

Johan van Benthem, Gerhard Heinzmann, Manuel Rebuschi, and Henk Visser (editors)

The Age of Alternative Logics: Assessing Philosophy of Logic and Mathematics Today

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REVIEW

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A conference took place at the Université de Nancy 2 in 2002 entitled “Philosophical Insights into Logic and Mathematics”. The proceedings were subsequently published in book form under the above title, and the following review is intended to provide a list of the papers and authors, together with scattered comments, followed by an overall perspective on the volume and its title. The length of comments for each paper is not a measure of its importance but also reflects its accessibility.

Van Benthem wrote the introduction, “Alternative Logics and Classical Concerns”. He argues that the wide variety of non-classical systems is not an indication of an ‘anything goes’ mentality, but that it is a symbol of natural growth. He looks at external and internal influences on the developments in logic and gives a hint to what is to come with referring to truth as a ‘zero-agent notion’ and proof as a ‘single-agent notion’. His claim is that we understand first-order logic better as a result of the emergence of competing systems and suggests that we may be in a position now to rethink decisions made around 1900. He even goes so far as to hope that these variations may help to keep mathematics from looking *sui generis*, to demonstrate, “[M]athematics is just common sense continued by other means.” (p. 7)

The book is divided into five main sections, of which the first is entitled, “Proof, Knowledge, and Computation.” In this section Mikaël Cozic leads off with “Epistemic Models, Logical Monotony and Substructural Logics”. He investigates alternatives to the principle of logical omniscience, which assumes that a reasoner believes the consequences of everything he believes. He looks at the reasoning process as

a ‘consumption-relation’ and wonders whether his substitute principles may just shift from one kind of omniscience to another.

The following essay by Jaroslav Peregrin, “Semantics as Based on Inference”, addresses the issue of whether the meaning of logical constants and connectives can be entirely furnished by their inference rules. He looks at some of the earlier attempts and criticisms, offering the suggestion that adding ‘and nothing else’ might help to avoid some of the earlier difficulties. A construction like ‘straightforwardly always only’ can give the reader something of a puzzle.

Stewart Shapiro’s “Effectiveness” is the next contribution, and he asks some historical questions to which readers may have something to contribute. The first is whether there were attempts previous to the annus mirabilis of 1936 to capture the general notion of computability. He notes that neither Babbage nor Leibniz was interested in the general questions. He asks whether the cardinality argument for the existence of noncomputable functions was known before Church’s thesis. Did Church and Turing feel, he wonders, that they were contributing to the discussion of Hilbert’s program or to generalizing Gödel’s work? Did anyone conjecture in print before 1936 that a decision problem might have a negative solution? He traces the issue of a general characterization of computability back to the questions about decision problems that Hilbert had asked (for example, in his Paris address). Shapiro suspects that most people felt that they could recognize an algorithm, but giving a negative answer to the existence of a decision procedure would involve characterizing what a possible algorithm would look like. Shapiro’s suggestive questions lead in to his proposal that the notion of effectiveness has more to it than simply the reduction to the various equivalent formalizations offered by Turing, Church, Gödel, and Post.

The next essay, by Joseph Vidal-Rosset, has the title “Does Gödel’s Incompleteness Theorem Prove that Truth Transcends Proof?”. This deals with some particular issues connected with deflationism, pitting Shapiro and J. Ketland against Hartry Field and Neil Tennant. Some of the linguistic difficulties the author has with English make the last section of philosophical conclusions less than transparent.

Henk Visser does not beat around any bushes in his “Transpositions”. He claims that the only importance of speculation about the nature of mathematical entities comes from influence it might have on how mathematics is done. He looks at some examples where translations of problems from one setting to another make the proofs easier or more transparent. It is strikingly different from most of the other essays in the book.

The second section, “Truth Values Beyond Bivalence”, begins with Jean-Yves Béziau’s contribution, “Many-Valued and Kripke Semantics”. His chief complaint is that Kripke semantics has been hijacked by modal logic, while there is no reason not to look at the interplay of Kripke and many-valued semantics in a wider range of contexts. He looks at certain specific many-valued systems of modal logic and defends their usefulness even without possible worlds.

The following essay by Newton C.A. da Costa and Décio Krause bears the title “The Logic of Complementarity”. In view of da Costa’s many publications on paraconsistent logics, it is not surprising that the authors claim that the proper way to understand the kind of logic that Bohr had in mind in the Copenhagen interpretation of quantum mechanics is via paralogic. They deny that there is one true logic, looking to different logical systems as applicable to different spheres of inquiry.

In “Semantics for Nave Set Theory in Many-Valued Logics”, Thierry Libert makes the familiar point that many-valued semantics can offer help with the set-theoretic paradoxes. Fixed-point arguments, he claims, bring out the essential features of avoiding paradoxes. He even offers a diagram of selected references before his bibliography, but it’s not clear how helpful that will be. His use of otherwise unexplained terminology continues through the last words of his article.

With Part III we turn to category-theoretic structures and some of the familiar contributors to arguments in the pages of *Philosophia Mathematica* and elsewhere. Steve Awodey starts the section with some strong claims in “Continuity and Logical Completeness: An Application of Sheaf Theory and Topoi”. He argues that categorical logic is the completion of the program of algebraicizing logic that began in the nineteenth century, culminating in Lawvere’s treatment of quantifiers as adjoint functors. Adjoints always admit an algebraic description, and this leads to the notion of a topos and the following ‘equation’:

Propositional logic/Boolean algebra = higher-order logic/*topos*.

The machinery of deductive systems is replaced by algebraic manipulations. Awodey provides two theorems: (1) A sentence is provable in higher-order logic if it is true in any topos model and (2) The logic of sheaves is classical deductive higher-order logic. Of these he points to the former as unsurprising, but the latter as a striking coincidence, since sheaves come from a geometric origin and higher-order logic has nothing apparently to do with that. It’s perhaps worth mentioning that he uses the plural ‘topoi’ instead of the slangier ‘toposes’.

The categorical theme continues with Geoffrey Hellman's "What is Categorical Structuralism?" This pursues an argument he has had with Colin McLarty and Awodey about whether category theory can serve as a foundation for mathematics. Hellman argues that foundations have to address certain kinds of questions, including the nature of the axioms, and claims that McLarty has a Fregean approach to axioms (they have genuine content), while Awodey takes a Hilbertian approach (the axioms are simply stipulatory). He refuses to accept either as satisfactory exclusively and concludes that mathematics requires both Fregean and Hilbertian axioms. Hellman's argument that his is the way out if one wishes to eschew formalism may leave room for both of his opponents to respond.

In Elaine Landry's "Category Theory as a Framework for an In Re Interpretation of Mathematical Structuralism", she claims that looking for a foundation can be too much, but that a framework that helps us to organize mathematics may be achievable. She suggests that arguments by Solomon Feferman and John Mayberry against category theory as a foundation fail when the role is reduced to the more modest framework. Her use of Leo Corry's work on the history of abstract algebra serves as a historical guide, although some of her section headings are confusing.

The last of the category-theoretic essays is Jean-Pierre Marquis's "Categories, Sets, and the Nature of Mathematical Entities". He ranges widely over philosophical issues and technical developments in category theory, although some of the former are taken for granted. His notion that the use of category theory in certain areas reveals what is fundamental there fits in with the claim that Landry made in the previous article.

Both of the last two sections of the book are related to Jaakko Hintikka's idea of independence-friendly logic. Of the essays in these sections, the broadest ranging is Hintikka's own with the title "Truth, Negation, and Other Basic Notions of Logic". He argues that his IF (as he abbreviates this kind of logic) arises from considering the basic questions often overlooked in the logic classroom. (It is not surprising that he repeatedly claims that 'classical' logic has that name only because it is the logic taught in the classroom.) Hintikka's aim is to capture the logic of ordinary language and to solve many of the outstanding problems of first-order logic in the process. For example, IF avoids the liar paradox by allowing statements to be neither true nor false and it works with two different kinds of negation. This is connected with the game semantics about which Hintikka has written at length.

Hintikka brings the perspective of IF to bear on some of the standard issues in the philosophy of mathematics. He claims that intuitionistic logic is the logic of our knowledge of mathematical and logical entities, which captures some of the essence in Brouwer's original characterization. As for logicism, he does not try to resurrect the Russellian or Fregean versions, but thinks that the prospects are good for defending the claim that all the different conceptualizations and modes of reasoning used in mathematics can be reconstructed by means of logic.

A briefer encapsulation of the remaining essays will suffice. Theo M.V. Janssen argues in "Signalling in IF Games: A Tricky Business" that Hintikka's claim that IF is a conservative extension of predicate logic is in error. Ahti-Veikko Pietarinen's "Independence-Friendly Logic and Games of Incomplete Information" tries to distinguish between games of imperfect information and games of incomplete information, the latter having to do with situations where the players have restricted information concerning aspects of the formal structure of the game itself. While this is connected with Grice's notions of conversational implicature, some readers may have enough problems with games when they do know the rules. Manuel Rebuschi's "IF and Epistemic Action Logic" seeks to reconcile the differences between Hintikka's logic and the creation of van Benthem.

The last section consists of three essays: Gerhard Heinzmann's "Naturalizing Dialogic Pragmatics", Kuno Lorenz's "Logic as a Tool for Science versus Logic as a Scientific Subject", and Shahid Rahman's "Non-normal Dialogics for a Wonderful World and More". The first pays tribute to the influence of Peirce, Gonsseth, Bernays, and Lorenz. The second is in agreement with Peirce's claim that pragmatics is the heir of ontology and semiotics of epistemology. Lorenz argues for the need 'to create a He/She- perspective towards the I/You-situation such that, on the one hand, He/She becomes a (secondary) You-perspective with respect to I/You as I, and, on the other hand, He/She becomes a (secondary) I-perspective with respect to I/You as You'. Rahman responds to the question of whether the alternative logics are real by claiming that the mere mental construction of them is enough to motivate their study.

There's an odd addendum in the form of an essay entitled "Louis Joly as a Platonist Painter" by Roger Pouivet. The painter in question was fascinated by the work of a Parisian group in the thirties led by a former participant in *De Stijl*. Joly himself felt that it was possible to create works of art from mathematical constructions, and Pouivet tries to look at the Platonist difficulties with that approach. The piece

is perhaps too brief to settle issues about Platonism, even just in this context.

The technical editing of the volume is imperfect (references missing, for example), but more discouraging is the lack of linguistic editing. Some of the contributors may have styles that would be recondite after any editing, but there are others where clarity is almost impossible to achieve across a linguistic border. It's not the fault of the editors that the majority of the contributors are not native English-speakers. The problem is that many of the points they are trying to make are subtle and they disappear in the approximation to English that results. Perhaps the publishers should have served as more of a linguistic court of last resort.

With regard to the book's accomplishments, the subtitle is over-ambitious. There are plenty of topics in the philosophy of logic and mathematics that are ignored entirely (there is no reference to Lakatos, for example, nor to neo-logicism at the other end of the spectrum). Of course, trying to tackle such a wide variety of figures and topics would have been beyond the scope of any single volume.

One fundamental question that emerges is whether the collection has made the case for 'alternative logics'. Hintikka makes the point that it is a singularly ill-defined term and recommends rejecting it. There is a tendency to regard classical logic as 'the enemy', although some of the complaints may come from the exclusiveness with which it is taught. Attempts such as Graham Priest's go some way to persuading even classical logicians that there is interesting and important work on the non-classical front. While some of the essays in this volume raise interesting points for logicians and mathematicians in general, they are not likely to persuade the classicist that it is time to give up on exclusivity.

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