reprint of the original Springer publication. There is the occasional altered exposition or rephrased theorem. Some typos have been corrected, others left in place, and yet new ones introduced. (Notable is the confusion from time to time between S and δ.) But by and large the classic text has remained intact.

In summary, the books *Diagonalization and Self-Reference* and *First-Order Logic* exhibit the hallmarks of Smullyan's style. They are accessible to beginners, yet present material of interest to specialists. They will, on the one hand, examine a single logical phenomenon from a variety of angles and, on the other hand, find a unifying setting for apparently different phenomena. Although the books differ from each other in age and scope (not to mention price), they are linked by a common approach to logic and its exposition — an approach that Smullyan has consistently and successfully pursued.

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M. Katětov and P. Simon (editors), T*he Mathematical Legacy of Eduard Čech,* Basel/Boston/Berlin, Birkhäuser, 1993. 441 + 4 pp.

Reviewed by

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Eduard Čech (1893-1960) was arguably the greatest mathematician that Czechoslovakia has ever produced, and during a lifetime of work in mathematics he made major contributions to algebraic and general topology and to differential geometry. The year 1993 marks the centenary of Čech's birth, and in honor of this occasion there have been at least two mathematical books published. One is the book under review, the other is *The Čech Centennial Homotopy Theory Conference* [Cenkl & Miller *1995*].

Čech was born on June 29, 1893 in Straučov in northeastern

Bohemia, about 100 miles from Prague. In 1912, at the age of 19, he began his studies in mathematics at Charles University in Prague. Čech spent five semesters there, where he acquired an impressive mathematical education, largely on his own. His studies were interrupted in 1915 by World War I, but by 1920 he had received his Doctor of Philosophy degree from Charles University, with a thesis entitled *On Curves and Plane Elements of the Third Order.*

Čech was immediately interested in mathematical research, and during his early years he devoted his efforts to a systematic study of differential geometry. He was acquainted with the work of the distinguished Italian geometer, G. Fubini, and he spent the year 1921-22 in Turin collaborating with Fubini. Eventually the two wrote three volumes on geometry, published in 1926, 1927, and 1931.

In 1922, Čech submitted a habilitation thesis on projective differential geometry and became a docent at Charles University. One year later, before the age of 30, he was appointed to a chair at Masaryk University in Brno, becoming a full professor in 1928.

It was about this time that Čech's research interests shifted from geometry to topology, both general and combinatorial (or algebraic). In 1932, Čech published two pioneering papers: one on the general theory of homology in arbitrary spaces and the other on the general theory of manifolds. After the publication of these two papers, his reputation as one of the leading mathematicians in algebraic topology was firmly established. In 1935, he attended a conference on combinatorial topology in Moscow; his report on his research was so impressive that he was invited to spend a year at the Institute for Advanced Studies in Princeton.

After returning to Brno from the U.S.A. in 1936, Čech founded a seminar with a special emphasis on general topology. Among the papers discussed were (1) the fundamental paper of Alexandroff and Urysohn on compactness and (2) Tychonoff's 1930 paper on the product topology, compactness, and completely regular spaces. Many questions were raised and solved in the seminar, and new areas of research were opened up. Of special interest is Čech's work on compactifications. The seminar continued until 1939, when all Czech universities were closed due to the German occupation. One of Čech's most gifted students, B. Popíšil, was arrested by the Nazi Gestapo in 1941 and not released until 1944. As a consequence of this long period of imprisonment, Popíšil died just a few months after his release.

In 1945, after spending twenty-two years in scholarly activity in Brno, Čech accepted an appointment at Charles University in Prague.

By then Čech was the leading figure in Czech mathematics. In 1947 he was appointed the director of the Mathematical Research Institute of the Czech Academy of Sciences and Arts. In 1950 the Central Mathematical Institute was established with Čech as the first director. In 1952, when the Czechoslovak Academy of Sciences was founded, this institute was incorporated into the Academy as the Mathematical Institute of the Czechoslovak Academy of Sciences, again with Čech as its first director. In 1954, he returned to Charles University as a director of the newly founded Mathematical Institute of Charles University.

By 1949 Čech has resumed his research in differential geometry, and during the period 1949-960 he wrote more than twenty papers on this subject. But Čech continued his interest in general topology, initiating the organization of the Symposium on General Topology. His untimely death on March 15, 1960 prevented him from attending the first meeting of the Symposium, which was held in Prague in September, 1961. Appropriately, the Proceedings of the Symposium were dedicated to Čech ([Novák *1961*]), and four speeches were given in his honor by the distinguished topologists, M. Katětov, M. H. Stone, P. S. Alexandroff, and K. Kuratowski. (As evidence of Čech's continuing influence in general topology, we note that the Eighth Prague Topology Symposium is scheduled to meet in August of 1996).

Katětov, himself a Czechoslovakian, emphasized in his speech not only the great mathematical work of Čech, but also his importance to the scientific community in Czechoslovakia. He described Čech as "the first Czechoslovak mathematician to make a fundamental contribution to world mathematics and, in his particular branch, influenced scientific work in other countries. It may be said that with E. Čech Czechoslovak mathematics assumed an active place in the general stream of world development of mathematical science" ([Novak *1961*, 25]). Čech is obviously a hero to contemporary Czechoslovakian mathematicians; again we quote from Katětov's speech ([Novak *1961*, 25]): "Čech presents us with an example difficult to emulate. His devotion to science knows no bounds. We shall always admire his exemplary diligence, the undauntable spirit with which he firmly upheld his views — arrived at on the basis of careful consideration — both in discussion and in practice."

The book under review consists of four main chapters as follows:

I. Čech-Stone Compactification [essay by P. Simon]
II. Dimension Theory [essay by Katětov]
III. Algebraic Topology [essay by E. G. Sklyarnko]

IV. Differential Geometry [essay by I. Kolář]

Each of these chapters includes at least one paper by Čech in the area of research under discussion, a brief essay on Čech's contributions to the field and subsequent developments in the field, and several papers by other mathematicians that are related to and influenced by Čech's work.

In addition to these four chapters, the book begins with a concise biography of Čech entitled "Life and work of Eduard Čech" and written by M. Katětov, J. Novák and A. Švec (many of the introductory remarks on Čech's life at the beginning of this review are taken from this biography). Following the biography there is a complete list of Čech's published works. Čech's list of publications is indeed impressive, and breaks down as follows:

Books 11 **Papers** 94
Topological (general, including dimension theory): 12
Topological (algebraic or combinatorial): 19
Geometry: 58
Other: 5

Finally, the book ends with a short essay by E. Kraemer entitled "Professor Čech and Didactics of Mathematics".

I will assume that the readers of *Modern Logic* are primarily interested in Čech's contributions to set theory and general topology and in his interest in the teaching of mathematics. So I will confine my review to Chapters I and II and the essay by Kraemer.

There is one paper by Čech in Chapter I, namely his famous paper "On Bicompact Spaces". This paper was obviously inspired by the earlier papers of Alexandroff-Urysohn on compact spaces and Tychonoff's paper on compact and completely regular spaces. In this paper Čech proved the existence of a maximal compactification for every completely regular space X. M. H. Stone proved this same result independently of Čech using completely different methods. Today this compactification is denoted by βX and is called the Čech-Stone (or the Stone-Čech) compactification. A precise statement of Čech's result is as follows:

Theorem. *For every completely regular space X there is a space βX such that*

(1) βX is a compact Hausdorff space;

(2) *$X \subseteq \beta X$ and X is dense in βX;*
(3) *every bounded real-valued continuous function defined on X extends continuously to βX.*

In this same paper, Čech introduced an important class of topological spaces now called the *Čech-complete* spaces. Every locally compact and every completely metrizable space is Čech-complete; the Baire category property holds for Čech-complete spaces.

There are seven reprinted papers in Chapter I related to the Čech-Stone compactification:

(1) B. Pospíšil "Remarks on Compact Spaces" (1937).
(2) I. Gelfand and A. Kolmogoroff, "On Rings of Continuous Functions on Topological Spaces" (1939).
(3) I. Glicksberg, "Stone-Čech Compactifications of Products" (1959).
(4) W. Rudin, "Homogeneity Problems in the Theory of Čech Compactifications" (1956).
(5) I. I. Parovičenko, "On a Universal Bicompactum of Weight $\aleph$" (1963).
(6) Z. Frolík, "Non-homogeneity of $\beta P - P$" (1967).
(7) K. Kunen, "Weak P-points in $\mathbf{N}^*$" (1978).

In (1), Pospíšil proved that for a discrete space D of cardinality κ, the cardinality of βD is exp exp κ. This answered a question raised by Čech in his own paper. In (2), Gelfand and Kolmogoroff proved that bX is homeomorphic to the space of all maximal ideals in the ring of all continuous real-valued bounded functions on X. In (3), Glicksberg proved that if $\beta(X \times Y) = \beta X \times \beta Y$, then $X \times Y$ is pseudocompact (a topological space X is *pseudocompact* if every real-valued continuous function on X is bounded).

Papers (4), (6) and (7) by Rudin, Frolík, and Kunen are concerned with the non-homogeneity of $\beta\omega - \omega$. (A space X is *homogeneous* if given distinct points p and q in X, there is a homeomorphism from $X - \{p\}$ onto $X - \{q\}$.) This area of research is described by Simon as the "field in which general topology, set theory, Boolean algebras and mathematical logic meet" (see p. 29). Rudin showed that, assuming CH, $\beta\omega - \omega$ is not homogeneous. He did this by proving, under CH, the existence of P-points in $\beta\omega - \omega$; since $\beta\omega - \omega$ obviously has points that are not P-points, this argument produces two points of $\beta\omega - \omega$ that have distinct topological properties. (A point x of a topological space X is a *P-point* if the intersection of every countable collection of neighborhoods of x is a neighborhood of x.) Frolík proved the non-homogeneity of $\beta\omega - \omega$ without assuming CH, but his proof, unlike that of Rudin, did not isolate a property of points of $\beta\omega - \omega$ that demonstrated non-homogeneity. Finally, Kunen proved in ZFC, the existence of weak-P-points in $\beta\omega - \omega$. (A point x of a topological space X is a *weak-P-point* if x is not a limit point of any countable subset of $X - \{x\}$.) Kunen's result established in an explicit manner, and without any additional set-theoretic axioms, the non-homogeneity of $\beta\omega - \omega$.

Chapter II on dimension theory contains two papers by Čech: "On the Dimension of Perfectly Normal Spaces" (1932); "Contribution to Dimension Theory" (1933). In these two important papers Čech gave the first formal definition of

- Ind(X) (the *large inductive dimension* of X, based on earlier ideas of Brouwer);
- dim(X) (the *covering dimension* of X, based on earlier ideas of Lebesgue).

Today, Ind(X) is also called the *Brouwer-Čech dimension*, and dim(X) is called the *Čech-Lebesgue dimension*.

Čech was obviously captivated by the beauty and power of general topology, and at times he expressed himself quite poetically about this area of mathematics. Consider, for example, the following comment that appears in his paper „Contribution to Dimension Theory“ (p. 151):

> It is not incidental that it was possible to choose so elementary from of exposition. Indeed, general topology in the course of its penetrating analysis of the space by no means gathers from the finely branching supply of knowledge of classical mathematics, still conquers its problems with success by a quite simple weapon, that is, by elementary rules of logical reasoning, applied directly to simple notions obtained from the intuition and axiomatically precised. Let the modest paper contribute somewhat to this beautiful mathematical discipline to enjoy even in our country such a favour, which it deserves for its inner grace and its significance in the totality of mathematical sciences.

Finally, let us turn to Čech's interest in the teaching of mathematics. Unlike many research mathematicians, Čech strongly believed that there should be close cooperation between university professors and secondary school teachers. Čech's first paper on this subject was written in 1938 and concerned the teaching of combinatorics and the calculus of probabilities in Czech high schools. In subsequent years he took a great interest in the teaching of mathematics. He audited classes of mathematics at grammer schools and discussed the possibilities of improving the teaching process with the teachers.

It was from 1939 until 1944, when the Czech universities were closed, that Čech was most active in the teaching of mathematics. During these years he wrote arithmetic textbooks for the first three years and geometry textbooks for the first four years of the Czech eight year grammar schools. Most remarkable were his seminars which took place from 1947 until 1954 at Charles University. In these seminars, many problems of teaching mathematics were discussed, for example, the solution of reasoning problems without the use of algebra, solving problems containing parameters, and the teaching of geometry.

As noted earlier, Čech published a total of eleven books; of this number, four are aimed at high school and college students:

- *Elementary Functions*
- *What is and what is the use of higher mathematics?*
- *Foundations of analytic geometry*
- *Numbers and operations with them.*

Finally, it was Čech who initiated the establishment of the Czech Mathematical Olypiad in 1951, a national competition in mathematics for pupils of high school age. In 1959, a number of Eastern European countries with separate competitions joined together to form what is now known as the International Mathematical Olympiad.

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