Grigorii E. Mints, *Selected Papers in Proof Theory*. Naples, Bibliopolis – Edizioni di Filosofia e Scienze and Amsterdam/Oxford/New York, North-Holland, 1992.

Reviewed by

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This collection brings together some of the more interesting and important of Grigorii Efroimovich Mints's papers exploring and exploiting the structure of proofs via the tool of normalization, i.e. putting proofs in normal form.*

In his "Introduction", Mints notes (p. 11) that the "structural theory of proofs [. . .] was born in the framework of Hilbert's program" and that a knowledge of the "definition and elementary properties of Gentzen-type systems" is the basis for an understanding of the work being carried out in these papers. There are four major influences in Mints's work. The first of these influences are the connections between Gentzen sequent calculi and Herbrand quantification, which Mints explored and developed and whereby he was able to give generalizations of Herbrand's Fundamental Theorem using Gentzen's *Hauptsatz* (some of Mints's work along these lines is noted in [Anellis *1994*, 220–221] and [Anellis *1994a*, 148–149]). The second influene is the constructivism of A. A. Markov. Using the extension of this method from Gentzen's classical sequent calculus **LK** to Gentzen's intuitionistic calculus **LJ**, Mints was able (as van Heijenoort [*1971*] showed in his review of several of Mints's papers) to obtain an analogue of Herbrand's Fundamental Theorem for intuitionistic predicate calculus. A third extemely impoirtant influence was

* Mints has also sometimes been transliterated as Minc.

the work of Mints's Leningrad teacher Nikolai A. Shanin. Shanin founded the Leningrad school of constructivism, which was centered at the Leningrad Branch of the Steklov Mathatematics Institute [LOMI] of the Academy of Sciences of the USSR and was most productive in the 1970s (before Shanin's group fell apart in the mid-1980s and Shanin went on in the late 1980s, to develop his own special brand of constructivism, called "Finite Mathematics", or constructivism without constructivist foundations). An important influence specifically in the publication of the book being reviewed here was Georg Kreisel. From Kreisel, Mints acknowledges (p. 16), came the initial proposal to publish a collection of Mints's papers. More important than furnishing the idea for the publication of Mints's papers was Kreisel's influence on the direction of Mints's research program and his choice of techniques in carrying out the program.

Mints was an important member of Shanin's Leningrad school before his "refusenik"** activities led him first to internal exile in Estonia, then to an an expatriate life at Stanford in California. Other members included V. P. Orevkov and Sergei Yure'evich Maslov. Maslov was killed in an automobile accident in Moscow, staged by the KGB, as he was leaving an "interview" with the KGB at the notorious "Lubyanka" (the KGB prison and home office at No. 2 Lubyanka). As a result of losing many of its members, Shanin's group has disbanded and Shanin himself went on to other work. A survey of the work of Shanin's Leningrad school was given by another of its members, Vladimir Lifschitz [*1986*] after Lifschitz emigrated to the United States (see also [Anellis *1994*, 223]).

While the Leningrad school was in operation, a fundamental aim of the group was to apply the techniques they developed to automated theorem proving, or, as Mints described it in the introduction to his book (pp. 11–12), to constructing a program from a deductive synthesis that concretizes normalization of proofs in intuitionistic logic.

In the first paper of this collection, "Finite Investigations of Transfinite Derivations" (pp. 17 –71), Mints developed the techniques of finitizing infinitary derivations, that is, of obtaining finite normalizations of infinite derivations. The original Russian paper, "Finitnoe issledovanie transfinitnykh vyvodov" [*ZNS LOMI* **49** (1975), 67–122; 177–178] was available as a 56-page preprint as early as 1972. The second paper,

** The term applied to dissident Soviet Jews in the 1970s and early 1980s who "refused to participate" in Soviet life or cooperate with the authorities and sought to emigrate from the USSR to the West.

"Normalization of Finite Terms and Derivations via Infinite Ones" (pp. 73–76), uses the finite treatment of infinite derivations to obtain a short proof of the normalization theorem for finite terms and derivations. Together with the third paper, "A New Reduction Sequence for Arithmetic" (pp. 77–96), translated from "Novaya posledovatel'nost' reduktsii dlya arifmetiki" [*Issledovaniya po konstruktivnoi matematiki i matematicheskoi logiki*, **VIII** (Leningrad, Nauka, 1979), 106–130)], normalization techniques different from the traditional ones are developed.

The next four papers deal with "unwinding proofs", that is with the extraction of an explicit realization of existential theorems from the proof. The best known paper of this group is the fifth in the collection, "On E-theorems" (pp. 105–115), translated from "O E-teoremakh" [*ZNS LOMI* **40** (1974), 110–118], which for years had circulated in manuscript form as "Über Existenzsätze" a translation by Egon Börger and M. Vogel. It began a series of studies on the stability of the numerical realization of existential theorems. The paper "Stability of E-theorems and Program Verification" (pp. 117 –121) applies the results of the "On E-theorems" paper to automated theorem proving, in particular, as the title states, to program verification. These two papers together concerned familiar systems of intuitionistic logic, whereas the paper "Heyting Predicate Calculus with Epsilon Symbol" (pp. 97–104) examines proof-theoretically a system of intuitionistic predicate logic devised by Albert G. Dragalin using model-theoretic tools. "Normalization of Natural Deduction and the Effectivity of Classical Existence" (pp. 123–146) applies normalization to extract numerical content from classical proofs.

The next papers apply normalization — more accurately, applies normal form theorems — to specific problems. In "On Novikov's Hypothesis" (pp. 147–151), Mints proves that the Gödel-Tarski translation, in which the modal operator $\Box$ of necessity is prefixed to all subformulæ, is "sound and faithful" for first-order arithmetic. The papers "Proof Theory and Category Theory" (pp. 157–182), "Closed Categories and the Theory of Proofs" (pp. 183–212) [a translation of "Zanknutye kategorii i teoriya dokazatel'stv" in G. E. Mints & V. P. Orevkov (eds.), *Teoreticheskie primeneniya metodov matematicheskoi logiki, II*, *ZNS LOMI* **68** (1977), 83–114], and "A Simple Proof of the Coherence Theorem for Cartesian Closed Categories" (pp. 213–220), applied proof theory to coherence theorems in category theory.

The last paper in the collection is "Lewis' Systems and System T (1965 – 1973)" (pp. 221–294), which seems to stand alone and to be out of place in this volume. It is an expository and historical survey of results in proof theory and model theory for modal logics for the period beginning with the publication of part II of Kripke's "Semantical Analysis of Modal Logic" [Kripke *1965*] and ending [actually] in 1974. It seems to be out of place in this collection, *not* because it is concerned with modal logics (it had long ago been shown by Boolos that modal logic is an excellent tool for studying provability, that "when modal logic is applied to the study of provability, it becomes provability logic" [Boolos *1993*, ix], thus a subject not completely out of place in a collection on constructing a program from a deductive synthesis that concretizes normalization of proofs), but because it is essentially and primarily an expository and historical survey rather than an original piece of research as are the other papers included in this collection — although it certainly does also include many results due to Mints himself. Thus, this survey paper on Lewis's systems and the system **T** stands alone in this collection. The principal concern in this paper are deductive questions relating to **S1**– **S5** and **T**; and that emphasis gives some justification to its inclusion in a collection of papers on proof theory. I would contend that, since one expository and historical survey was included, it would also have been at least equally worthwhile to open the collection with Mints's survey "Teoriya dokazatel'stv (arifmetika i analiz)" [*Itogi Nauki i Tekhniki (Algebra, Topologiya, Geometriya)* **13** (1975), 5–49; English translation as "Theory of Proofs (Arithmetic and Analysis)" in *Journal of Soviet Mathematics* **7** (1977), 503–531], since the proof theory paper is also a general expository survey and provides a crucial historical and conceptual-expository background for the other papers included here as well as to the subject as a whole as it existed in the mid-1970s.

If I have any other "complaints" about this collection, they are minor. A complete bibliography of Mints's writings would have been a useful tool to locating the papers chosen for the collection within the general framework of Mints's research program. And, at the very least, biblio-graphical information on those papers that were included in this selection should have been provided.

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