Sun-Joo Shin. *The Logical Status of Diagrams*. Cambridge/New York, Cambridge University Press, 1995.

Reviewed by

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2nd grade teacher: Genny, you must try not to use your fingers when you count.

9th grade teacher: Genny, remember the triangle you drew is just a crude picture of a real triangle. Real triangles can't actually be seen. Geometry is in the mind — not the eyes!

College logic instructor: Now, keep in mind, Genny, that these Venn diagrams are nothing more than heuristic devices, visual aids to the understanding. When you have mastered the techniques of formal inference you can dispense with diagrammatic crutches.

Friend: Genny, I can't believe you're still wearing that old watch; analogue is, like, so passé.

Supervior, engineering firm: Genny, work up a flow chart for the new computer programme and attach a wiring diagram for the next console line. When you finish, drive over to Mr. Peirce's office for our meeting. Use this map to get there.

In spite of the fact that charts, maps, family trees, and diagrams of all sort are a common part of our daily lives, mathematicians and logicians have generally denigrated the use of visual devices. Genuine formal reasoning, so the claim goes, takes place in the head. Pictures may help the novice to get the right idea, but they are really incidental to the process. Most particularly, they say, such devices could never serve as a medium for reasoning *per se*. A diagram is never to be taken as a proof.

This general prejudice against diagrammatic methods of proof is fairly recent (at least among mathematical logicians). There is good textual evidence that Aristotle made use of some form of diagrams in his original account of syllogistic, as witness his use of (Greek equivalents of) such expressions as 'term' (= 'end point'), 'middle', 'extreme', 'figure', and so forth. (My own view is that Aristotle's diagrams were probably linear [Englebretsen *1992*].) Not only is there no evidence of a prejudice against diagrams in the work of the first logician, there is a decided emphasis on the power of diagrams to affect the understanding. Consider this well known passage concerning the Pythagorean Theorem: "If, then, the line parallel to the side had been already drawn upwards, the reason would have been evident to any one as soon as he saw the figure" (*Metaphysics* 1051a26-26). Ever since there have been repeated attempts by logicians to mechanize logical thinking (often by means of diagrams). From Ramon Lull in the 13^{th} century, to Lambert, Euler, Venn, Carroll, Marquand, and Peirce, and finally in our century to researchers such as Karnaugh and those now exploiting the powers of computers for generating diagrammatic proofs. Indeed, it is possible to view Frege's unique symbolism for the predicate calculus as a kind of linear diagram system. (For the now classic survey of the development of logic diagrams see [Gardner *1982*].) Still, as I said, there is a clear reluctance on the part of most logicians (after Peirce) to take logic diagrams seriously. One very important reason for this has been the absence of a proof of the soundness and completeness of any diagrammatic system.

Sun-Joo Shin offers the first systematic attempt to defend logical diagrams against what she terms "the general prejudice against diagrams". Thus, a key element of her*The Logical Status of Diagrams* is her set of proofs for the soundness and consistency of her two main diagrammatic systems. Shin begins with a brief, clear account of Euler, Venn, and Peirce diagrams and then, in the body of the book, develops two diagrammatic systems, Venn-I and Venn-II. She then compares the latter system with the first-order predicate calculus, and she concludes with an extended discussion of diagrams (*vs.* linguistic representations) as devices for conveying information.

It is important to note that while the system chosen for elaboration here is based on Venn diagrams, there is nothing to suggest that a non-Venn type system might not enjoy the same advantages. The simple fact is that Venn diagrams constitute the system with which logicians today have the most familiarity. The version developed here is not just Venn. Venn himself developed his system as an improvement on Euler diagrams. Euler diagrams represent the information conveyed by a categorical proposition by modelling the corresponding set inclusions, exclusions, and intersections as pairs of closed curves (circles) with one wholly contained in the other, wholly separate from the other, or partially overlapping the other. There are a variety of well known weaknesses in the Euler system. Prominent among these, as Venn saw, was the inability of Euler diagrams to represent facts about which we have only partial information, leaving us free to add further information later. But that is just what would be required of a diagram system designed to map Boolean algebra, Venn's primary goal. To overcome the limitations of Euler diagrams Venn allowed a pair of intersecting circles (a "primary diagram") to convey no particular information. Such information as is conveyed by a categorical proposition was then indicated by the use of shading in one or more of the closed areas ("cells") of the primary diagram. Thus additional information could be accumulated on the same diagram. Still, the system of diagrams originally developed by Venn faced its own limitations. Most important among these is its inability to represent existential propositions. The version of Venn diagrams most of us are now familiar with is the result of modification made by Peirce. Peirce saw his own diagrams ("Existential Graphs") as extended Venn diagrams. One of the important ways in which Venn diagrams are extended is by use of the character '*x*' in a cell to represent the nonemptiness of that set, i.e., the representation of an existential proposition. When two or more contiguous cells are known to be nonempty, but not *which* each is inscribed with an '*x*' and these are then connected by lines. Thus Peirce's version of Venn allows the representation of some kinds of disjunctive information. But Peirce's contribution was more substantial than this suggests. He was the first to recognize that logic diagrams could be taken as a logical system in its own right. Consequently he provided his system with axioms and rules for transforming one diagram into another in order to represent logical inference. Nevertheless, Peirce's version of Venn is insufficient as an independent system of logic. Some of his rules are unclear, the system is incomplete, and there is no way to establish its soundness since Peirce, like his predecessors, was unable to get clear about the syntactic/ semantic distinctions required.

In attempting to overcome the limits still surrounding Peirce's extension of Venn diagrams, Shin formulates Venn-I. In doing so she is careful to demarcate syntax and semantics. Venn-I can be seen as a formal language consisting of rectangles, closed curves, *x*'s, lines, and shadings. The formation rules for a well-formed-diagram, wfd, are, briefly, as follows: any rectangle is a wfd; if *D* is a wfd, the addition of a closed curve interior to the rectangle of *D* is a wfd; if *D* is a wfd, shading a region of *D* is a wfd; if *D* is a wfd, adding an '*x*' to a region of *D* is a wfd; if *D* is a wfd, connecting existing *x*'s in different regions of *D* by lines is a wfd. There are, of course, certain refinements and restrictions to these rules. Most important among these are the "partial-overlapping rule" and the "avoid-*x* rule". The former requires that any added closed curve must overlap a proper part of every existent nonrectangle minimal region of the diagram to which it is added once and only once. A minimal region is an enclosed region having no proper parts which are enclosed regions. The latter rule requires that no closed curve added to an existing diagram passes through an existing *x*. The semantics for Venn-I is equally simple. Each region of a diagram represents a set. Empty sets are represented by shading;

nonempty sets are represented by inscriptions of 'x'. Set relations, such as intersection, union, and exclusion are represented by the overlapping of regions and shadings. Rectangles represent the relevant domains of discourse ("background sets"). In nondiagrammatic systems of logic, such as the sentential calculus and first-order predicate calculus, a given variable is presumed to be unambiguous across its various tokens in any given discourse. For example, in '$p \& q \vee p \& \sim q$' each of the two tokens of 'p' represents the same proposition (being a variable, 'p' might very well represent a different proposition when occurring in a different context). The problem of guaranteeing unambiguous representations is less transparent in a diagrammatic logic. In using Venn-like diagrams to analyze syllogisms separate diagrams are drawn for each premise and conclusion. Validity depends upon whether the diagram for the conclusion can be "read off" the diagrams for the premises. In this process a given set may be represented more than once in the various diagrams. To prohibit ambiguity Shin formulates a counterpart relation ("cp") among closed curve tokens: cp is an equivalence relation on a set of basic regions of a set of diagrams such that if A and B are cp-related then A and B are either both closed curves or rectangles, and either A is identical to B or A and B are in different diagrams. Semantically, then, any two cp-related regions must represent the same set. Finally, just as models provide interpretations for first-order formulæ, "set assignments" assign a set to each basic region of a diagram. The consequence relation among diagrams can then be defined as follows: a given wfd, D follows from a set of wfds if and only if every set assignment satisfying every member of that set also satisfies D.

The proof theory for Venn-I requires six transformation rules which allow one to obtain a diagram from given diagrams. I refrain from repeating the rules here, but suffice it to say that generally speaking they are formulations of rules we implicitly use when manipulating Venn diagrams. Thus, for example, given any diagram we can add a new closed curve (as long as we keep in mind the partial-overlap restriction). Also, e.g., pairs of distinct diagrams can be "unified", essentially, by copying the closed curves not cp-related to one another from one diagram onto the other. This is what we do when we diagram a syllogistic premise and then use that same diagram in adding the representation of the second premise to yield, in valid cases at least, a unified diagram from which the conclusion can be read. Shin's presentation of Venn-I concludes with careful, detailed, clear proofs that if there is a proof via transformation rules of wfd D from a given set of wfds then D is a consequence of that set (soundness) and that if D is a consequence of a given set of wfds then there is a proof via transformation rules of D from that set (completeness).

In spite of its strengths relative to earlier versions of Venn diagrams, Venn-I diagrams are severely limited in their expressive power. In particular Venn-I cannot express propositions resulting from the application of truth-functions. In order to formulate a system of Venn diagrams matching (monadic) first-order language, Shin refashions her system to yield Venn-II. Venn-II is achieved by simple modifications to Venn-I. Syntactically, only one additional formation rule is required, *viz*., that if D_1 and D_2 are wfds then the result of connecting D_1 and D_2 by a straight line is a wfd. Where the straight line had been used only to connect x's, indicating that either the region with the first connected x is nonempty or the region with the second connected x is nonempty, or . . . , Venn-II allows entire diagrams to be so connected, indicating the disjunction of the information represented in the connected diagrams. Since any proposition can be formulated by an equivalent formula in disjunctive normal form, Venn-II, in effect, expresses all compound propositions in disjunctive normal form. "Atomic" diagrams have a single rectangle; compound diagrams consist of two or more atomic diagrams connected by lines. The definition of the cp-relation can now be modified to allow that if A and B are cp-related then either A and B are identical or A and B are in different *atomic* diagrams. The transformation rules of Venn-I are, in effect, augmented by rules that amount to diagram versions of the rule of disjunctive addition in the propositional calculus. For example, given a wfd with an empty region (neither shaded nor inscribed with 'x'), we can derive a compound diagram consisting of two connected atomic diagrams, each exactly like the original except that one has the counterpart of that region shaded and the other has the counterpart of that region inscribed with 'x'. As with Venn-I, Venn-II is provided with proofs of its soundness and completeness.

A minimal requirement for the acceptance of a system of diagrams as an adequate medium for logical reckoning is that such a system be expressively equivalent to a first-order language. To that end Shin establishes that Venn-II is equivalent to a monadic first-order language without identity (*LO*). Thus, for every wfd of Venn-II there is a wff of *LO* such that the set assignments that satisfy that wfd are isomorphic to the structures (models) that satisfy that wff. Moreover, for every wff of *LO* there is a wfd of Venn-II such that the structures that satisfy that wff are isomorphic to the set assignments that satisfy that wfd. Consequently, Venn-II diagrams and *LO* formulae are mutually intertranslatable.

Shin has built Venn-II as an example of a nonlinguistic representational system which is sound and complete. Her claim is that logicians ought to be interested in any such system. But a possible objection might be that such diagrammatic systems are really only more visual but less expressive *linguistic* systems. To counter this objection Shin provides a very clear and convincing case for distinguishing between diagrammatic and linguistic systems of representation. The distinction turns on the thesis that representation systems in general are characterized both by a degree of convention and a degree of reliance on "perceptual inference". Her claim is that conventionality is inversely proportional to perceptual inferences. We could think of a photograp as a representation that requires virtually no conventions for inferring information. Suppose I view a photograph of Clinton standing to the left of his wife. I need heed no particular conventions in making the (perceptual) inferences that she is to the right of him and that he is taller than her. On the other hand, having been told that Clinton is standing to the left of his wife, I can make no such perceptual inferences (all I perceive are a few sounds). The inferences I *can* make are those governed by linguistic and logical conventions (e.g., that Clinton is not to the right of his wife). Pictures, photographs, etc., tend to have a fairly high degree of resemblance to their objects. Diagrams have a smaller degree of resemblance to their objects. Consequently, their use tends to require more conventions. Linguistic systems enjoy no degree of resemblance to their objects; they depend very heavily upon conventions for their use. In comparing diagrammatic and linguistic systems of representation, Shin tries to show that the former can, with the aid of no or few conventions, provide the foundations for perceptual inferences similar to those made given immediate perceptions of reality. In this sense, diagrammatic representation is more natural than linguistic representation. Thus, relations among objects (especially geometric ones) are more naturally represented by diagrams, which, by trading on our geometric intuitions, use the spacial arrangements of symbols to map those of objects. Conjunctive information is more naturally represented by diagrams than by linguistic formulæ. For example, a single Venn diagram can convey the information that all S are M and that all M are P, while two separate formulae are required. As well, the perceptual inferences made in such cases are more immediate and direct than the logical inferences depending on formal conventions. Thus, for example, a single diagram can represent the information that x is to the left of y, which, in turn, is to the left of z. The inference, based on perception, that x is to the left of z is natural and immediate. A first-order language can conjoin the two formulæ into a single conjunctive formula, but the inference will require familiarity with the syntactic and semantic conventions governing the conjunctive device. Finally, diagrammatic systems can represent tautologies and contradictions more perspicuously than can linguistic systems. Since contradictions convey conflicting pieces of information, the capacity of diagrammatic systems to represent conjunctions of information more naturally than linguistic systems do gives diagrammatic systems a greater degree of naturalness. Consider the Venn diagram of 'there is no A and something is an A'. This is simply diagrammed by both shading and x-inscribing the A region. Tautologies can only be represented linguistically by an appropriate string of symbols, but since they convey, in effect, no information, diagrams can represent them simply by not depicting any fact at all.

Needless to say, there are limits on systems of diagrams. Virtually all systems of logic diagrams make use of either closed curves or lines to represent sets. Information about objects is taken to be information about relations among sets of objects. Those relations are modelled by appropriate geometric relations among the closed curves or lines of the diagrams. So the key concept for successful systems of logic diagrams is containment. Intuitively, members and subsets are contained in sets; surfaces determined by closed curves are contained in other surfaces determined by closed curves, and line segments are contained in longer line segments. Leibniz struggled to bring out the pivotal role of containment for reasoning, especially in his "General Inquiries About the Analysis of Concepts and of Truth" [Parkinson *1966*, 47–87]. Leibniz's goal for logic was the unification of all kinds of inference (including those involving categoricals, truth-functions, relationals, and singular sentences). He says ([Parkinson *1966*, 66]): "If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance." Taking categoricals as having the general logical form: subject contains predicate, he went on to construe conditionals (hypotheticals) as having a similar form: antecedent contains consequent. Indeed, valid arguments can be viewed as: premises contain conclusion. One who, like Leibniz, takes containment to be the key logical concept, and who recognizes the obvious way in which lines and closed curves literally contain lines and closed curves, could not ignore Shin's call to the view that diagrams can constitute a viable medium for logical reckoning.

Still, not all relations can be viewed as membership or inclusion. Shin has been careful throughout her book to restrict herself to monadic systems. Relations per se (polyadic predicates) are not considered. And while it may be true that the formation of a system (such as Venn-II) that is provably both sound and complete would help mitigate the prejudice among logicians against diagrams, it will not eliminate that prejudice. What is still required is a system of logic diagrams that can, like the first-order predicate calculus with identity, handle categoricals, truth-functions, relationals, and singulars. (For an attempt to do this using linear diagrams see Englebretsen *1992*], for a nonlinear system see [Rybak & Rybak *1976*; *1984*; *1984a*].)

I have, as well, a less important reservation about this book. In establishing her claim that Venn-II offers more perspicuous representations of set relations, conjunctive information, tautologies and contradictions when compared with the language *LO*, Shin relies on the fact that diagrams, while sharing some features with linguistic representations, also share important features with pictures. Indeed, these latter features, as we have seen, account for our ability to make perceptive inferences. But, of course, the concept of perceptive inference rests on the concept of perception. In her discussion of perception she shows that disjunctive information is not representable in *any* system. In doing so she relies on Barwise and Perry's [*1983*] distinction between the "primary secondary senses of 'show'." Since I take their distinction to be flawed, I take her exploitation of it to be unproductive. Briefly, my complaint with the distinction turns on Barwise and Perry's demonstration of the distinction with the following example. In the sentence 'I saw that the tree was whipping around, so I saw that the wind was blowing', the first token of 'saw' is supposed to be used in its primary (perceptual) sense; the second token is used in its secondary sense. The secondary sense seems to be something like what is known by virtue of perceptual inference from what is perceived (i.e., seen in the primary sense). Now the object expression for the first token of 'saw' is propositional, 'that the tree was whipping around'. This is an expression for the sort of things Barwise and Perry call "situations" (they have often been called 'states', 'states of affairs', 'circumstances', etc.). One who admits that we can perceive trees, clouds, cats, and cupboards, but not situations or states, will shy away from this version of how to distinguish senses of perception and will question theses depending upon it. (I, of course, do not make the stronger (false) claim that there is *no* way to draw distinctions among different senses of perception.)

Finally, one, even less important, complaint. Throughout the book Shin shifts back and forth between 'I' and 'we'. Either one will do. But just one.

I will conclude by offering general praise for a work that really does deserve praise. Even more, it deserves to be read by those mathematicians and logicians who adhere to the general prejudice against diagrams. Shin has gone much farther than anyone in showing how a diagrammatic system can hold its own as a medium for reasoning. For the most part, this book is clear and convincing. And, though I have omitted most of the technical aspects of her work, I should say that Shin's mastery and manipulation of her technical tools is always thorough and lucid. All in all, this is a very impressive, valuable piece of work.

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Raymond M. Smullyan, *Diagonalization and Self-Reference*. Oxford Logic Guides **27**, New York, Oxford University Press, 1994; xv + 396 pp. and Raymond M. Smullyan. *First-Order Logic*. New York, Dover Publications, 1995 (originally published in 1968 by Springer-Verlag.); xii + 158 pp.

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Here are two books, written 26 years apart. The older one deals with a very specific area of logic, the newer one with a common thread that runs through a variety of logical fields. Yet they are recognizably by the same author and display the features, both mathematical and stylistic, that typify Smullyan's writings.

The focus of *Diagonalization and Self-Reference* is the development of a unified framework for the fixed-point theorems that occur in different areas of mathematical logic, such as recursion theory, combinatory logic, and proof theory. To this end Smullyan introduces the notion of a sequential system. To quote his definition, "By a *sequential system* S we shall mean a triple (N, Σ, $\rightarrow$), where N is a set, Σ is a collection of functions of various numbers of arguments, all arguments and values being in N, and $\rightarrow$ is a transitive binary relation on the set of all finite non-empty sequences of elements of N."

Of course, this definition is highly general, which provides for the flexibility to deal with disparate applications. For example, consider the statement: