J. Rybak and J. Rybak. 1976. *Venn diagrams extended: Map logic*, Notre Dame Journal of Formal Logic **17**, 469–475.

—. 1984. *Mechanizing logic I: Map logic extended formally to relational arguments*, Notre Dame Journal of Formal Logic **25**, 250–264.

—. 1984a. *Mechanizing logic II: Automated map logic method for relational arguments on paper and by computer,* Notre Dame Journal of Formal Logic **25**, 265–282.

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Raymond M. Smullyan, *Diagonalization and Self-Reference*. Oxford Logic Guides **27**, New York, Oxford University Press, 1994; xv + 396 pp. and Raymond M. Smullyan. *First-Order Logic*. New York, Dover Publications, 1995 (originally published in 1968 by Springer-Verlag.); xii + 158 pp.

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Here are two books, written 26 years apart. The older one deals with a very specific area of logic, the newer one with a common thread that runs through a variety of logical fields. Yet they are recognizably by the same author and display the features, both mathematical and stylistic, that typify Smullyan's writings.

The focus of *Diagonalization and Self-Reference* is the development of a unified framework for the fixed-point theorems that occur in different areas of mathematical logic, such as recursion theory, combinatory logic, and proof theory. To this end Smullyan introduces the notion of a sequential system. To quote his definition, "By a *sequential system* S we shall mean a triple (N, Σ, $\rightarrow$), where N is a set, Σ is a collection of functions of various numbers of arguments, all arguments and values being in N, and $\rightarrow$ is a transitive binary relation on the set of all finite non-empty sequences of elements of N."

Of course, this definition is highly general, which provides for the flexibility to deal with disparate applications. For example, consider the statement:

(1) For each $a \in N$, there exists $b \in N$ such that $b \rightarrow a, b$.

When the sequential system chosen is one appropriate for combinatory logic, this statement becomes in that context:
(2) For each combinator a, there exists a combinator b such that $b = ab$,

i.e., every combinator has a fixed point. Somewhat more subtly, under Smullyan's interpretation of sequential systems in recursion theory, (1) translates as:

(3) For every binary recursively enumerable relation R, there exists a number b such that $\omega_b = \{x \mid \mathrm{R}(b, x)\}$,

where, as usual, ω_b denotes the b^{th} r.e. set. Now (3), a standard corollary of the Recursion Theorem, holds uniformly in that b can be computed effectively from an r.e. index for R. Likewise, the uniform version of (2) is that there exists a fixed-point combinator, i.e., a combinator Y such that for all a, the combinator Ya is a fixed point for a. These two uniform results have a common sequential system translation:

(4) There exists $f \in \Sigma$ such that for all a $\in N, f(a) \rightarrow a, f(a)$.

Naturally, the point of sequential systems is not just to provide shared formulations of results from different fields of logic. Smullyan uses the notion of sequential system as a vehicle for getting at the essence of various fixed-point results and their proofs. For example, what makes the Recursion Theorem tick? Recursion theory, after all, deals with a very rich structure. Both the recursive and partial recursive functions are closed under composition, there are universal r.e. relations, *s-m-n* theorems hold, and so on till the computable cows come home. Which of these or other properties ensure that the Recursion Theorem or some other fixed-point theorem holds? Do such properties somehow correspond to combinators whose existence guarantees the existence of fixed points or of fixed-point combinator? The study of sequential systems addresses such matters.

Diagonalization and Self-Reference examines a whole battery of properties sequential systems might have and explores what kinds of fixed-point results follow from these properties. The properties often involve the existence of some sort of diagonalizing object. And the fixed-point results come in several types, including not only garden-variety and even higher-order versions. Again, the bottom line is that all these properties and results for general sequential systems have (often very familiar) interpretations in specific areas such as recursion theory and proof theory.

Although the study of sequential systems forms the heart of *Diagonalization and Self-Reference*, it by no means constitues the whole book. Indeed, sequential systems are not even introduced until after the halfway point. Since this monograph was designed to be self-contained, several chapters are devoted to relevant background material. This leads to a large amount of overlap with previous books of Smullyan. The obvious disadvantage of such overlap to readers of the earlier volumes is somewhat mitigated by the fact that often the reworked material receives a "new, improved" handling — results are generalized and strengthened, more direct proofs are presented, etc. And interspersed are sections on self-reference, based on material from Smullyan's journal articles that had not found its way into his books previously.

As with Smullyan's previous books in the Oxford Logic Guides series, there are inordinately many typos and minor errors. Occasionally, more serious errors arise. For example, the claim in Chapter 6 that the minimalization of a partial recursive function must be partial recursive is false under the definition of minimalization given there. (In fact, disproving that claim occurs as an exercise in both Hartley Rogers' and Robert Soare's recursion-theory textbooks.) The mistakes form an unfortunate distraction from the many interesting ideas presented in *Diagonalization and Self-Reference*.

First-Order Logic originally appeared in 1968. An influential and much-cited book in its area, it had nevertheless been out of print for quite a while. Thanks to Dover, it has now reappeared in a slightly altered version and costs less than the hardback of 27 years ago.

As its preface states, the book "is intended to serve both as an introduction to Quantification Theory and as an exposition of new results and techniques in 'analytic' or 'cut-free' methods." As could be expected of such a book, then or now, Smullyan includes such results as compactness, completeness, the Löwenheim-Skolem Theorem, cut elimination, Craig's Interpolation Lemma, and Beth's Definability Theorem. However, *First-Order Logic* broke new ground with its detailed presentation of analytic tableau methods. This review will not go into the history of tableaus. The role of Smullyan and his book, along with that of other figures such as Hintikka and Beth, in the development of this area is discussed at length by Irving Anellis in the very first issue of this journal. It suffices to note that in the *Handbook of Philosophical Logic* (1983), *First-Order Logic* is cited as the "*locus classicus*" for tableau theory.

Not that the monograph deals exclusively with tableaus. Chapters are also devoted to proof systems of the Hilbert and Gentzen types. And much of the book works with a general framework that is independent of the particular formal system in which proofs are constructed. In particular, the introduction of the notion of an analytic consistency property leads to the so-called Unifying Principle, of which many of the standard theorems are special cases. And then a related notion, that of a synthetic consistency property, is used to explore the role of cut conditions.

As mentioned above, this edition of *First-Order Logic* is not an exact reprint of the original Springer publication. There is the occasional altered exposition or rephrased theorem. Some typos have been corrected, others left in place, and yet new ones introduced. (Notable is the confusion from time to time between S and δ.) But by and large the classic text has remained intact.

In summary, the books *Diagonalization and Self-Reference* and *First-Order Logic* exhibit the hallmarks of Smullyan's style. They are accessible to beginners, yet present material of interest to specialists. They will, on the one hand, examine a single logical phenomenon from a variety of angles and, on the other hand, find a unifying setting for apparently different phenomena. Although the books differ from each other in age and scope (not to mention price), they are linked by a common approach to logic and its exposition — an approach that Smullyan has consistently and successfully pursued.

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M. Katětov and P. Simon (editors), T*he Mathematical Legacy of Eduard Čech,* Basel/Boston/Berlin, Birkhäuser, 1993. 441 + 4 pp.

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Eduard Čech (1893-1960) was arguably the greatest mathematician that Czechoslovakia has ever produced, and during a lifetime of work in mathematics he made major contributions to algebraic and general topology and to differential geometry. The year 1993 marks the centenary of Čech's birth, and in honor of this occasion there have been at least two mathematical books published. One is the book under review, the other is *The Čech Centennial Homotopy Theory Conference* [Cenkl & Miller *1995*].

Čech was born on June 29, 1893 in Straučov in northeastern