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LETTER FROM OUR FAR-FLUNG CORRESPONDENT

HIGHLIGHTS OF ICM 2006: THE VIEW FROM HISTORY OF MATHEMATICS, LOGIC AND FOUNDATIONS OF MATHEMATICS, AND MATHEMATICAL ASPECTS OF COMPUTER SCIENCE

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Introduction.

The International Congress of Mathematicians met in the Palacio Municipal de Congresos located in a more secluded section of the north-eastern part of Madrid called Campo de las Naciones for eight exciting days of lectures and discussions. Under heavy security, King Juan Carlos opened the Congress on August 22$^{\text{nd}}$. For the occasion, the Spanish Post issued a special stamp in the amount of a standard letter in the European Union. There were twenty plenary and 160 invited lectures scheduled. American mathematicians gave twelve of the plenary lectures (two each from NYU, Stanford University, and UCLA), and seventy of the invited lectures. Of the remaining eight plenary lectures, French mathematicians delivered three, and mathematicians from Denmark, Israel, Japan, Spain, and Switzerland gave one each. All were delivered in English except for one invited address given in French. Of the two relevant cultural events, one was several organized visits to the El Escorial Library which holds about 40,000 editions including important mathematical collections of Arabic, Greek, Hebrew, and Latin manuscripts. The other, the exhibition curated by Karl Sigmund and John Dawson and first presented in Vienna on April 26, 2006 honoring the centenary of Kurt Gödel's birth, was installed in the Exhibits Hall of the Botanical Gardens of the Universidad Complutense for the duration of the Congress. The conference was well organized and the weather couldn't have been better: bright, sunny, relatively cool days, especially for this time of the year in the Iberian peninsula. On the last day of the conference, we learned that India will host the 2010 Congress.

The Fields Medals and the Rolf Nevanlinna Prize.

Four Fields Medals were awarded. Grigori (Grisha) Perelman (Steklov Institute of Mathematics, St. Petersburg, Russia), who was not present, received, but did not accept the medal for his work on the analytic geometry and structure of Ricci flows, specifically his proof of William Thurston's Geometrization Conjecture which includes the famous Poincaré Conjecture established in 1904 by Jules Henri Poincaré (1854-1912): every compact simply connected 3-dimensional manifold is homeomorphic to the 3-dimensional sphere. This problem is one of the seven outstanding problems in mathematics designated by the Clay Mathematical Institute. Andrei Okounkov (Princeton University, Princeton, NJ) received the medal for his work in bridging the gap between probability, representation theory, and algebraic geometry. Terence Tao's (UCLA, Los Angeles, CA) medal was awarded for his work in extending PDEs, combinatorics, harmonic analysis, and additive number theory. For his work on the geometry of 2-dimensional Brownian motion, and applications of conformal field theory, Wendelin Werner (Université de Paris - Sud, Paris, France) earned the medal. The Nevanlinna Prize was awarded to Jon Kleinberg (Cornell University, Ithaca, NY) for his mathematical theory of global information, especially the theory of search engines.

The Plenary and Special Lectures.

Two plenary and one special lecture are of particular interest. These are: Richard Hamilton's (Columbia University, New York, NY) "The Poincaré conjecture," Avi Wigderson's (Institute for Advanced Study, Princeton, NJ) "P, NP and mathematics: a computational complexity perspective," and John W. Morgan's (Columbia University, New York, NY) special lecture, "A report on the Poincaré conjecture." Since Morgan's and Hamilton's lectures are linked, I will discuss them first.

In his talk, Morgan gave an overview of the history of attacks on the Poincaré conjecture, showing that it had been the main motivating problem in topology for the entire 20^{th} century, and emphasizing that its solution uses concepts from PDEs. Perelman's solution depends on the program developed by Hamilton in which an arbitrary notion of distances and angles are imposed on the space. Then the mathematical object, a tensor which is a Riemannian metric, is allowed to evolve using a non-linear version of the classic heat equation. In the same way that the temperature distribution is smoothed out by the heat equation, the metric is smoothed out by the evolution equation to one of constant curvature. Such a manifold is the sphere according to a classical result in differential geometry.

Hamilton presented the program that he and S.-T. Yau developed using the Ricci Flow (Gregorio Ricci - Curbastro, 1853-1925), a non-parabolic system of PDEs. Hamilton discussed estimates and procedures to do several things: bound higher derivatives of curvature, complete the 2-dimensional case, analyze the formation of singularities, show that the singularity models have nonnegative curvature in the 3-dimensional case, and classify these singularities (pinching necks and degenerate necks) except for one case, that of the cigar type, which Perelman's work rules out. Perelman solved the "neck" problems using a sequence of mathematical surgeries—cutting out the singularities and patching up the raw edges. Most importantly, Perelman constructed what he calls the Ricci flow with surgery which allows him to extend the flow in a natural way past the singularities.

The focus of Wigderson's lecture was on the influence of the "P vs NP" question, another of the seven outstanding problems in mathematics designated by the Clay Mathematical Institute, on many areas of mathematics besides computational complexity theory which was developed to resolve this question. For example, one implication is that if we accept as true the conjecture that NP requires exponential time/size, then every probabilistic polynomial algorithm has a deterministic counterpart. He discussed the complexity of the factoring problem which is exp $n^{1/2}$, the pseudo-randomness paradigm: can specific algorithms be de-randomized without assumptions (*e.g.*, for $x \in (2^n, 2^{n+1})$, is x a prime?), and the connection between ordinary mathematical proofs with those that are probabilistically checkable and with zero knowledge proofs under certain "hardness" assumptions.

The Invited Lectures.

I have listed the titles of all the invited lectures, and selected three of them to describe briefly because I believe their topics are relevant to the variety of interests of the readership of *The Review of Modern Logic*.

In the Logic and Foundations section there were five invited lectures. These were: Rod Downey (Victoria University, New Zealand) "Algorithmic randomness and computability;" Thomas Scanlon (UC at Berkeley, USA and University of Leeds, UK), "Model theory of p-jets;" Simon Thomas (Rutgers University, USA), "Borel superrigidity and the classification problem for the torsion-free Abelian groups of finite rank;" Itay Neeman (UCLA, USA), "Determinacy and large cardinals;" and Michael Rathjen (Ohio State University, USA and University of Leeds, UK), "The art of ordinal analysis," which I shall now discuss.

Ordinal-theoretic proof theory, whose origins go back to David Hilbert's second problem (which asks for a consistency proof of the arithmetical axioms of the real numbers), is a fundamental part of proof theory. In the first part of his lecture, Rathjen presented the development of ordinal analysis beginning with Gerhard Gentzen's work (his use of transfinite induction on ordinals in his 1936 consistency proof for Peano arithmetic; his sequent calculus and the main theorem about it, the *Hauptsatz*, *i.e.*, the cut elimination technique that he used in his consistency proof). Then Rathjen reviewed the ordinal representation systems that K. Schütte used in his work in the 1960s on infinitary proof theory. In the second part of his lecture, Rathjen discussed subsystems of second order arithmetic, and the migration of ordinal analysis from the setting of second order arithmetic to that of set theory in the 1980s; an ordinal analysis of Kripke-Platek set theory; and admissible proof theory starting in 1990. In the final two parts of his lecture, Rathjen briefly discussed current developments to establish the consistency of analysis, and the use of large cardinal axioms in modeling projection functions, bringing his survey up to 2005.

One of the two invited lectures in the History of Mathematics section, Leo Corry's (Tel Aviv University, Israel) "On the origins of Hilbert's sixth problem: physics and the empiricist approach to axiomatization," is also relevant. The other lecture was Niccolò Guicciardini's (Università di Siena, Italy) "Method versus calculus in Newton's criticisms of Descartes and Leibniz."

Hilbert's sixth problem (not really a problem, but a general task) deals with the axiomatization of physics. The problem emerged from his research on the foundations of geometry. The central place of the ideas in the sixth problem in Hilbert's general scientific outlook has been brought to light in recent research. Corry's main points were directed at our understanding of the historical roots of the development of the sixth problem. In particular, Corry illuminated the extent to which similar parallel developments in physics (basic laws and principles in physical theories) and axioms in mathematics shaped Hilbert's views on axiomatization. Corry pointed out that of Hilbert's requirements for an axiom system—independence, simplicity, completeness, and consistency—that appeared in his *Grundlagen der Geometrie*, the first two came directly from the demands of physical theories.

In the Mathematical Aspects of Computer Science section there were seven invited lectures. Luca Trevisan (UC at Berkeley, USA), "Pseudorandomness and combinatorial constructions;" Omer Reingold (Weizmann Institute of Science, Israel), "On expander graphs and connectivity in small space;" Jon M. Kleinberg "Complex networks and decentralized search algorithms;" Tim Roughgarden (Stanford University, USA), "Potential functions and the inefficiency of equilibria;" Alexander Semenovich Holevo (Steklov Mathematics Institute, Russia), "The additivity problem in quantum information theory;" Manindra Agrawal (Indian Institute of Technology Kanpur, India), "Determinant versus permanent;" Ronitt Rubinfeld (MIT, USA), "Sublinear time algorithms." Trevisan's talk was interesting particularly because it is related to Wigderson's work.

In his lecture, Trevisan explored the connections between the conditional de-randomization results in the computational theory of pseudorandomness and unconditional explicit constructions of such combinatorial objects as error-correcting codes. The computational theory of pseudorandomness depends heavily on work done by Wigderson and his colleagues in the 1990s. For example, as a result of work by Wigderson and Russell Impagliazzo, randomness extractors (procedures originally designed to generate truly random bits) can be constructed unconditionally. Concluding his very technical paper, Trevisan states that by starting from worst-case complexity assumptions, he has discussed how very strong pseudorandom generators can be constructed and how conditional de-randomization results can be derived for *all* probabilistic algorithms.

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