps of S^3 into S^n.

In his seminar in 1951–52 Borsuk made us first learn about fibre bundles. We were reading the newly published Steenrod's book "The Topology of Fibre Bundles"; the paper by Stiefel on parallelizability of manifolds; the classical papers by Hopf on maps of spheres and on fiberings of spheres. From his first visit to the United States after the war, Borsuk brought back an early mimeographed version of Eilenberg-Steenrod's "Foundations of Algebraic Topology" which was about to appear at the Princeton University Press. As for the textbooks for topology (among those which existed at that time), Borsuk liked best Seifert and Threlfall "Lehrbuch der Topologie" and Alexandroff and Hopf "Topologie".

In his lectures and seminars Borsuk never concealed the fact that his interests lay in geometry and not in the algebraic tools used to study it. This attitude was typical for him and it manifested itself throughout his entire work. Borsuk was familiar, of course, with the basic tools of algebraic topology. He appreciated their potential, he was skillful in using them, he invented some of them himself or adapted them for his own purposes. He opened doors for some important ideas in algebraic topology such as cohomotopy groups. For him, however, algebraic topology, or algebra in general, had always remained only a tool, albeit a very powerful tool, which played only a subservient role for him in his study of "the real world", as he saw it: GEOMETRY. He always liked "elementary" proofs of theorems which otherwise required algebraic topology, such as fixed point theorems, his own "Antipodensatz", results involving the topological degree or duality properties in Euclidean spaces. For Borsuk, the truly real spaces in topology (those created by God?) were metric spaces, especially compact metric spaces; most other objects looked suspicious and artificial to him. He did not like pathologies in topology; but in order to eliminate them, he preferred to proceed from the top downwards, from the general to the special, by narrowing down the classes of spaces. An excellent, and a truly successful, illustration of this approach is his theory of retracts which he created to eliminate various pathologies appearing in compact metric spaces. The idea of constructing well-behaved spaces from simpler objects, as it is done for CW-spaces, did not appeal to him too much. He loved simplicial complexes and simplicial homology for their geometric flavor, but he did not seem to think too much of singular homology; and the CW-complexes looked much too artificial to him. He liked to use a homology theory which was constructed in as "elementary" a way as possible and one which could be applied to all compact metric spaces. For these reasons, he always preferred to use the Vietoris homology. It was a very happy news to him when Mardesič proved the uniqueness of homology for the class of compact ANR's, something that Borsuk must always have expected. His attitude towards manifolds was somewhat ambiguous. He appreciated their importance; but they may have seemed a bit too elusive for him. He would have much preferred to have manifolds defined as (perhaps compact?) metric spaces satisfying certain, obviously quite strong, conditions. He probably knew that this was not realistic. In a similar spirit, he made some attempts to define a class of spaces which would be as much close to the class of compact polyhedra as possible.

Borsuk was a highly disciplined man, at the same time ascetic and kind. He did not discourage an independent approach to topology among his pupils; he rejoiced in seeing their originality and creativity. However, his overwhelming personality and strong mathematical convictions did not make it easy for them not to follow in his footsteps.

Borsuk had a remarkable and rare ability to initiate new and fruitful directions of research. An essential quality of his own mathematical contributions, some two hundred in number, was their originality and relevance. He had clear ideas of what, according to him, was important in mathematics and he pursued these ideas relentlessly. Already as a young man he had such a large vision of the sort of work he would like to do, that its accomplishment and extensions would be sufficient for his entire productive career.

Borsuk enjoyed lecturing at all levels and was an admirable lecturer. His manner of speaking, delivery and even his hand-writing were fascinating. The courses and seminars he gave were always a source of inspiration for his students and colleagues alike. He was always helpful and encouraging to his students; if criticism was required, he knew how to find a form that could not hurt even the most sensitive. Many Warsaw dissertations owed much to his supervision and inspiration.

Borsuk had two compelling loyalties: one to mathematics and another to the mathematical community. While he always had the highest hopes for the Polish Mathematical School, he was at the same time internationally minded: P.S. Aleksandroff, R.H. Bing, E. Čech, H. Hopf and S. Lefschetz were among his best friends. He also always unhesitantly accepted the public responsibilities that came his way. Over the years he did much of the unrewarding administrative work for various institutions. During the war he carried his share in participating in the Underground Resistance Movement and, because of that, was imprisoned by the Gestapo for 3 months in the Pawiak prison in Warsaw. Only through a series of lucky circumstances did he escape execution.

Borsuk pursued his life of scientific activity steadily and without rest. He was a hard worker, working constantly (often during late hours and even during vacations) and periodically with great intensity. These habits did not change much with advancing age. His personal life was also well integrated and free of excentricity. His principal outside interests were reading and travelling, both of which he shared with Mrs. Zofia Borsuk. Some of those trips (to Greece, Sicily, Egypt and Mexico) were "non-mathematical"; they were linked to his interests in the ancient history, in which field he grew to be quite a connoisseur. Zofia and Karol Borsuk also had a cottage, "Radachówka", in the countryside some 60 km. from Warsaw. They loved it and used to spend weekends there with their family and close friends. For many years the cottage was accessible only by a small, unpaved, sandy road, and Borsuk had to go in a hired horse cart about 13 km. from the train station. His friends, visitors from abroad, and his students were often invited to Radachówka. It was a delight to visit him there. In Radachówka, Borsuk was generally much more relaxed and informal than he used to be at the University or at the Institute. We felt truly privileged and happy when we visited him there. He valued and enjoyed simple things in life. Comfort and superficial satisfaction were secondary to him, and luxury was in a direct conflict with his basically ascetic nature.

During the last years of his life Borsuk knew that his heart was no longer as strong as it had been, but he never slackened up in his scientific and other work. Some weeks before his death Borsuk was elected a member of the Papal Academy of Sciences, a particular honour, since there are only a few of those from all nations and scientific subjects. For those who knew him, Borsuk appeared as an idealist never deviating from the highest standards for himself and yet fully appreciating the qualities of others who had different views and ideas. His generosity, modesty and selfless dedication to science have left their impression on all who knew him. The memory of this great man and mathematician will be cherished with deep and warm gratitude.

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