

Selected Logic Papers, Enlarged Edition by W. V. Quine. Cambridge, Mass., Belknap Press of Harvard University Press, 1995. xii + 307 pp., paperback.

Reviewed by

JONATHAN WEINBERGER

University of California-Berkeley
Group in Logic, 731 Evans Hall
Berkeley, CA 94720-3840, USA

email: jonathan@math.berkeley.edu

Quine's updated text is remarkable in its ability to fulfill the traditions of two noble styles of exposition at once. *Selected Logic Papers* is both a retrospective of a century of hot and heady development in logic and a work that is valuable for a practitioner in the field.

In its first rôle, as a compilation of some of the century's best published opinions and proofs in formal logic, this work shines. And it shines more brightly than the 1966 edition of *Selected Logic Papers* in virtue of its extension, composed of eight added essays representing ideas communicated since 1982. "MacHale on Boole" first appeared in the *Times Literary Supplement*; "Free Logic, Description, and Virtual Classes" was, writes Quine, written for a celebration of Hugues Leblanc's seventieth birthday.

The first essay of the work, unchanged from a shorter edition, is "Whitehead and the Rise of Modern Logic". It dates from 1941, but remains an artfully constructed introduction to logic as it grew out of notions of universal algebra. Quine is a philosopher and does not shy from philosophical opinions of others. Early in "Wittgenstein and Modern Logic" he cites Wittgenstein's *Tractatus*: "Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing" [1922, 139]. He works to undercut this sentiment in a manner reminiscent of that in "New Foundations for Mathematical Logic" [1937], which is absent, still, from this compendium. Quine addresses this omission, noting that

“New Foundations” is widely available elsewhere.

“Peirce’s Logic” is a foray into the history of mathematics. Quine argues that Pierce, rather than Frege, should be seen as the “father” of quantification. The essay examines work leading up to Russell and Whitehead’s *Principia* and points to Peirce’s slow approach to the theory of quantification. The rationale for putting Pierce above Frege is an interesting insight into Quine’s personality:

But it remains instructive to trace the slightly subsequent [to Frege’s *Begriffsschrift*] and independent emergence of quantification through Peirce’s writings. Frege affords no such genetic insight, since quantification is already full fledged in his maiden publication. In an important sense, moreover, Peirce and not Frege was indeed the founding father; for Peirce’s influence was continuous through Schröder’s work, with side channels into Peano, culminating in *Principia Mathematica*. Frege had been a voice crying in the wilderness. (p. 259)

Despite Frege’s earlier publication and in virtue of his influence, Peirce is seen as fatherly. This is probably a fair assessment, for, as Quine notes, Frege’s work was poorly received in its time. Papers like this one give the reader a sense for the subject that can’t be had reading only the technical publications.

The thirty-first and final essay of the volume is notable: rather than to a logical topic, Quine brings his time-tested insight and rhetorical pre-eminence to bear on the recently renewed Fermat’s Last Theorem. Quine retells the story of FLT as given in his *Quiddities*, but with less detail, and presents a sorting problem analogous to FLT. In an addition to what he reported in the 1987 work, Quine shows a new expression for the general form of the Pythagorean triples. Every triple of the form, and it is a nicely symmetrical form, $\langle x, y, z \rangle = \langle a + \sqrt{2ab}, b + \sqrt{2ab}, a + b + \sqrt{2ab} \rangle$ satisfies $x^2 + y^2 = z^2$. This is but one example of the import to Quine of elegance. As Georg Kreisel wrote in his review of the first edition of this work “The author’s style is remarkable: Though he obviously puts a lot of effort into the choice of terminology, he always has enough energy left to give complete and detailed proofs” [1967, 1355]. Elegance permeates his mathematics and this exposition.

But for the sake of elegance Quine has never neglected his duty to relevance. Through his reviews and his creative work, his career has been, in this modern age, one of nearly unprecedented addition. Quine more than anyone since Frege, save perhaps Tarski, has advanced mathematical logic. This aspect of Quine is displayed adeptly in

Selected Logic Papers. Notable papers include his “Definition of Substitution”, “Set-theoretic Foundations for Logic” (recall that in 1970, he argued persuasively that set-theory should not be seen as *part of logic*), and “Logic Based on Inclusion and Abstraction” (the last two originally published in the then-new *Journal of Symbolic Logic*), and “Completeness of the Propositional Calculus”. This paper, while appearing seventeen years after Post’s proof of the completeness of the propositional calculus, is notable in that it proceeded in such a way as to make every formula recursively definable in terms of a primitive notion of falsehood; it stands as an example of the utility of the Tarski-Bernays formulation of propositional logic.

Of particular interest is the essay “Free Logic, Description, and Virtual Classes”. In language at time reminiscent of the 1948 “On What There Is”, Quine takes up the quantification of free logics. This essay shows us that Quine, while staying disappointingly quiet on issues of deviant logics, remains concerned with logical issues of current import. Free logics (those that countenance non-denoting singular terms and don’t require any existent entities at all) find their way into popular formalizations of, for instance, modal logics, where their flexibility in dealing with non-denoting terms makes them ideal for discussing the truth of predicate assignments in a particular, potentially other-than-actual, world.

Quine closes “The Inception of ‘New Foundations’” with a tribute to those many who followed him, and then continued forward along the path his bright insight suggested. He writes: “I am dazzled by the deep and ingenious discoveries that various of you have made regarding ‘New Foundations’” (p. 289). Intellectual foment followed other works too. Quine’s logic re-legitimized ontology; the analytical tradition in American philosophy which owes so much to him in fact honors him in its language and attitudes. Such is our acknowledgment of the bedazzling opus he has shared with us. *Selected Logic Papers* takes its rightful place alongside his other collections, *The Ways of Paradox* and *From a Logical Point of View*, as an indispensable verbal portrait of the analytical landscape, as crafted by one of the most able masters.

BIBLIOGRAPHY

KREISEL, Georg. 1967. Review of Quine’s *Selected logic papers*, *Mathematical Reviews* 34 (MR# 7333).

QUINE, W. V. 1976. *The ways of paradox and other essays*, Cambridge, Mass., Harvard University Press.

— . 1980. *From a logical point of view*, 2nd ed., Cambridge, Mass., Harvard University Press.

— . 1987. *Quiddities*, Cambridge, Mass., Harvard University Press.

WITTGENSTEIN, Ludwig. 1922. *Tractatus logico-philosophicus* (F. P. Ramsey and P. K. Ogden trans.), London, Routledge & Kegan Paul.

Tadeusz Kwiatkowski, *Skicze z Historii Logiki ogólnej*. Lublin, Instytut Wydawniczy "Daimonion", 1993.

Reviewed by

IRVING H. ANELLIS

Modern Logic Publishing
2408¹/₂ W. Lincoln Way (Upper Level)
Ames, IA 50014-7217, USA

email: F1.MLP@ISUMVS.IASTATE.EDU

The present work is titled an "outline" or "sketch" of the history of general logic. Nevertheless, it is *not*, as one might expect from the title, a continuous, if abbreviated or condensed, narrative history of logic such as one finds, e.g., in the text of Kneale & Kneale (*The Development of Logic*). It is instead better — i.e. more accurately — viewed as a collection of disparate essays on various aspects and phases of the history of general logic, where by "general" is meant in this context traditional or classical logic, i.e. syllogistic logic or the logic of terms.

The first section, roughly 30% of the nearly 400 pages of text, is devoted (according to its title) to the history of "the logic of the ancients", with the focus, however, being entirely on Aristotle. The second section, of roughly 40%, examines several topics in the history of logic in Poland, including in particular, logic in the school program of the Public Education Commission of the mid-18th century; from thence, attention shifts to recent times and consideration of the character and