REVIEWS

Charles S. Peirce, *Writings of Charles S. Peirce: Chronological Edition. Volume 5, 1884 – 1886*. Edited by Christian J. W. Kloesel. Bloomington/ Indianapolis, Indiana University Press, 1993. xlvii + 623 pp. $65.00.

Reviewed by

JAMES W. VAN EVRA

Department of Philosophy
University of Waterloo
Waterloo, Ontario
N2l 3G1, Canada

The years covered in this volume were among the most difficult in the life of Charles Sanders Peirce. First, Johns Hopkins failed to renew his contract, thereby bringing Peirce's academic career to a quick halt. Facing bleak prospects on that front, he turned to science as the natural alternative. But catastrophe lay in wait there too, for after a short tenure as head of the Office of Weights and Measures, a federal administration bent on reducing government involvement in the public sector, plus other problems within the larger organization of the Coast and Geodetic Survey combined to deliver Peirce into the wilderness. All of this left a heavy mark on him; his health afterward would never be as good as it had been in earlier years. Yet somehow he managed, in the midst of chaos, to produce over one hundred papers and reports during the period. And while the papers cover the usual daunting array of topics, among them are some logical works of particular importance, for it was in this period that Peirce developed his full theory of modern quantification.

Accounts of the origin of modern quantification often let Frege steal the show. They point out that he got there first (*Begriffsschrift* (1879)), with a full rendering of quantificational logic which contained, within a single theory, both the quantification of individuals in first-order, as well

as second-order quantification of functions. Given the additional (often implicit assumption that our current version of quantification stems uniquely from him, it seems reasonable to ask why we should be concerned with another version of something which had already been discovered.

In addition to the fact that anything worth discovering is worth discovering again, making even the n^{th} independent discovery interesting, there are two main reasons for affirming the importance of Peirce's version of quantification. First, as Gregory Moore has shown ("The Emergence of First-Order Logic," *Minnesota Studies in the Philosophy of Science*, Vol. 11 (1988), pp. 95–135), Peirce's version of quantification left a clear mark on the development of logic well into the twentieth century, as is evident in the appearance of his distinctive notation in the works of Schröder, Lewis, Skolem and Löwenheim.

The second (and more immediate) reason for paying attention to Peirce's version of quantification is that it is possible, by working through a series of papers in this volume, to trace the course of a scientific idea as it is being developed.

In a paper published in *Studies in Logic* (1883), Peirce's student O. H. Mitchell provided some initial ideas which served as a starting point for Peirce's subsequent development of quantification. Specifically, Mitchell placed $\prod$ and $\sum$ (product and sum) before Boolean expressions containing the component expressions F_1 and F_u (All U (the universe of class terms) is F (where F is identified as a polynomial of class terms), and Some U is F respectively.) Mitchell also generalized De Morgan's concept of a universe of discourse by his recognition of a "hyperlogic," *i.e.* one interpretable in potentially many universes.

For his part, Peirce began by producing the basic components of the full theory. Thus in a fragment and the start of another paper (item 21, titled (by the editors) "Fragment on the Algebra of Logic," and item 22, "On the Algebra of Logic (Second Paper") written in the summer of 1883, he recognized that Mitchell's notation could be used selectively to characterize portions ("dimensions," in Peirce's terminology) of the universe of discourse. This permitted the construction of expressions such as

$$\prod_i a,$$

which stands for 'Every i (*i.e.* every member of the universe of discourse in dimension i) has the mark a,' and

$$\Sigma_i \Pi_j \, l$$

for 'Some i is in the relation l to all js.' In other words, Peirce was on the verge of recognizing pronomial cross reference to individuals linked to quantifiers. In addition, in these works Peirce also recognized that $\Pi \prec \Sigma$ holds in a non-empty universe.

The full theory of quantification appeared a year later in a remarkable work entitled "On the Algebra of Logic: A Contribution to the Theory of Notation" (item 30). In it, Peirce explicitly recognized the significance of what Mitchell had only glimpsed, *i.e.* that Mitchell's notation was ideally suited not only to gross quantification, but also to keeping straight, through the use of "indices," *i.e.* pronouns and other similar pointers, which items in the expression are associated with which quantifier. This permitted the formulation of complex expressions such as

$$\Pi_i \Sigma_j \, l_{ij} \, b_{ij} \, ,$$

which stands for 'Everything is at once the lover and benefactor of something.'

Having formulated first order quantification, Peirce then went on to introduce a version of second order quantification by first defining the identity relation (1_{ij}) in a typically Leibnizian fashion:

$$1_{ij} = \Pi_k \, (q_{ki} \, q_{kj} + \bar{q}_{ki} \, \bar{q}_{kj} \,)$$

where q is a predicate token. Peirce tied the introduction of this second order notation to the need for a way to treat classes collectively, which in turn permits the possibility of creating relative terms with infinitely many indices.

Tracing the development of quantification in this volume also makes it clear that while the roots of the theory lie within standard algebraic logic, by the time it was fully worked out, Peirce viewed it as a new and independent stage in the development of the subject. In his words, "The algebra of logic should be self-developed, and arithmetic should spring out of logic instead of reverting to it" (p. 169).

In addition to the works already mentioned, there are other significant logical works in the volume. "Qualitative Logic," (item 54), for instance, is a lengthy portion of Peirce's projected general logic (commonly known as the "Grand Logic"), and there are two other

shorter works ("The Logic of Relatives: qualitative and quantitative," and "An Elementary Account of the Logic of Relatives" (items 55 and 56)) which somewhat extend Peirce's theory of quantification. All are valuable additions to the logical corpus being reconstructed in the first four volumes.

Still more of value is to be found in the non-logical portion of the book (which consists of approximately three-quarters of the entire volume), including articles on thinkers and their works (Kant, Clifford, Abbot and others), articles on key aspects of Peirce's own thought (the categories One, Two, Three, for instance), and much else as well.

Like its predecessors, this volume is executed with editorial precision and acumen. The selection of works clearly reveal patterns of development in the many lines of Peirce's thought, and the mechanics of their presentation is of excellent quality. The introduction by Nathan Houser is outstanding. Houser describes the events in Peirce's life during the period, and provides a guided tour of the works in the volume, pointing out significant accomplishments, and placing them in the larger context of Peirce's intellectual development. All of this is done in a clear, measured and objective manner.

There is, finally, a sad irony concerning the publication of the series of which this volume is so valuable a part. The same political conditions which played a prominent role in forcing Peirce out of a promising career in science now threaten the publication of his works. The National Endowment for the Humanities rejected the penultimate application made by the Project for continued funding, and recently informed the Project that it has been excluded from the current competition for funds within the Program for Editions. Charles Sanders Peirce is one of a small number of individuals responsible for shaping American thought. It is inconceivable that the only scholarly edition of his works in existence would be allowed to languish.