Gaisi Takeuti
Memoirs of a Proof Theorist: Gödel and other Logicians
Singapore: World Scientific Publishing, 2003
xviii + 135 pp. ISBN 9812382798
and
John W. Dawson, Jr.
Logical Dilemmas: The Life and Work of Kurt Gödel
Wellesley, Massachusets: A.K. Peters, 1997
xiv + 361 pp. ISBN 1568810253

DOUBLE REVIEW

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The year 2006 has seen a wealth of material devoted to the memory of Kurt Gödel, the logician, the centenary of whose birth it is. In fact, it may already have seen a wealth of articles starting off with some variation on the previous sentence. To the best of this reviewer's knowledge, it has not yet seen a plurality of articles with the above second sentence, but he is prepared to be undeceived.

It is appropriate to look back at a couple of books that have been largely devoted to Gödel, his career, and his mathematical contributions. Of the two considered here, the first listed above will receive more protracted consideration, despite its brevity, as the more recent to reach an English-speaking audience. The second volume has already achieved the status of a biographical classic and serves as a contrast to the other.

Gaisi Takeuti is an eminent logician, whose texts on set theory adorn the shelves of many mathematicians. This book, however, is put together in rather an eccentric fashion and does not give the reader the best opportunity to appreciate his views. It is a collection of articles, almost all previously published in Japanese, and translated here by Mariko Yasugi and Nicholas Passell. The translators inform the reader in their preface that their intention is to try to preserve the stylistic features of the original 'even at the cost of some awkwardness'. For a non-Japanese speaking reader, however, this attempt is likely to end up creating confusion as to what the author was trying to say. Whether

it's a matter of idiom or of use of prepositions, the resulting text is more baffling than it needed to be.

There is also the rather minimal amount of editing that went into trying to avoid duplication of material from one essay to the next. Again, the intention was to preserve 'the sense of the time and the author's feelings', but there is surely a limit to the number of times it makes sense, especially in so short a volume, to trot out a summary of Hilbertian formalism. This occurs at least four times, almost word for word, and the reader wonders whether in book form some cross-reference might not have been possible.

One difficulty is just that there is too much repetition on a larger scale of subjects. It is clear that the author feels that the relationship between Hilbert and Gödel deserves a little more elucidation than it standardly receives. He reiterates his own view in article after article, not to the benefit of the reader of the entire sequence. It is possible that the repetition was designed to appeal to those who might consult just one essay in the volume, but that's less likely to happen in view of the absence of an index. In various essays the level of background expected of the reader varies considerably (and not just from one essay to another).

The sloppiness of production gets in the way of the reader's ability to use it effectively and to trust judgments based on historical details. On page 103 one finds a reference to 'Todorčević'. In the very next line the name has become 'Todorochevich'. Three lines later it's back to the correct form. This sort of absence of editing is discouraging, as is the reference in the list of 'Major Figures in Logic' to J.H.C. Whitehead instead of A.N. Whitehead. The correct dates are given for the younger Whitehead, but no one bothered to notice that this would have given him credit for *Principia Mathematica* before he turned ten.

Takeuti is a mathematician and not an historian. As a result, his approach to history generally involves either anecdotal reminiscences or philosophical reconstructions, rather than the detailed justification of a claim by textual evidence. He notes in the first essay in the volume that he does not really have the time to indulge in the sort of checking of facts which would be required for history. Perhaps this indicates that the book belongs more in the genre of 'heritage', as defined by Ivor Grattan-Guinness in a number of recent publications, than 'history'. The question, for the historian seeking to be more than a collector of anecdotes, is what use one can make of what Takeuti has assembled.

In the first place, there is his first-hand assessment of Gödel's personality. He is less inclined to see Gödel as the dark, isolated figure characteristic of most biographical portraits of him, if not quite as

the jovial figure in the movie *I.Q.*. His claim is that the Gödel close to paranoia was an exaggeration, and that even in his last few years Gödel was just inclined to see death as welcome. This portrait of Gödel is based on his many encounters with the elder logician, who expressed an interest in the work Takeuti was doing. One of the appendices to this book is a discussion of Takeuti's own work in proof theory, partly with a view to explaining why Gödel was interested.

Takeuti asserts that Gödel was the only genius he had ever known among logicians. What he points to as the three great achievements of Gödel's life were the completeness theorem in his thesis, the incompleteness theorem, and his work with constructible sets that established the consistency of the axiom of choice and the continuum hypothesis with ZF. With regard to the first of these, Takeuti comes down on the side of those who find Gödel guilty at least of forgetfulness in not having remembered consulting Skolem's article from 1923 that came close to establishing the same result. On the subject of the incompleteness theorem, Takeuti quotes the verdict of J. Robert Oppenheimer on its importance a number of times, without explaining why Oppenheimer might have been especially well placed to comment on it. He also quotes von Neumann's verdict on the importance of Gödel's incompleteness work at length (three pages running). The readiness of the author (or the translators) to leave in such lengthy passages from others without commentary again seems to detract from the central points being made.

Takeuti pays tribute to Paul Cohen's extension of Gödel's work, not just to the consistency of the continuum hypothesis but to its independence. What may strike some readers as odd is the repeated claim that Cohen carried out his construction in an 'imaginary' universe, while Gödel did his in the 'real' universe. This form of implicit realism might be puzzling to those who wonder where a line is being drawn.

One of the tasks to which Gödel devoted much effort in his later years was finding an axiom that might settle the truth of the continuum hypothesis. Takeuti asked Gödel whether the validity of an axiom resides in its mathematical consequences, anticipating the perspective Penelope Maddy has taken in some of her articles about realism and set theory. Gödel replied, 'Not necessarily,' but went ahead to claim that there was something about the continuum hypothesis that disturbed the beauty of mathematics. Aesthetic issues are also raised in a contrast between the world of ordinal numbers and the world of the power set axiom, but scarcely explored. This is another case where it might have been helpful to avoid repetition in favor of some commentary.

The 'secret' of Gödel's success, according to Takeuti, was his application of the constructive approach to objects that were not defined constructively. Hilbert's followers, Takeuti argues, were so convinced of the importance of not attaching meaning to the symbols they were manipulating that they did not see how much could be gotten by tying those meaningless symbols into things as familiar as the integers. Gödel was not bound by Hilbert's program, but was able to use ideas from that program to explode it. Takeuti even juxtaposes the devastation of Hilbert's program at the hands of Gödelian incompleteness with the disintegration of the mathematical community at Göttingen in the days of the Third Reich.

Emblematic of the difficulties a reader will have in using this book is the chapter entitled 'Short Biographies of Logicians'. Whatever the title may have conveyed in its Japanese original, most would turn to it for details like dates of birth and death, or perhaps summaries of their work. Instead, the thirty-five page chapter starts with eighteen pages on Gödel, much of it repeated from earlier essays, and then observations of various sorts on Fraenkel, Bernays, Erdös, Tarski, Heyting, Church, Kleene, and Kreisel. In some cases explanations of their mathematics are provided, but in other cases there's at most a recollection of the person's appearance (or that of his wife and daughter). This chapter scarcely makes it even to the level of 'heritage'.

One of the most interesting features of the book is the range of the author's differences of opinion with Kreisel. It was a request from Kreisel which led to the composition of the first essay in the book, although it's perhaps not surprising in view of the rivalry between the proof theorists that Takeuti turned Kreisel down. There's a certain humor in Takeuti's suggesting that the reason both he and Kreisel were hard to read was that his own English was too poor while Kreisel's English was too good. A harsher criticism emerges from his recollection of Gödel's reluctance to write a letter on Takeuti's behalf because he had not had the chance to read all of Takeuti's papers in detail. Takeuti observes that Gödel had understood his papers far more accurately than Kreisel 'who writes reviews of them'.

Takeuti explains the paucity of work (especially in published form) from Gödel's later years on the grounds that the logic community had not caught up with Gödel until he was too old to continue active research. Interestingly enough, he also explains (in the appendix about his own work) that the proof theorists had not caught up with his (Takeuti's) work until too late in his career. This seems like a reflection of his historical judgment on the life of the author.

One of Hao Wang's comments after Gödel's death was that he wished he had been listening more carefully in his conversations with Gödel but that his attention wasn't turned to what was interesting Gödel. Takeuti does not make quite the same point, but he does mention that he doesn't remember the details of many conversations with Gödel. Despite all the shortcomings of the way Takeuti's book was produced in English, it does offer a view at first hand from one of the few people who can claim to have been a sounding board for Gödel at any time of his career.

John W. Dawson, Jr., has put the entire historical community in his debt with the extent of his research into Gödel's life and the transformation of that research into what remains, amid all the volumes of more recent years, the most important biography. One point that Takeuti made in his book was his inability to appreciate the details of the manuscript notes Gödel left behind because of his inability to read the Gabelsberger shorthand in which they were written. Dawson gives consideration both to the evolution of German shorthand and to the reasons for Gödel's having employed it. It is a different generation of scholarship.

More than that, it is historical scholarship, and the more impressive in that Dawson was trained as a mathematician rather than an historian. Dawson does not float along with the currents of contemporary historical scholarship, but produced his biography by dint of examination of archival materials in many places and comparison of different accounts of the same event. This is of lasting historical value, and the small changes that have been made in the text in the course of almost a decade are just evidence for the author's care to avoid making claims without evidence.

Dawson depicts the environments in which Gödel grew up and the intellectual currents swirling around Vienna in his student days. He captures the extent to which the work leading to the incompleteness theorem was in the air and the extent to which Gödel was an innovator. There are different ways of describing Gödel's intellectual antecedents, but Dawson is careful to make the explanation of the importance of Gödel's work accessible to non-mathematicians. It is still easier reading for a mathematician, but Dawson went to the trouble of trying to get the explanations correct.

One of the crucial historical issues about the incompleteness theorem was their reception within the logical community and more broadly within the mathematical community at large. Even Takeuti recognizes the value of Dawson's investigation into the response at the time, rather than what later generations might imagine it must have been. Just as

Gregory Moore in his volume on Zermelo put paid to the notion that the axiomatization of set theory was simply a result of the paradoxes, so Dawson's work (carried over from earlier publications) points out why even the logicians of the time were not immediately overwhelmed. Hilbert, for example, still carried on with the hope that his program could survive incompleteness.

Most scholars of Gödel's life and work recognize the importance of the material left unpublished at the time of his death, and the subsequent publication in the sequence of volumes by Oxford University Press has made a good deal accessible. Dawson's biography was the first to capitalize on the wealth of unpublished material and consequently provides a better-rounded intellectual portrait. He also uses his critical judgment to assess the various stories that have long floated around within the logic community about Gödel. Those who want a reasonably complete version of the citizenship hearing can find it in Dawson's book.

For the non-mathematicians, Dawson provides an appendix of 'biographical vignettes', much more helpfully presented than the sketches in Takeuti's volume. One feature missing from Dawson's volume that Takeuti could supply is first-hand recollections of the subject by the author. On the other hand, perhaps it is precisely the absence of first-hand acquaintance with Gödel that forces Dawson to play the historian rather than the memoirist. Dawson's volume has footnotes and endnotes, a bibliography, and an index. These contribute to making the volume useful for a reader of any degree of historical and mathematical sophistication.

Dawson's book does have its share of typos, most of which have been corrected for subsequent printings. There are aspects of Gödel's life and views into which Dawson does not go so far as some readers might like. In 2005 the centenary of Albert Einstein's *annus mirabilis* brought forth a popular volume by Palle Yourgrau on Gödel, Einstein, and relativity, which gets rather shorter shrift from Dawson. The controversy over Gödel's philosophical views continues unabated, and Dawson does not try to wrestle out the precise nuances of Gödelian Platonism.

Still, Dawson's book has become the rock on which, together with the publication of Gödel's collected papers, subsequent scholars will build. It manages to tell a story of several hundred pages that reads easily and puts the reader in touch with the central features of his subject's life. Even with all the popularizations of Gödel to coincide with his

centenary, Dawson's book remains a starting point for our view into the life and work of the man who gave the world incompleteness.

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