

DISCUSSION AND QUERIES

A NOTE ON "THE DEVELOPMENT OF MULTISSET THEORY"  
[BLIZARD 1991]"

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Blizard's survey paper [Blizard 1991] is an up-to-date and lucid account of the development of multiset theory. However, to my knowledge, a few significant pieces of work from the early history of multiset theory may be added Blizard's account.

Angelelli found a very early reference to the multiset concept in the work of Marius Nizolius (1498 – 1576). In particular Angelelli has vindicated a number of reflections of Leibniz and others on Nizolius' notion of *multitudo*, saying [Angelelli 1965, 319] that "...it is not perhaps clear whether Nizolius' *multitudo* comes closer to 'class' or 'heap'," but adding in a footnote [Angelelli 1965, 320] that "Nizolius' *multitudines* might still be heaps in the sense of Quine and Goodman."

Brink [1987] and Hailperin [1986] find the notion of multiset present in the work of Boole. It is noted in [Brink 1987, 1], with emphasis (rightly, I think) that [Hailperin 1976] "...is the link to the historical background of multisets. For indeed multisets do have a history," and that the credit for introducing them "should go to George Boole's [1854] *Laws of Thought*." This emphasis is missing in Blizard's [1991, 330] "attempting to 'make sense' of Boole's algebra of logic." Hailperin has justified, by introducing 'signed heaps' the assertion that Boole's *Laws of Thought* may be interpreted as a treatise dealing with multisets. More specifically, in [Hailperin 1976, 88], we are told that "*To obtain a meaningful interpretation of Boole's system we have to use not the notion of a class but that of a heap*," and in [Hailperin 1986, 136], that "*To obtain a meaningful interpretation of Boole's system we have to use not the notion of a class (class = set) but that of a multiset*." However, on the question of whether Boole himself really had this

interpretation in mind, Brink [1987, 2; cf. Brink 1978] notes that "Hailperin is silent on this point, but strong evidence comes from another quarter: at least one eminent 19<sup>th</sup> century logician read Boole in precisely the right way, and that was Charles Sanders Peirce." In this connection, [Brink 1987, 2; cf. Brink 1978] states that "the axiom system for Boole's original system is an axiom system for signed heaps," where "a heap is a collection of objects in which more than one example of an object may occur... [and] can be represented notationally by attaching a numerical coefficient to each of its elements, this coefficient giving the multiplicity of that element in the heap" [Brink 1978, 293] and, "by allowing the coefficients to take on negative as well as positive values one obtains the notion of a *signed heap*" [Brink 1978, 293–294]. Brink [1987, 1] suggests calling the elements of multisets 'multiples' instead of 'copies'.

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