NON-FREGEAN APPROACH TO V. A. SMIRNOV'S COMBINED LOGICS

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In his paper "Internal and External Logics" [*1988*] V. A. Smirnov considers systems of two-levelled logic in which the extrinsic level (external logic) would be a propositional logic while the intrinsic level (internal logic) would be an algebra of events (the latter are the terms). By extending the system **CM** (De Morgan's logic with the classical external one) at the expense of assertions concerning the identity of events, Smirnov introduces the rule $\frac{\theta a \leftrightarrow \theta b}{a = b}$ which he, in accordance with R. Suszko's ideas, calls the *Frege principle*. It denotes that an algebra of events might be modified within wide ranges.

As is generally known, non-fregean logics suppose an abolition of Frege's principle which results in introducing a new identity connective into the syntax and impels the use of so-called situational semantics. Following again Suszko's ideas let us replace Frege's principle with the rule $\frac{a = b}{\theta a \leftrightarrow \theta b}$, which in turn would be called *Suszko's principle*. For such a system of combined logic we can yield the R. Wójcicki-type situational semantics [Wójcicki *1986*] in case of acceptance of the conception of events as the collection of situations instead of possible worlds, to wit, treating an identity of events as to be determined by the identity of situations in which those are occurring. And if in the case of accepting Frege's principle we conclude from an assertion act to the state of affairs, then in the case of accepting Suszko's principle we conclude from a state of affairs to the assertion act, i.e. ontological aspects become decisive.

The lawfulness of exploitation of the situational semantic becomes more evident if we consider the systems which are weaker than **CM**. The fact of the matter is that in the general case we can confine ourselves to accepting for internal logic only the usual type axiom $a = a$ and rules

$$\frac{a = b}{b = a}, \quad \frac{a = b \quad b = c}{a = c}$$

describing the properties of the identity of events and the rule

$$\frac{a_1 = b_1, \ldots, a_{s(i)} = b_{s(i)}}{R_1(a_1, \ldots, a_{s(i)}) \rightarrow R_1(b_1, \ldots, b_{s(i)})}, \quad i = 1, \ldots, m$$

describing the substitutional properties of terms-events. But according to Smirnov's approach $R_i\,(a_1, \ldots, a_{s\,(i)})$ will be not a formula but the sentential term and therefore the last axiom could not be approved. In this case our proposal are the rules

$$\frac{\theta\,(a = b)}{\theta a \leftrightarrow \theta b}$$

$$\frac{\theta(a_1 = b_1), \ldots, \theta\,(a_{s(i)} = b_{s(i)})}{\theta R_1(a_1, \ldots, a_{s(i)}) \rightarrow \theta R_1(b_1, \ldots, b_{s(i)})}, \quad i = 1, \ldots, m$$

an axiom $\theta(a{=}a)$, and rules $\dfrac{\theta(a = b)}{\theta(b = a)}$, $\dfrac{\theta(a = b) \quad \theta(b = c)}{\theta(a = c)}$, which would hardly be interpreted in the framework of possible-worlds semantics of combined logic considered by Smirnov. Since in such semantics an interpretation of $R_i\,(a_1, \ldots, a_{s\,(i)})$ will be an event, then following the course of Wójcicki's situational semantics we arrive at the situational treatment of the sentential terms, i.e. we say that $(R_i\,, a_1, \ldots, a_{s\,(i)})$ is a situation such that $R_i\,(a_1, \ldots, a_{s\,(i)})$. Moreover, $a = b$ in an obvious way would also be a sentential term and hence could be treated as an elementary situation. As a consequence, $R_i\,(a_1, \ldots, a_{s\,(i)}) = (R_i\,, a_1, \ldots, a_{s\,(i)})$ would be understood as an elementary situation. Indeed, the passage to situational semantics does not mean the abolition of the possible-worlds treatment: we always may refer to the possible worlds as the maximally large situations.

In order to be more correct in our non-fregean approach, instead of the rules in question we need to accept the following rules:

$$\frac{\theta (a = b)}{\theta a \leftrightarrow \theta b \quad \theta \sim a \leftrightarrow \theta \sim b}$$

$$\frac{\theta(a_1 = b_1), \dots, \theta (a_{s(i)} = b_{s(i)})}{\theta R_1(a_1, \dots, a_{s(i)}) \rightarrow \theta R_1(b_1, \dots, b_{s(i)}) \quad \theta \sim R_1(a_1, \dots, a_{s(i)}) \rightarrow \theta \sim R_1(b_1, \dots, b_{s(i)})},$$

$i = 1, \dots, m$

where there is an internal negation.

Since Wójcicki's version of situational semantics presupposes that every set of elementary situations relates with the unique situation { } and the other way round, then, in effect, the difference between situations and events dissappears; we are always able to assign the respective situation to our event which relates with the set of situation (i.e. our *universum* of situations will be the transitive set). Hence, the standard interpretation list of situational semantics would be completed with the following condition:

$D(\theta R_i\ (a_1, \dots, a_{s(i)})$ is the fact whenever $R_i\ (a_1, \dots, a_{s(i)})$.

Developing our approach, we can adopt B. Wolniewicz's [*1981*] ordering of situations when $a \leq b$ means "a obtains in b" and accept weak Suszko's principle in form of the rule

$$\frac{\theta (a \leq b)}{\theta a \rightarrow \theta b \quad \theta \sim a \rightarrow \theta \sim b}$$

or

$$\frac{\theta(a_1 \leq b_1), \dots, \theta (a_{s(i)} \leq b_{s(i)})}{\theta R_1(a_1, \dots, a_{s(i)}) \rightarrow \theta R_1(b_1, \dots, b_{s(i)}) \quad \theta \sim R_1(a_1, \dots, a_{s(i)}) \rightarrow \theta \sim R_1(b_1, \dots, b_{s(i)})},$$

$i = 1, \dots, m$

along with the usual type axiom $\theta (a \leq b)$ and the rule

$$\frac{\theta(a \leq b) \quad \theta(b \leq c)}{\theta(a \leq c)}.$$

But in this case we need a much more elaborated situational semantics because our *universum* of situations becomes the non-wellfounded set (see [Barwise *1989*], etc.).

In a sense, the systems in question seem to be much too amorphous regarding the situational aspects, since we are not concerned with imposing any constraints on the structure of situations. On the one hand, it leads to the contingency of the links among the situations, and on the other hand to the lack of the firm belief that we deal with an ontological ordering of situations only the consequence of our experience (one may claim that the ordering adopted is the only consequence of our experience because of the sense of the θ-operator). In order to overcome such difficulties, we shall resort to Smirnov's general logic of sentences and events from [Smirnov *1989b*, 23]. There is an operator [–] in the language of this calculus such that if a is a formula then [a] is a sentential term. By means of such an operator we enrich our system with the axiom

$$\theta\,[\alpha] \leftrightarrow \alpha.$$

The list of admissible interpretation conditions ought to be enlarged by the following condition:

D ([α]) is a fact if $\{D\,(\alpha)$: $D\,(\alpha)$ is a fact$\}$ is a fact.

Loosely speaking, we relate to every formula the set of factual situations under any admissible interpretation — the maximal fact. It is easy to see that we then no longer need our axiom $\theta\ (a\ \leq\ b)$ and the rule $\frac{\theta(a \leq b) \quad \theta(b \leq c)}{\theta(a \leq c)}$ because the respective statements become the indispensible conditions of the structure of our algebra of events.

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