

Jarmo Pulkkinen, *The Threat of Logical Mathematism. A Study on the Critique of Mathematical Logic in Germany at the Turn of the 20th Century*, Frankfurt/Berlin/Bern/New York/Paris/Wien, Peter Lang, 1994, 187 pp.

Reviewed by

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From the subtitle of Pulkkinen's book we might expect to learn only about the fortunes of mathematical logic in the German academy at the turn of the century. However, a cursory glance at its contents reveals that only one chapter out of eight (Chapt. 5) is devoted to its public reception. The remaining chapters treat the history of logic in Germany between 1830 and 1920 and discuss the relationships logic was seen to bear not just to mathematics but also to such various disciplines as psychology and linguistics. The investigation of those relationships is a prerequisite to the main issue because a good number of arguments against the mathematization of logic were seen to depend on psychological or linguistic considerations. This becomes particularly evident in the last three chapters, which examine the criticisms of mathematical logic advanced by Fritz Mauthner, Heinrich Rickert and Theodor Ziehen.

The author traverses the history and philosophy of logic, psychology, and linguistics and in doing so pays close attention to matters epistemological and ontological. One might therefore be tempted to compare the treatment Pulkkinen offers, say of the history of psychology or linguistics, with other discussions of them in the literature and then find the present work decidedly scanty. Any such judgment would be unfair though because Pulkkinen's interest lies principally in the history

of logic, and he explores a period of it that is frequently neglected. To be sure, every recent book on the history of logic devotes much space to Frege, for example, those by the Kneales [1962], Bochenski [1956] and Blanché [1970]. But what about the logic of Frege's German contemporaries? After the scientific community accorded full and public recognition to Frege's *Begriffsschrift* as a masterpiece, if not the masterpiece of nineteenth century logic, nearly all interest in the work of his German contemporaries on their own account has waned. If we limit our attention to the books by the Kneales, Bochenski and Blanché, only Blanché's deals with their work, and then only briefly. (Other books on the history of logic that treat their work fully were published before Frege was seen to be the dominant logician of his time.)

Who then are these figures neglected in the recent historical literature? They are Christoph Sigwart, Wilhelm Wundt and Benno Erdmann, whom Pulkkinen presents as psychologistic logicians (see pp. 27–31); they are the formal logicians Hermann Lotze, Julius Bergmann and Friedrich Albert Lange (see pp. 31–33); finally there are Ernst Schröder, Andreas Heinrich Voigt, Reinhold Korselt, Karl Eugen Müller, and Joseph Hontheim, who are classified here as mathematical logicians (see pp. 33–35). Frege belongs to this last group.

Turning to the philosophical literature, the logic of Frege's German contemporaries, almost totally ignored by Dummett [1973], has recently received a good deal of attention in Sluga [1980], Picardi [1987], [1994] and Carl [1994]. However, all these works are specifically concerned with illuminating Frege's own views. To the best of my knowledge, the only work dedicated specifically to the logic of his close contemporaries is Ferriani's [1982], which examines the debates about logical psychologism in Germany up to the work of Ziehen. And it is with Ziehen that Pulkkinen concludes his own book. But, whereas Ferriani discusses that issue in its own right, Pulkkinen's concern is to show how psychological conceptions of logic were used against mathematical logic.

The psychologistic, formal, and to some extent, mathematical logicians all viewed logic as a normative discipline and called it 'technology of thought' (*die Kunstlehre des Denkens*) (see p. 21 and p. 171). This conception of logic was used to argue against the acceptability of mathematical logic. For instance, Lotze thought traditional logic superior to the mathematical variety (the Boolean one in particular) because it offered a better representation of the laws of thought (see p. 99). A similar argument was used by Wundt (see p. 105). Moreover, algebraic logic, *viz.* Boolean logic, represented and developed in Germany mainly by Schröder, is basically the logic of

class, that is, an extensional logic and as such misses, or at least fails to do justice to, the intensional character of thought. This was the reaction of the early Husserl and of Carl Prantl (see Chapt. 5.3), although later Husserl, like Frege before him, went on to criticize vehemently the psychological conception of logic (see Chaps. 2.3 and 2.4).

Why was mathematical logic seen as a threat to philosophy anyway? According to Pulkkinen (see Chapt. 5.4), it was because mathematics threatened to absorb logic, a central part of traditional philosophy, maybe even its very core. So some philosophers, notably Melchior Palágyi, Paul Natorp and Martin Heidegger, advanced a number of arguments that insisted on a sharp separation between logic and mathematics. One of the main figures of the neo-Kantian Baden school, Rickert, was among them and Pulkkinen devotes Chapter 7 of his book to his views. In particular he explores Rickert's two main arguments against mathematical logic.

The first of these is that neither the objects of logic nor those of mathematics are real, but whereas the objects of logic belong to the realm of values (the realm of the transcendental 'ought'), the objects of mathematics belong to the realm of the ideal. This ontological distinction was supposed to make it clear that logic can not be treated mathematically. The second argument is based on a conception of logic as a 'theory of science'. This conception was shared by the later Husserl and by Natorp, and it led Rickert to advocate the separation of logic from mathematics, failing which a theory of science, a metascience, would be confused with a science proper.

While Rickert is a relatively well-known figure in the German intellectual firmament, the same can not be said of Mauthner or Ziehen, and Pulkkinen deserves our gratitude for charting the thoughts of such minor writers and enabling us to appreciate how widespread was the hostility to mathematical logic in Germany at the beginning of the century. In Chapter 6, we are told that Mauthner claimed that mathematical logic is unable to generate any new knowledge, and also that it is incapable of overcoming the shortcomings of natural language. Pulkkinen does not mention the fact that the former is an old criticism addressed even to the traditional (syllogistic) logic at least from the time of Descartes. However, his treatment of the second criticism and the accompanying discussion of Mauthner's philosophy of language and its connection with profound changes in linguistics deserves high praise. The final chapter deals with Ziehen's critique of mathematical logic and its dependence both on the *Kunstlehre*-conception and on the theory of science-

conception.

The Threat of Logical Mathematics is a stimulating book to read, being full of engrossing details, social and political as well as philosophical, about one of the most intricate periods in the history of philosophy. Logic, psychology and linguistics were becoming autonomous sciences: they constituted threats to philosophy as well as sources of stimulation for it. Even readers who are familiar with the history of logic will find ideas for illuminating numerous issues such as the indifference or downright hostility that greeted Frege's work. Because of its broad sweep, the book will appeal to a wider audience, being of interest to anyone willing to explore the often overlooked tensions between philosophy, logic, mathematics, psychology and linguistics that characterized German thought at the turn of our century.

Now, I turn to a number of reservations I have about Pulkkinen's book. First, he does not mention Kant's conception of logic as 'closed and completed' [1787, B viii]. This conception may be one of the reasons for the negative German view of mathematical logic. If logic is properly Aristotelian, as Kant believed, and if it is closed and completed, then no development of it is possible, be it mathematical or not. Support for this view is provided by Prantl, who spent his life writing his four volume *Geschichte der Logik im Abendlande* [1855-1870]. This work, whose importance led to reprintings in 1927 and 1955, is not mentioned by Pulkkinen, yet it constitutes a huge part of history of logic devoted to showing that Kant was right, and effectively that logic has no history!

Secondly, in his conclusive remarks Pulkkinen says (p. 170) that "[l]ogical algebra was introduced to the wider German philosophical public for the first time in 1878 when the neo-Kantian Alois Riehl published his article, 'Die englische Logik der Gegenwart'. In his article Riehl discusses mainly the logical algebra of W. S. Jevons. The fate of this article brings forward the difference between the developments in Germany and Britain. In Britain Jevons achieved a considerable contemporary reputation and helped logical algebra to get a larger philosophical audience. In Germany, however, Riehl's article was not able to win the sympathies of the philosophers. On the contrary, it prompted for the most part only hostile reactions". This judgment about the difference between the development in Germany and in Britain is quite incorrect: when mathematical logic made its first appearance in Britain in the 1840s, thanks to Boole and De Morgan, it was heavily criticized. The public response to Boole's two books (*The Mathematical Analysis of Logic*, 1847, and *An Investigation of the Laws of Thought*,

1854) was so dismal as to lead him to envisage a third book in which mathematics was to be relegated to the footnotes. So, when it first appeared mathematical logic enjoyed no better treatment in Britain than it received in Germany.

Thirdly, Pulkkinen's presentation of the thought of some of his authors is considerably less than exhaustive. For example, he claims that Frege had basically two arguments against psychologism. First, that logic should be distinguished from psychology because the former is a normative, and the latter a descriptive, science (see pp. 50–52). And second, that only if we establish that distinction shall we be able to recognise 'the objective and not actual' (the third world), beside the 'objective and actual' (physical world) and the 'subjective and actual' (the psychological world) (see pp. 52–54). This picture is too narrow. Frege had more arguments than just these: he was set against the genetic explanations of logical concepts, against the identification of logical contents with psychological ones, against any denial of the necessity of logical laws, and against the provision of any psychological justifications for them (see Vassallo [1995]).

Then, too, some of Pulkkinen's exposition is confusing. For example, he claims that "Frege does not believe that the laws of logic have any specific relation to thought" (p. 51) and that "Frege believes that his conceptual notation . . . reflects correctly the structure and relations of thoughts" (pp. 82–83). The two claims seems quite contradictory. Was Frege himself inconsistent? Not at all. The problem is that Pulkkinen translates both 'Das Denken' and 'Der Gedanke' by 'thought'. It is better to translate 'Das Denken' by 'thinking' and to reserve 'thought' for 'Der Gedanke'. According to Frege, the thought (Der Gedanke) is the logical content which is to be distinguished from the psychological content, *viz.* from ideas or representations. The second claim just cited is correct enough: there 'thought' is 'Der Gedanke'. But the first claim is surely wrong if Pulkkinen has in mind 'thought' as 'Der Gedanke'. And, it is still imprecise if by 'thought' he means 'Das Denken', *viz.* 'thinking'. According to Carl [1994, Chaps. 1 and 2], Frege agrees with his contemporaries (Lotze, Sigwart, Wundt) on logic having a specific relation to thinking. But, unlike them, he requires that a sharp distinction be drawn between logical thinking and psychological thinking. So Frege does believe that the laws of logic have specific relations to logical thinking, but not to psychological thinking. En passant, all this suggests the need to translate 'Kunstlehre des Denkens' not, as Pulkkinen does, as 'technology of thought' but as 'technology of thinking'. Furthermore, this may be a reason why the standard books on

the history of logic show little interest in the logic of Frege's contemporaries. In fact, the conception of logic as a 'technology of thinking' is worthy of attention in any history of logic only in so far as it is clear that it is intended as a 'technology of logical thinking'. But at least for Lotze, Sigwart, and Wundt it is not at all clear that this was how they saw it (see Carl [1994] Chapt. 1). If logic is viewed at bottom as a 'technology of psychological thinking', this is much more interesting for a psychologist than for a logician. So, Pulkkinen is only partially right in attributing the slight interest in the logic of Frege's contemporaries to the fact that "these studies have usually be written from the viewpoint of mathematical logic, presenting only those logicians who have been important to its development, while the rest have been forgotten" (p. 11). The point is that these studies have usually be written from the strict viewpoint of logic, and not of psychology. Of course, those psychological conceptions of logic, as well as any other conception of it, deserve to be carefully examined in any book specifically concerned with the history of philosophy of logic. As far as I know, this has not yet been done.

The last grumble I have is that Pulkkinen very rarely assesses the criticisms of mathematical logic which he reports, and leaves his reader with a sense of frustration. To cite one instance, he writes:

In his *Der Gegenstand der Erkenntnis* Rickert distinguishes himself from the 'ontological' tradition in logic which treats logical as something ideal. In his opinion the tradition was started by Bolzano and its most original contemporary representative was Husserl. To Rickert the reason why these thinkers had misunderstood the essence of logic was that they had based their logics not on sentences . . . but on mere words. In Rickert's opinion the meaning of words can be understood as something which exists ideally. However, Rickert believes that we should take, not the meaning of words, but the meaning of sentences as the basis of logic. He believes also that the true character of logical formations as values and the impossibility of the 'ontological' point of view in logic become evident as soon as we analyze the meaning of true sentences. (pp. 142-143)

Well, any reader not well acquainted with Rickert, but familiar enough with Frege, cannot fail to recall that although Frege analyzed the meaning of sentences (recall his famous context principle), he still espoused the 'ontological' point of view and considered logical contents to be somehow ideal. So, how is it possible to justify these claims of Rickert's given that Frege testifies to the possibility of maintaining the

'ontological' point of view together with the analysis of the meaning of sentences? Avoiding any assessment of Rickert's beliefs, Pulkkinen does not help the reader to gain a full understanding of this neo-Kantian thinker.

Considering the obvious strengths of the book, such flaws are distinctly minor. Pulkkinen has very usefully captured the main lines of German criticism of mathematical logic from its first appearance to the beginning of the First World War.

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