ℵ Modern Logic ω

IS THE CONCEPT *HORSE* AN OBJECT?

ANTONI DILLER

School of Computer Science
University of Birmingham
Edgbaston, Birmingham B15 2TT
England, U.K.

1. An Awkwardness of Language.

In his paper "Über Begriff und Gegenstand" ([*1892*] hereafter referred to as "On Concept and Object"[1]) Frege replies to a number of criticisms of his views about concepts made by Kerry. The details of Kerry's criticisms of Frege's *Grundlagen* need not concern us here, but in the course of the discussion Frege [*1892*, 196] comes across what he calls 'an awkwardness of language' that arises when we try to talk about concepts. One of Kerry's examples was the proposition, 'The concept "horse" is a concept easily attained.' He said that the concept 'horse' was an object which fell under the concept 'concept easily attained.' This has the consequence that the concept 'horse' is not a concept, but an object, and this is the awkwardness that Frege refers to. This result has come to be known as Frege's paradox, but it is only a paradox in the sense that it is highly counter-intuitive; it is not — nor does it entail — a self-contradictory proposition.

Given that this result is counter-intuitive, why did Frege accept it? The answer lies in the criterion that he laid down in *Grundlagen* for distinguishing between expressions that stand for objects and those that stand for concepts. He, of course, expressed this for German phrases, but it can be reformulated for English phrases as follows: A singular substantival phrase governed by the definite article stands for an object, whereas a word 'used with the indefinite article or in the plural without any article' is a concept-word.[2] According to this criterion the expression 'the concept "horse"' is a singular term that stands for an object and as no object is a concept in Frege's scheme of things it follows that the concept 'horse' is an object and not a concept.

Dummett has claimed — on p. 54 of *Frege* [*1973*] — that Frege's criterion is inexact both ways and he adds that

> Frege was perfectly well aware that there are expressions satisfying this criterion which he would not wish to admit as proper names, and others which fail the

[1]Page numbers in the citations are to the German originals.

[2]Section 51. See also section 57 and the footnotes to sections 66 and 68.

> criterion which he would wish to admit: but he was content to allow the whole distinction between proper names and expressions of other kinds to depend upon intuitive recognition, guided only by the most rough and ready of tests.

That is a misrepresentation of Frege's attitude. He thought the criteion was almost watertight. The only exception he could think of as regards the indefinite article was an obsolete German formula for a councillor and as regards the definite article the only troublesome cases he mentions are propositions like, 'The horse is a four-legged animal' [Frege *1892*, 195–196]. It is because Frege thought that the criterion had hardly any exceptions that he held the proposition that the concept 'horse' is an object to be true. If he really did have a low opinion of this criterion, there would have been little reason for him to categorize phrases like 'the concept "horse"' as singular terms.

Although Frege accepted this consequence of his criterion it is clear that he was not entirely happy with it, for he wrote [*1892*, 204]:

> I admit that there is a quite particular obstacle in the way of an understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way — who does not begrudge a pinch of salt.

This further shows the high regard he had for his criterion; he was willing to accept this consequence of it rather than give it up.

Before proceeding I just want to make two points:

(I) When Frege's paradox is discussed it is usually presented as a problem having to do with his understanding of concepts, but it is a problem that also affects his account of relations and, indeed, functions in general. In "On Concept and Object" Frege explicitly says that the problem arises for relations[3] and also for functions.[4]

(II) Up to now I have used expressions like 'the concept "horse".' This was because I was following Kerry's usage in discussing his example, as indeed does Frege. From now on, however, I shall either use expressions of the form 'The concept horse' or of the form 'the concept x is a horse.' The second alternative is more precise but it is also more awkward. It is probably for this reason that Frege preferred the first alternative in cases where it was possible to isolate a concept-word.

---

[3] He says [*1892*, 205] that 'the words "the relation of an object to the concept it falls under" designate not a relation but an object.'

[4] He says[*1892*, 198, in footnote 3] that the expression 'the function $f(x)$' does not stand for a function; clearly, the only thing it can stand for for him is an object.

2. The Geach–Dummett Solution.

The proposition 'the concept *horse* is not a concept' is the negation of 'the concept *horse* is a concept' and this proposition, from its surface structure, appears to be analyzable into the singular term 'the concept *horse*'and the predicate 'ξ is a concept.' Stated simply, the Geach-Dummett solution says that this analysis is incorrect, because the categorization of the constituents is incorrect. I shall first look at the categorization of the apparent predicate 'ξ is a concept' and then at that of the apparent singular term 'the concept *horse*.'[5]

'ξ is a concept' appears to be a predicate which has been constructed by analogy with 'ξ is an object.' This latter predicate has the property that when its argument-place is filled with a non-vacuous singular term it always yields a true proposition, irrespective of what object the singular term refers to. In constructing 'ξ is a concept' it was hoped to obtain a predicate which yields a true proposition whenever its argument-place is filled with an expression that stands for a concept, but expressions which stand for concepts are not fitting for its argument-place. In fact, it looks as if 'ξ is a concept' is a predicate which yields a false proposition whenever its argument-place is filled by a non-vacuous singular term. In order to construct a predicate which yields a true proposition whenever its argument-place is filled by a non-vacuous predicate, we have to go up to the second-level and for this role Dummett [*1973*, 216–217]:

$$(\forall x)(\phi(x) \vee \neg \phi(x))$$

which can be expressed in ordiny language as 'ϕ is something which everything either is or is not.' Dummett advocates the banishment of the phrase 'is a concept' from our language and its replacement with this second-level predicate.

The expression 'the concept *horse*' appears to be a singular term, but both Geach and Dummett argue against this categorization. Surprisingly, rather than conducting their discussion in terms of the expression 'the concept *horse*,' which Frege uses, they prefer the form of words 'what "ξ is a horse" stands for.' For their discussion to be a discussion of Frege's paradox we must assume that these two expressions are equivalent in every way. Geach [*1976*, 56–57] writes:

> The result of inserting an English expression in the blank between the quotes in the context:
>
> what ' ' stands for
>
> will stand for, *bedeutet*, whatever that very English expression stands for...

---

[5]Dummett says in *Frege*, pp 211–217, that the solution was discovered by Frege himself after the publication of "On Concept and Object," but there is no textual support for this claim in Frege's writings. In the poshumously published paper "Comments on Sense and Meaning" Frege [*1979*], says a few things which are superficially similar to parts of the Geach-Dummett solution. See also Anscombe's *An Introduction to Wittgenstein's Tractatus* [*1959*, 111–112], and Geach's "On What There Is" [*1951*, 132–134], where the solution was first published.

As an example he considers the predicate 'ξ killed Caesar.' He sys that the expression 'what "ξ killed Caesar" stands for' is a predicate. It could be said against this position that the expression 'what "ξ killed Caesar" stands for' produces nonsense when substituted for 'ξ killed Caesar' without alteration. For example, making this substitution in the sentence 'Brutus killed Csesar' results in the string of words 'Brutus what "ξ killed Caesar" stands for.' Geach says, however, that the string of words 'Brutus is what "ξ killed Caesar" stands for' is acceptable and nothing of logical significance hangs on the fact that grammar demands the copula 'is'. Applying these insights to the expression 'the concept *horse*' we see that this expression, although looking on the surface like a singular term, is actually a predicate. This has the consequence that the proposition 'Red Rum is the concept *horse*' or perhaps more idiomatically 'Red Rum falls under the concept *horse*' are just alternative ways of expressing the proposition 'Red Rum is a horse.'

Putting together these accounts of the expressions 'the concept horse' and 'is a concept' we see that it is no longer possible to construct such pseudo-propositions as 'the concept *horse* is a concept' or 'what "ξ is a horse" stands for is a concept.' According to Dummett [*1973*, 216–217], what we hoped to convey by means of these pseudo-propositions is correctly expressed, respectively, by the propositions 'a horse is something which everything either is or is not' and 'what "ξ is a horse" stands for is something which everything either is or is not.'[6] It should be noted that, whereas for Frege — at least when he wrote "On Concept and Object" — it was true to say that 'the concept *horse* is not a concept,' for Dummett the proposition 'a horse is not something which everything either is or is not' is false.

Dummett [*1973*, 217] urges that the pseudo-predicate 'ξ is a concept' be banished from our language; then it will be impossible to construct such paradoxical sounding propositions as 'the concept horse is not a concept.' But it is possible to rehabilitate this instead and turn it into a legitimate predicate. If we reject Frege's criterion for distinguishing between singular terms and concept-words, then just because the indefinite article occurs in the phrase 'is a concept' it does not follow that the word 'concept' is a concept-word. It is possible to construe the phrase 'is a concept' as a second-level predicate. In fact, we can regard the expression '...is a concept' as referring to the same second-level concept to which the expression '... is something which everything either is or is not' refers. If either Geach or Dummett had put forward this option, then their discussion of Frege's paradox would amount to an interpretation of the proposition 'the concept *horse* is a concept' in which it was true. But, because this pouibility has not been used by either Geach or Dummett, I shall not pursue it further here.

2.1. Further Aspects of Dummett's Solution.

Dummett says that the problem with talking about unsaturated entities begins 'with the words "concept", "relation" and "function" [*1973*, 213], and that solutions similar to the one he gives for the version of the paradox involving the word 'concept' can be given for the versions of the paradox which make use of the words 'relation' and 'function' [*1973*,

[6]Dummett replaces the phrase 'the concept *horse* here by 'a horse,' no doubt for idiomatic reasons.

213, 218]. His solution does indeed work for the pseudo-predicate 'ξ is a concept' and 'ξ is a relation' if these are construed as meaning 'ξ is a first-level concept' and 'ξ is a two-place first-level relation,' but it does not work for them if they are construed generically. Similarly, his solution does not work for the pseudo-predicate 'ξ is a function' if this is construed generically. It does, however, work for each element in the potentially infinite list of pseudo-predicates: 'ξ is a one-place first-level function,' 'ξ is a two-place first-level function,' 'ξ is a three-place first-level function,' and so on. It also works for all the specific pseudo-predicates which appear to be true of particular types of second-level functions and so on. Considering some of these examples shows us why the solution fails for the pseudo-predicate 'ξ is a function.'

Dummett would have to replace the pseudo-predicate 'ξ is a one-place first-level function' by a second-level predicate which is true of all one-place first-level functions. A suitable candidate for this second-level predicate is

$$(\forall x)(\phi(x) = x \vee \phi(x) \neq x).$$

And he would have to replace the pseudo-predicate 'ξ is a two-place first-level function' by a second-level predicate which is true of all two-place first-level functions. A suitable candidate for this second-level predicate is

$$(\forall x)(\psi(x, x) = x \vee \psi(x, x) \neq x).$$

The generic term 'function' is used as if it were true of all one-place first-level functions and also true of all two-place first-level functions and also true of all three-place first-level functions and so on. But Dummett cannot account for this generic term being in our language. This is because he would require 'ξ is a function' to be replaced in both the propositions: 'every one-place first-level function is a function' and 'every two-place first-level function is a function' (as well as many others). In the first of these 'ξ is a function' would have to be replaced by something of the same category as that which replaces 'ξ is a one-place first-level function' and in the second by something from the same category as that which replaces 'ξ is a two-place first-level function.' Quite clearly, these two replacement second-level predicates cannot be combined to form a single unified second-level predicate, because they have different sorts of argument-place. An expression which was fitting for one of them would not be fitting for the other. Thus, there is no consistent replacement for 'ξ is a function' which would work in both these cases. Hence, there can be no replacement for the pseudo-predicate 'ξ is a function,' which appears to be true of all first-level functions and all second-level functions and so on. Unlike the pseudo-predicate 'ξ is a one-place first-level function,' which he says should be banished from our language and replaced by a certain second-level predicate, in the case of the generic term 'function,' he has to say just that it be banished from our language. His account cannot be modified in such a way as to find a suitable replacement for the generic pseudo-predicate 'ξ is a function."[7]

[7]Consideration of this argument shows that there can be no replacement in Dummett's scheme of things for the generic notion of a concept, namely that notion which appears to be employed in the

A similar argument shows that there can be no replacement for the pseudo-predicate 'ξ is an entity,' which almost everyone who writes about Frege's ontology finds so useful. Currie [*1982*, 85], for example, says that according to Frege 'every entity is either a function or an object.' In an account such as Dummett's there cannot be a genus *entity* of which *one-place first-level function* and *two-place first-level function* are species, let alone *function* and *object*. To be fair to Dummett, he does recognize this, for he writes that 'the word "entity" is un-Fregean, in that it trespasses over the bounds dividing one level from another' [*1982*, 523], but rather than suggesting the banishment of the word 'entity' he suggests construing a proposition in which it occurs as being 'something like a typically ambiguous formula within the theory of types' [*1982*, 523]. A similar argument to that given above shows this to be imposdble, since in the proposition 'every entity is either a one-place first-level function or an object' we would have to assign two distinct types to the term 'entity' simultaneously, which cannot be done. The most that Dummett's suggestion would allow us to do is to assert, for example, 'some entities are objects' and 'some entities are one-place first-level functions.'

But even if we banish the pseudo-predicate 'ξ is an entity' from our language, we will still be left with many troublesome pseudo-propositions. An example occurs in Frege's writings: 'I count as objects everything that is not a function...' [*Grundgesetze*, *1893*, 35–36]. For this to make sense the domain of quantification must be understood as including both objects and functions and — on Fregean principles — this is clearly impossible. (This string of words would still be meaningless and not false if we replaced 'function' by 'one-place first-level function,' because there is no logically acceptable translation along the lines suggested by Dummett.)

So, if we accept Dummett's solution to Frege's paradox, we have to make radical changes to our ontological vocabulary. Generic terms such as 'function' and 'entity' would simply have to go, whereas specific terms such as 'one-place first-level function' and 'two-place first-level function' would have to be replaced by second-level predicates. Furthermore, we would have to outlaw many constructions such as 'everything which is not an object is a function.'

After presenting his solution to Frege's paradox Dummett [*1973*, 217] writes:

> The terminology that would be required for speaking, in a logically correct manner, about the referents of predicates and relational expressions is...cumbrous and verbose; it is therefore best, when there is no danger of misunderstanding or of antinomies, to revert to the logically erroneous vocabulary of 'concept', 'relation' and 'function'.

What I have shown, however, is that in a number of cases there is no 'logically correct manner' of speaking; in particular, propositions containing the term 'function' construed generically do not have a proper form, nor do many propositions containing the word 'entity.'

---

phrases 'is a first-level concept' and 'is a second-level concept.' Dummett's account does, however, work for the specific notion of a concept, that is to say, the notion of a one-place first-level concept. My discussion of the notion of a concept in the body of the text is restricted to the specific notion and does not apply to the generic notion.

2.2. Additional Aspects of Geach's Solution.

Geach's position is slightly different. Unlike Dummett, he does not first urge us to banish the troublesome words 'concept', 'relation' and 'function' from our language but then permit us revert to the logically erroneous terminology, provided we use it carefully. Right from the start Geach realises that many of the propositions that we would like to assert about the differences of type between entities in Frege's ontology violate the principles on which those very differences of type are based. As an example he considers the proposition 'there is a difference between what "Brutus" stands for and what the predicate "— killed Caesar" stands for' (in "Showing and Saying in Frege and Wittgenstein," [*1976*, 57]). On his account of the expression 'what "..." stands for' this translates into the proposition 'there is a difference between Brutus and killed Caesar' and this is manifest nonsense. Therefore, the original proposition is also meaningless and it could not even be formulated in a language such as Frege's *Begriffsschrift*. Geach believes, however, that such nonsensical propositions can be 'didactically useful,' as he puts it [*1976*, 58), for they can be helpful in teaching someone to understand the *Begriffsschrift*. Unfortunately, Geach does not develop the idea of didactically useful nonsense in any great detail; so, for example, it is impossible to know if he would classify the string of words 'unsaturated entities totter on the brink of the abyss of nothingness' as didactically useful or as didactically useless nonsense.

But there is a more serious objection to the notion of didactically useful nonsense and that is that it makes disagreement impossible. Imagine, for the sake of argument, a student who has listened to Geach's lectures on Frege taking an examination in which there is a question on Frege's paradox. The student asserts, amongt other things, that 'the concept horse is an object.' Geach cannot but take this as evidence that the student has not understood his solution of Frege's paradox: by his account of didactically useful nonsense there is no way in which somebody could understand, for example, the difference between concepts and objects and yet express the view that it is mistaken.

2.3. Semantic Ascent.

Frege's paradox is usually formulated in ontological terms and so far I have discussed it in such terms. It might be thought that this paradox could easily be solved by means of the manoeuvre that Quine (*Word and Object*, [*1960*, 271]) calls semantic ascent and which he describes as follows:

> It is the shift from talk of miles to talk of 'mile'. It is what leads from the material (*inhaltlich*) mode into the formal mode, to invoke an old terminology of Carnap's. It is the shift from talking in certain terms to talking about them.

To use this manoeuvre in an attempt to solve Frege's paradox is the approach that a follower of Carnap, for example, would favour. Carnap (in *The Logical Syntax of Language*, [*1937*, 286]) distinguished between three kinds of sentence, namely syntactical sentences, object sentences and pseudo-object sentences. An example of an object sentence is 'water is not an acid but an alkali.' One species of pseudo-object sentence occurs frequently in philosophy. An example is '5 is not a thing but a number.' Such sentences give the impression of being about objects, but — according to Carnap — they are really disguised syntactical sentences. He claims that it is more precise to say that 'the numeral "5 " is not a thing-word but a number-word.' Carnap goes in for a wholesale translation of similar philosophical sentences. He translates Kronecker's statement that 'God created the natural numbers (integers); fractions and real numbers, on the other hand, are the work of man' into the syntactical sentence that 'the natural number symbols are primitive symbols; but the fractional expressions and the real-number expressions are introduced by definition' [*1937*, 304–305].

Someone standing in this philosophical tradition might think that if we stop talking about entities, and talk exclusively in terms of linguistic expressions and the categories to which they belong, then Frege's paradox will never arise. Unfortunately, this is not the case, similar difflculties also occur on the linguistic level. This s not really very surprising. Geach ("Names and Identity," [*1975*, 142–143]) articulates well in general terms what is wrong with such an approach:

> Language, after all, is not something set over against the whole world, like the Divine Mind; languages are part of the world, linguistic facts and structures are facts and structures in the world. This sets a limit to the usefulness of semantic ascent in solving philosophical problems. We cannot solve the problem of universals by talking about the word 'pig' instead of The Pig; for there is exactly the same problem about the relation of the word 'pig' to its tokens as there is about the relation of The Pig to Jones's pigs.... So also in our case; the business of function and argument and value cannot be shelved by talking about expressions of different category, because it reappears if we consider, as we must, the ways of forming expressions out of expressions.

His own criticism of those people who believe that Frege's paradox can be solved by the manoeuvre of semantic ascent consists in pointing out that they think of predicates and functional signs as complete expressions and what he does is to show that predicates and functional signs are unsaturated expressions (for example, in "Saying and Showing in Frege and Wittgenstein," [*1976*, 58–61]).

It is more interesting, however, to construct a version of the paradox on the linguistic level analogous to the ontological version. Frege is aware that something like this is possible, but he expresses himself badly. He says (in "On Concept and Object," [*1892*, 196, footnote †]):

> A similar thing happens when we say as regards the sentence 'this rose is red': The grammatical predicate 'is red' belongs to the subject 'this rose.' Here the words 'The grammatical predicate "is red"'are not a grammatical predicate but a oubject. By the very act of explicitly calling it a predicate, we deprive it of this property.

He expresses himself badly because he talks of *grammatical* predicates. This is probably why Geach says [*1976*, 60] that this comment is irrelevant, but reformulated in terms of Fregean predicates, that is to say, unsaturated expressions which make a proposiition out of a singular term, it becomes as problematical as the ontological version. Doing this we have the paradoxical proposition: 'The predicate "ξ is a horse" is not a predicate but a singular term.' Dummett [*1973*, 212] tacitly performs such a reformulation; yet he says:

> But this latter fact is no more paradoxical than the fact that the expression 'the city of Paris' io not a city: the case would be comparable with 'The concept *horse* is not a concept' only if we had reason for saying that the predicate 'ξ is a horse' is not a predicate.

However, we have exactly the same reason for saying that the expression 'the predicate "x is a horse"' is a singular term as for saying that the expression 'the concept *horse*' is one, namely Frege's criterion for distinguishing between singular terms and concept-words. The definite article followed by a singular substantival phrase figures in both of them.

Not surprisingly, because of the parallelism between the linguistic and the ontological levels in Frege's thought, a solution similar to that offered by Geach and Dummett to the ontological version of the paradox can be constructed for the linguistic version. It would go something like this:

> The problem begins with the predicate 'ξ is a predicate'. The predicate 'ξ is a singular term' is true of all singular terms and in constructing 'ξ is a predicate' we were hoping to devise a predicate that was true of all predicates; but the argument-place of 'ξ is a predicate' is not fitting for predicates, it is only fitting for saturated expressions. In order to construct a suitable predicate we have to go up to the second level and devise a second-level predicate which is true of all first-level predicates. This is no trickier than for the ontological predicate 'ξ is a concept' and a suitable candidate is: '...is something which yields a proposition when applied to a singular term'.
>
> And the apparent singular term 'the predicate "ξ is a *horse*"' is really an incomplete expression which refers to a predicate and is fully equivalent with ' "ξ is a horse" '.
>
> Thus, what we hoped to convey by means of the pseudo-proposition 'the predicate "ξ is a horse" is a predicate' is correctly conveyed by the logically more accurate 'the predicate "ξ is a horse" is something which yields a proposition when applied to a singular term' or ' "ξ is a horse" is something which yields a proposition when applied to a singular term'.

To be made fully analogous to Dummett's solution to the ontological paradox this linguistic version would have to be combined with the view that predicates like 'ξ is a predicate', 'ξ is a one-place first-level functional sign', 'ξ is a two-place first-level functional sign', and so on, be banished from our language and be replaced by their logically correct translations. The generic pseudo-predicates 'ξ is a functional sign' and 'ξ is an incomplete expression' (and others) would simply have to be banished.

The purpose of this subsection has been to show that Frege's paradox cannot be resolved by means of semantic ascent: there is an analogous version of the paradox at the linguistic level. Having established that, I will continue discussing the ontological version of the paradox; but before doing that I just want to say something about nested quotation marks.

2.4. Nested Quotation Marks.

In constructing the above linguistic version of Frege's paradox and its solution I have assumed that an expression enclosed in two sets of quotation marks — such as ' "ξ is a horse" ' or 'the predicate 'ξ is a horse" ' — is unsaturated and that it refers to something unsaturated, namely 'ξ is a horse.' Needless to say, the varieties of unsaturatedness associated with such expressions are very different from those associated with expresoions enclosed in only one set of quotation marks.

This seems to me to be the correct assumption to have made, but it differs from both Dummett's and Frege's views about expressions enclosed in two sets of quotation marks. Concerning the predicate 'ξ is what "ξ is a horse" stands for' Dummett writes (in *Frege*, [*1973*, 214]):

> The first 'ξ' indicates the argument-place of this predicate; the second 'ξ', being, in this predicate, between quotation marks, does not indicate an argument-place, but is a constant part of the expression.

This cannot be correct, because it has the consequence that something complete, namely '"ξ is a horse"', refers to something incomplete, namely 'ξ is a horse.'

Similarly, Frege writes concerning nested quotation marks:

> While '( ) · 3 + 4' is a function name, ' "( ) · 3 + 4" ' is a proper name, and its meaning is the function name '( ) · 3 + 4'.[8]

Again, this cannot be correct, because it has the consequence that something complete, namely '"( ) · 3 + 4"', refers to something incomplete, namely '( ) · 3 + 4'.

Both Dummett's and Frege's accounts violate the principle that only saturated things can refer to saturated things and that only unsaturated things can refer to unsaturated things. And ii we give up this principle when one of the things involved is enclosed in two sets of quotation marlts and the other in one, then why should we retain it when one of the things involved is enclosed in one set of quotation marks and the other is not enclosed in any? There is nothing magical about nested quotation marks. Once the above principle is violated the isomorphism between expressions and their referents — which is characteristic of Frege's philosophy — breaks down as well. There is nothing special about nested quotation marks which creates an immunity to being involved in this isomorphism. Thus, if we want to retain this isomorphism, then we cannot accept either Dummett's or Frege's account of nested quotation marks.

---

[8]This is from a letter to Russell dated 29 June 1902. In the English version [Frege *1980*, 135] there is a typographical mistake — the second occurrence of '4' is missing — which I have rectified.

2.5. Further Difficulties in the Geach-Dummett Solution.

In order to continue my explanation of Frege's paradox I unfortunately need to introduce some symbolic notation; this is, however, based on the standard mathematical way of expressing type information. I partition linguistic expressions into *syntactic categories* and their referents into *types*.

N is the syntactic category of singular terms and P is the category of propositions. $N \to P$ is the category of incomplete expressions which make propositions out of singular terms. I write x:: σ to show that x belongs to the syntactic category σ. Thus, for example, we have:

$$\text{`}\xi \text{ snores'} : N \to P.$$

We can define the hierarchy of syntactic categories $\mathfrak{C}$ as follows, where P and N are taken as the basic categories:

(i) $P, N \in \mathfrak{C}$.
(ii) If $\alpha, \beta \in \mathfrak{C}$, then $\alpha \to \beta \in \mathfrak{C}$.

A similar notation is used for types. J is the type of objects and H is the type of truth-values. The notation:

$$\xi \text{ snores}: J \to H,$$

indicates that x snores is a function that maps objects to truth-values. We can define the hierarchy of types $\mathfrak{T}$ as follows, where J and H are taken as the basic types:

(i) $J, H \in \mathfrak{T}$.
(ii) If $\alpha, \beta \in \mathfrak{T}$, then $\alpha \to \beta \in \mathfrak{T}$.

Having introduced this notation I will continue my discussion of Frege's paradox.

I want to consider the operator 'what "..." stands for' in greater detail. As explained by Geach and Dummett this is an operator that cannot be assigned to a single syntactic category. According to Geach's rule (quoted above) the expression 'what "Socrates" stands for' is a singular term and is just a long-winded way of referring to Socrates. Similarly, the expression 'that function of 2 which "the square of" stands for' is a long-winded way of referring to the square of 2 (see [Geach *1976*, 57]). Thus, we must regard 'what "the square of ξ" stands for' in this case as being an unsaturated functional sign, that is to say, an expression of category $N \to N$.[9] We also have that the expression 'what "ξ killed ζ" stands for' is a relational sign and 'what "the brother of ξ and ζ"stands for' is a

[9]Geach changes 'what' to "which,' no doubt for idiomatic reasons, and does not use Frege's ξ-notation in this example.

two-place functional sign. Furthermore, 'what "everything" stands for' is a quantifier of category $(N \rightarrow P) \rightarrow P$. These categorizations can be presented as follows:

(1) 'what "Socrates" stands for': N,

(2) 'what "ξ is a horse" stands for': $N \rightarrow P$,

(3) 'what "ξ killed ζ" stands for': $N \rightarrow (N \rightarrow P)$),

(4) 'what "the square of ξ" stands for': $N \rightarrow N$,

(5) 'what "the brother of ξ and ζ" stands for': $N \rightarrow (N \rightarrow N)$,

(6) 'what "everything" stands for': $(N \rightarrow P) \rightarrow P$.

Generalizing, we can say that given an expression of arbitrary category the operator 'what "..." stands for' yields an expression of the same category which is everywhere substitutable with the original expression *salva congruitate* and *salva veritate*.

Although both Geach and Dummett make use of this operator, neither of them stops to consider the implications of its introduction or whether its existence can be justified. Had they done so they would have realised that in a language constructed on Frege's principles, in which expressions are assigned to distinct non-overlapping syntactic categories, there can be no such operator. This is because if an unsaturated expression has an argument-place that is fitting for singular terms, then *ipso facto* it is not fitting for predicates and if an unsaturated expression has an argument-place that is fitting for one-place first-level functional signs, then it is not fitting for quantifiers, and so on.

The upshot of this discussion is that the operator required by Geach and Dummett in their solution of Frege's paradox does not exist. They require a single operator, whereas the best that can be accomplished in a Fregean language is a large number of similar operators. In a Fregean language, each of the operators represented by the form of words 'what "..." stands for' in (1) to (6) has to be different. In order for the Geach-Dummett solution to be successful, they would require a trans-categorial operator of the form 'what "..." stands for, but such operators are illegitimate on Fregean principles.[10]

Another example of a trans-categorial operator is '... refers to__'. This is trans-categorial because we would — in our uncritical moments — accept the following propositions to be true:

(7) 'Socrates' refers to Socrates,

(8) 'The concept *horse*' refers to the concept horse,

[10]The word 'trans-categorical' simply means that what it qualifies can belong to several categories. A propositionin which a trans-categorical operator occurs is, therefore, similar to a typically ambiguous formula in the theory of types. But But as I partition the expressions of a language into *syntactic categories* (and their referents into *types*) I prefer the phrase 'trans-categorical' to 'typically ambiguous.'

(9) 'ξ loves ζ' refers to ξ loves ζ,

(10) The universal quantifier refers to a function from concepts to truth-values,

(11) Conjunction refers to a truth-function.

And we would also accept the following as being true:

(12) Proper names refer to objects,

(13) Predicates refer to concepts,

(14) Relational signs refer to relations,

(15) Second-level quantifiers refer to functions from concepts to truth-values,

(16) Binary truth-functional connectives refer to two-place truth-functions.

The best that Dummett can do faced with the propositions (7) to (11) and (12) to (16) is to banish the alleged trans-categorial operator '... refers to __' from our language and replace it with lots of signs for all the different reference relations that are necessary. He would require at least as many signs as there are different categories of expression in our language. This is because the argument-place on the right of '... refers to __' in (7) is fitting for a proper name, whereas that on the right in (8) is fitting for a predicate. And that on the right in (9) is fitting for a relational sign and so on.[11] And the best that Geach can do is to say that whenever we use a proposition involving the words 'refers to' we are talking gibberish, which yet might be useful in teaching people about Frege's philosophy.

This investigation of the solutions offered by Geach and Dummett to Frege's paradox shows that the views they are forced to adopt are far worse philosophically than the paradox they use them to solve. Neither of them can account for the terms that are indispensable for talking about language and ontology. Dummett's solution of Frege's paradox involves banishing at least the following expressions from our language: 'refers to', 'entity', 'function', 'incomplete expression' and 'what "..." stands for'. Geach's solution has the consequence that whenever we use any of these terms we are talking gibberish. That would mean, for example, that a large number of the sentences in Dummett's book *Frege* are meaningless. It seems to me that in the light of these considerations we should investigate alternative ways of looking at Frege's paradox. And what I propose to do is to take seriously Frege's view that 'the concept *horse*' and similar expressions are genuine singular terms which refer to objects and explore its consequences. Dummett has claimed that Frege repudiated this position, but it seems to me that there is some evidence to suggest that he continued to think in these terms. I will first present this

[11] And even then he would have difficulty with "propositions" like '"Socrates" does not refer to a concept' and '"the concept *horse*" does not refer to a relation.'

evidence and then I will look in greater detail at the consequences of treating expressions such as 'the concept *horse*' as genuine singular terms.

3. Frege's Definition of Numbers.

In the *Grundlagen* [*1884*, § 68] Frege at one point gives the following definition:

> the Number which belongs to the concept *F* is the extension of the concept "equal to the concept *F*".

If we accept that the expression 'the concept *F*' is a predicate, then 'the Number which belongs to...' is an incomplete expression which makes singular terms out ofpredicates and the expression '... is equal to __' is an incomplete expression which makes a proposition out of two predicates. Thus, the concept which is the referent of the expression 'equal to the concept *F*' is a second-level concept and Frege is defining the expression 'the Number which belongs to the concept *F*' as a singular term which refers to a class of first-level concepts.

Or is he? In order to justify this definition he has been considering the definition of the direction of a line *s*, say, as the extension of the concept *parallel to line x*. To understand 'the Number which belongs to...' as an incomplete expression which makes singular terms out of predicates makes the correspondence with the definition of the direction of a line inexact. Because, in the numerical case the equivalence relation involved is one which holds between first-level concepts, whereas for directions the equivalence relation involved, namely the relation *parallel to*, is one which holds between objects.

The interpretation of Frege's definition that I have just given is, in fact, anachronistic, for it is based on the Geach-Dummett account of the expression 'the concept *horse*' and similar expressions. As he makes explicit in "On Concept and Object," Frege considered such expressions to be singular terms, therefore they should be so understood in the *Grundlagen.* If we interpret them thusly there, then the correspondence between the detinition of numbers and directions becomes exact. The expression 'the Number which belongs to...' is then a functional sign which makes singular terms out of singular terms and the equivalence relation equal to is then just a relation between objects.

In his discussion of the hermeneutical principles to be used in giving a correct exegesis of Frege, Dummett says (in *The Interpretation of Frege's Philosophy*, [*1981*, 8]) that

> what Frege wrote in work he did not submit for publication bears less weight than what occurs in his published writings; and, above all, what is said in *Grundgesetze* carries more weight than anything else.

But as a result of a careful structural analysis of the *Grundgesetze* [Dummett *1981*, 9] adds that the '*Grundlagen*...is in a somewhat different position,' that is to say, from Frege's published writings other than the *Grundgesetze*. Simply stated, this is because Dummett regards the *Grundlagen*, in effect, as a prose section of the *Grundgesetze*. He says [*1981*, 10] that the progression of ideas in the corresponding passages of the two books is very similar, 'apart from the fact that in *Grundgesetze* numbers are defined as classes of classes, rather than as classes of concepts as in *Grundlagen*." But, as I have shown above, in the *Grundlagen* numbers are defined as classes of objects of a particular kind, namely those objects that can be the referents of expressions of the form 'the concept *F*.' That Frege did not go over this material again in prose in the *Grundgesetze* leads me to presume that he still regarded expressions of the form 'the concept *F*' as singalar terms.

The correspondence between relevant sections of the two works suggests two possible candidates for the type of object that could be the referent of such an expression. It could either be the class of *F*s or it could be the referent of the expression which is formed by applying the functional abstraction operator to the unsaturated expression 'ξ is an *F*.' There is a real choice here, because the operator in the *Begriffsschrift* which is usually called the class abstraction operator can just as validly be thought of as a lambda abstraction operator. This is because in the *Grundgesetze* Frege did not distinguish between singular terms and propositions. If the operator in question is applied to a predicate, it makes sense to call it a class abstraction operator; but when it is applied to a functional sign, there is greater justification in calling it a functional abstraction operator.

I must admit that I do not regard this as conclusive proof that Frege still thought that expressions such as 'the concept *horse*' were singular terms when he wrote the *Grundgesetze*, but I do think that it constitutes æ at least æ a strong *prima facie* case for so regarting them. In any case, even if Frege did not think this, it is still possible that treating such expressions as singular terms has consequences that are preferable to those of the position that treats them as predicates, and I shall explore that possibility next.

4. An Alternative Proposal.

The solution that Dummett and Geach put forward in order to deal with Frege's paradox has — at first sight — a lot of plausibility, but upon investigation it leads to several unacceptable consequences. In the first place, the operator that they use, namely 'what "..." stands for,' cannot be accommodated in their accounts and yet it is essential to them. It cannot be accommodated because, in order to do the job required of it, it must belong to more than one syntactic category. It must, for example, make a proper name out of a proper name and also a predicate out of a predicate. It thus violates Frege's requirement that no expression be assigned to more than one category. The alternative, in which there are as many operators of the form 'what "..." stands for' as there are syntactic categories in our language, does not have much to recommend it.

This is not the only unacceptable consequence of the Geach-Dummett solution. Dummett's account leads him to a position in which a large part of our vocabulary for talking about type and category differences has to be banished from our language. Alternatively, we are allowed to use this logically erroneous terminology, if we do so carefully. Neither of these positions are very attractive philosophically. Geach's account, on the other hand, leads him to a position in which meaningless strings of words are given definite and precise uses in teaching people about Frege's philosophy. This — again — is not very attractive philosophically.

The inadequacies of both Geach's and Dummett's solution to Frege's paradox should encourage us to look for a better solution and in this section I explore the possibility that a sensible theory can be forinulated in which expressions like 'the concept *horse*' are genuine singular terms that really do refer to objects. The best place from which to start in trying to construct such a theory is Frege's discussion (in "On Concept and Object," [*1892*, 204–205]) of an attempted refutation of his doctrine that concepts are unsaturated.[12] He considers the translation of the sentence 'the number 2 is a prime number' into 'the number 2 falls under the concept *prime number*.' The first sentence contains the predicate 'ξ is a prime number' which refers to an unsaturated concept, whereas the second contains the singular term 'the concept *prime number*' which refers to an object. So it looks as if the troublesome notion of unsaturatedness has been removed. Frege points out, however, that it has not been removed. What has happened is that the second sentence contains the relational expression 'ξ falls under ζ' which is unsaturated and which refers to an unsaturated relation. He writes in "On Concept and Object" [*1892*, 205]): 'It is thus easy for us to see that the difficulty arising from the "unsaturatedness" of one part of the thought can indeed be shifted but not avoided.' This also applies to the linguistic and the ontological levels. The word 'shifted' is unfortunate, because the same expression cannot be both the unsaturated predicate 'ξ is a prime number' and the saturated singular term 'the concept *prime number*' (and similarly on the ontological level). What we are dealing with is a procedure which translates unsaturated predicates into their corresponding saturated singular terms.

The picture of language that Frege has here is that language can be divided into at least two parts. One part of it contains a rich multiplicity of unsaturated expressions. In this part of language there are predicates such as 'ξ rides,' relational signs such as 'ξ loves ζ,' functional signs of various levels, quantifiers of several levels, truth-functional connectives and so on. Because of the variety of unsaturated expressions in this part of language, we can call it the *rich* part.

But there is another part of language. Here there are singular terms and little else. Corresponding to each variety of unsaturated expression in the rich part of language there is a singular term here. We might, therefore, call this the *austere* part of language. Clearly, the austere part cannot just consist of singular terms. There has to be at least one unsaturated expression. In fact, there is no need to have more than one unsaturated expression in the austere part of language and I shall assume that there is only one.[13] In order to avoid confuuon I call the category of austere singular terms 'E'. In fact, it is best to consider E as being the category of all the complete expressions in the austere part of language. It thus contains propositions as well as singular terms like '2' and 'Jack' and like 'the concept *horse*.' Singular terms such as '2' and 'Jack' are, in fact, the only expressions common to the rich and austere parts of language. The single unsaturated expression in the austere part of language is of syntactic category $E \to (E \to E)$ and it has a number of linguistic

[12] The discussion is actually carried out by Frege in the realm of sense, but — because of the parallelism that exists in Frege's mature philosophy between his ontology, his account of the logical structure of language and his views about the senses that various expressions have — it also applies in the realm of reference and on the linguistic level.

[13] Although I shall often write this unsaturated expression as 'ξ falls under ζ,' the expression I use is more general than Frege's operator. I deliberately generalized it in the direction of the functional application operator of combinatory logic. This is justified because the two operators have the same meaning in all of the cases that Frege discusses and he gives no indication of how his views should be generalized.

incarnations. In the example Frege considers, the rich proposition 'the number 2 is a prime number' is translated into the austere sentence 'the number 2 falls under the concept *prime number*' and the unsaturated expression from the austere part of language is represented as 'ξ falls under ζ.'[14] It can sometimes be more idiomatically expressed as 'ζ applies to ξ' and its most succinct representation is '$(\zeta\xi)$.'

One of the diffIculties that results from allowing propositions such as 'the number 2 is a prime number' to be translated into 'the number 2 falls under the concept *prime number*' and then letting the latter be analyzed into the two singular terms 'the number 2' and 'the concept *prime number*' and the austere unsaturated expression is that it is possible to rearrange the austere constituents in such a way that no proposition constructed from rich components corresponds to it. In order to illustrate this, it should be observed that as well as allowing the above translation, Frege would also have to allow the proposition 'Bucephalus is a horse' to be translated into 'Bucephalus falls under the concept *horse*.' And now it is possible to combine both the singular terms 'the concept *prime number*' and 'the concept *horse*' by means of the unsaturated expression 'ξ falls under ζ' to yield:

(17) The concept *prime number* falls under the concept *horse*.

But there is no way in which the corresponding unsaturated expressions 'ξ is a horse' and 'ξ is a prime number' can combine together because the argument-place of each of them is not fitting for the other.

Another example of this sort of austere proposition can be constructed from an example that Frege discusses. He considers the proposition 'the number 2 falls under the concept *prime number*' and says (in "On Concept and Object," [*1892*, 205]) that a similar problem to that of 'the concept *horse*' can be constructed here, since the 'words "the relation of an object to the concept it falls under" designate not a relation but an object.' We can, therefore, fill up both argument-places of the relational sign 'ξ falls under ζ' by the singular term 'the relation of an object to the concept it falls under' to give us the proposition:

(18) The relation of an object to the concept it falls under falls under the relation of an object to the concept it falls under.

There is a way, however, of isolating those combinations of austere symbols that do have propositions constructed out of rich expressions corresponding to them. In order to

[14]Long argues that the decomposition of 'the number 2 falls under the concept *prime number*' into a relational sign 'ξ falls under ζ' and two singular terms is incorrect (see, for exaample, his articles "Universals" [*1984*] and "Formal Relations" [*1982*].) He says that it is much more plausible to decompose this sentence into the singular term 'the number 2' and the predicate 'ξ falls under the concept *prime number*' and to regard this predicate as just a long-winded version of 'ξ is a prime number.' Viewed as an account of what goes on in the rich part of language, this position has much to recommend it, but it cannot be a correct account of the way in which the austere part of language functions. If there was no unsaturated expression there, then the austere singular terms could not combine with one another.

do this I need to introduce the notion of an austere expression's functional character and also that of stratification.[15]

Every incomplete expression in the rich part of language has a category, for example, '$\xi > \zeta$' is of category $N \rightarrow (N \rightarrow P)$. The austere saturated expression 'the relation *greater than,*' which is its translation, is not of this category. It is in fact a singular term, but we can assign to it what I propose to call a *functional character* which tells us what its powers of significant combination are. In this case the name of the functional character is written '$N \Rightarrow (N \Rightarrow P)$' and its extension to other cases should be obvious: to obtain the name of a functional character from a category name silnply replace every single-shafted arrow '$\rightarrow$' by a double-shafted one '$\Rightarrow$'. The relation between category names and names of functional characters generated by this trandation induces a relation between categories and functional characters. A functional character x' *corresponds* to a syntactic category x iff [if and only if] the name of x' has bese obtained from that of x by replacing every single-shafted arrow in the name of x by a double-shafted one.

The notion of stratification can now be defined as follows:

(i) Every saturated austere expression without any logically relevant internal structure which has a rich counterpart is stratified and its functional character corresponds to the syntactic category of its rich counterpart.

(ii) An expression $\ulcorner X$ falls under $Y\urcorner$ or $\ulcorner Y$ applies to $X\urcorner$ or simply $\ulcorner(YX)\urcorner$ is stratified and has the functional character w iff X is stratified and has the functional character v and Y is stratified and has the functional character $v \Rightarrow w$.

An austere expression has a logically relevant internal structure if it is made up from austere expressions which have functional characters. Thus, 'the concept *horse,*' under this account, has no logically relevant internal structure. If an expression made up from austere constituents is stratified, then the rich components that correspond to those constituents can combine together to form a syntactically coherent expression in the rich part of language.

Under this definition the strings (17) and (18), although syntactically coherent expressions of the austere part of language, are unstratified. Therefore, the rich components that correspond to their austere constituents cannot combine together. In other words, there are no rich propositions corresponding to (17) and (18).

Similarly, it is possible to assign *valencies* to the referents of all expressions of category E in the austere part of language. In order to avoid confusion, I shall call these entities *obs* (the name comes from combinatory logic). Every entity that is the referent of some expression from the rich part of language has a type. The referent of the linguistic function '$\xi > \zeta$,' for example, is of type $J \rightarrow (J \rightarrow H)$. The *ob* which is the referent of the austere counterpart of this unsaturated expresdon, namely the relation *greater than*, is not of this type, but we can assign to it what I propose to call its *valency.* This tells us what its powers of meaningful combination are. In this case the name of the valency is written '$J \Rightarrow (J \Rightarrow H)$' and its extension should be obvious. To obtain the name of a valency from that

[15]The notion of stratification wa introduced by Quine in "New Foundations for Mathematical Logic" [*1937*], but my account derives from that of Curry and Feys in *Combinatory Logic* [*1958*, 289*ff*].

of a type simply replace every single-shafted arrow by a double-shafted one. The relation between names of types and valency names generated by this translation induces a relation between types and valencies. A valency x' *corresponds* to a type x iff the name of x' has been obtained from that of x by replacing every single-shafted arrow in the name of by a double-shafted one.

Analogous to the notion of linguistic stratification defined earlier it is possible to define a notion of ontological stratification:

(i) Every referent of an expression of category E which has no logically relevant internal structure and which has a rich counterpart is stratified and its valency corresponds to the type of the referent of its rich counterpart.

(ii) The referent of an n expression which has either the form $\ulcorner X$ falls under $Y\urcorner$ or $\ulcorner Y$ applies to $X\urcorner$ or simply $\ulcorner(YX)\urcorner$ is stratified and has valency w iff the referent of X is stratified and has valency v and the referent of Y is stratified and has valency $v \Rightarrow w$.

Expressions like 'the concept *prime number* falls under the concept *horse*' which are not stratified do not have stratified referents.

It is now possible to give a coherent account of such terms as 'entity' and 'function.' To be precise, it is possible to give an account of the predicate 'ξ falls under the concept *entity*' and 'ξ falls under the concept *function*.'[16] It is possible to give a coherent account of these although we can neither assign a functional character to 'the concept *entity*' nor to 'the concept *function*.' These expressions do not have any logically relevant internal structure, but, as shown earlier in this paper, they do not have rich counterparts. Furthermore, their referents do not have valencies.

The satisfaction-condition of the predicate 'ξ falls under the concept *entity*' is that $\ulcorner X$ falls under the concept *entity*$\urcorner$ is true or $\ulcorner X$ is an entity$\urcorner$ is true iff X is stratified. The satisfaction-condition of the predicate 'ξ falls under the concept *function*' is that $\ulcorner X$ falls under the concept *function*$\urcorner$ is true or $\ulcorner X$ is a function$\urcorner$ is true iff X is stratified and the name of its valency (a) contains at bast one double-shafted arrow '$\Rightarrow$' and (b) contains no occurrence of the valency name 'H'.

There is no problem about incorporating the operators '... refers to __' and 'what "..." stands for' into the austere part of language. '...' refers to __' is just a relation between complete austere expressions with a functional character and *obs* that have a valency and 'what "..." stands for' is an operator which makes complete austere expressions out of complete austere expressions. The satisfaction-conditions of these operators are only slightly more complicated than those of the predicates 'ξ is an entity' and 'ξ is a function.'[17] In order to make the following account concise, I shall write '$(\zeta\xi)$' instead of

[16] The first of these is constructed by filling the second argument-place of 'ξ falls under ζ' with the complete expression 'the concept *entity*' and the second results from filling the same argument-place with 'the concept *function*.'

[17] Needless to say, the operator '... refers to __' is built up out of two tokens of the unsaturated expression 'ξ applies to ζ' and a suitable singular term, which I propose to call the saturated reference operator.

either 'ζ applies to ξ' or 'ξ falls under ζ.' I also make use the disquotation operator. The disquotation of an expression X or $\Delta(X)$ is simply the thing named by the expression X.

In order to give the satisfaction-conditions of '... refers to __', we first require every proposition of the form

(19) X refers to $\Delta(X)$,

to be true, where X is an austere saturated expression without any logically relevant internal structure which has a rich counterpart. Then, we require the following to hold:

(20) $\ulcorner(YX)\urcorner$ refers to $\Delta(\ulcorner(YX)\urcorner)$,

for all stratified Y and X such that $\ulcorner(YX)\urcorner$ is also stratified. (The disquotation of $\ulcorner(YX)\urcorner$ is simply the disquotation of Y applied to the disquotation of X.)

The result of inserting a stratified expression in the gap of 'what "..." stands for' is an expression which is everywhere substitutable *salva congruitate* and *salva veritate* with the original expression.

Having explained how the predicates 'x is an entity' and 'x is a function,' the relational sign '... refers to __' and the operator 'what "..." stands for' can be accommodated in the austere part of language, it should not be too difficult to see how other similar expressions can be handled there.

Before concluding, I just want to make it explicit that the austere part of lanuage has two distinct classes of complete expressions in it. The first is the class of all those austere expressions which have analogues in the rich part of language. Members of this class are expressions such as 'the concept *horse*'and 'the addition function' and 'the number 2 falls under the concept *prime number*.' Every expression in this class has a functional character and its referent has a valency. The other class of complete expressions in the austere part of language consists of such expressions as 'the concept *function*', 'the concept *entity*' and the saturated reference operator. These do not have rich counterparts, they do not have functional characters associated with them and their referents do not have valencies assigned to them.[18]

5. Conclusion.

Frege's notion of unsaturatedness is very attractive and so is the related idea of a language whose expressions are partitioned into a multiplicity of discrete non-overlapping syntactic categories. Associated with each category is a distinct kind of unsaturatedness which unambiguously determines the powers of combination of those expressions. Frege's *Begriffsschrift* is a language built on these ideas. Although these ideas are very attractive, we should be clear about the limitations of a language constructed in accordance

[18] I showed earlier in this paper, in discussing Dummett's views, that 'refers to' and 'entity' have to be banished from the rich part of language, although I had not then introduced the 'rich' / 'austere' terminology.

with them. It is impossible in such a language to express the category differences on which it is built and it is impossible to accommodate trans-categorial operators in it. An important question to consider is: 'To what extent is natural language a *Begriffsschrift* ?' Dummett assumes that it is and proposes a thoroughgoing revision of our ontological vocabulary. Geach assumes it is, but rather than outlawing all category differentiating "propositions" and all "propositions" involving trans-categorial operators, he retains them as meaingless strings of symbols. What I have shown is that by building a theory in which phrases such as 'the concept *horse*' are genuine singular terms it is possible to accommodate these kinds of proposition as being *meaningful.* Such an advantage outweighs the initial unnaturalness of the theory. The resulting language still needs an unsaturated expression, but a single one will suffice. Because there is just one unsaturated expression, we can discuss it explicitly when necessary. Such a language does not violate any of Frege's principles. Its syntactic simplicity is achieved by not including the multiplicity of unsaturated expressions to be found in the *Begriffsschrift.* This language has much in common with the language of combinatory logic. My own contention is that natural language is closer in syntax to such a combinatory language than to Frege's *Begriffsschrift*, but the elaboration of that idea is reserved for future papers of mine.

Bibliography

ANSCOMBE, G.E.M. 1959. *An Introduction to Wittgenstein's Tractatus*, London, Hutchinson University Library; reprinted 1963, 1967, 1971.

CARNAP, R. 1931. *The logical syntax of language* (A. Smeaton, translator), London, Kegan Paul, Trench, Trubner & Co. Ltd.

CURRIE, G. 1982. *Frege: an introduction to his philosophy*, Brighton (Sussex), Harvester.

CURRY, H.B., and R. FEYS. 1958. *Combinatory logic,* vol. I, Amstertam, North-Holland.

DUMMETT, M. 1973. *Frege: Philosophy of language,* London, Duckworth, 2nd ed., 1981.

—. 1981. *The interpretation of Frege's philosophy*, London, Duckworth.

FREGE, G. 1884. *Die Grundlagen der Arithmetik: Eine logisch mathematische Untersuchung über den Begriff der Zahl*, Breslau, Koebner. English translation by J.L. Austin, *The foundations of arithmetic, a logico-mathematical Enquiry into the concept of number*, Oxford, Basil Blackwell, 1950; 1953.

—. 1892. *Über Begriff und Gegenstand*, Vierteljahrsschrift für wissenschaftliche Philosophie **16**, 192–205. English translation as "On Concept and Object" in [*1980*, 42-55].

—. 1893. *Grundgesetze der Arithmetik: Begriffsschrifftlich abgeleitet*, Band 1, Jena, Pohle.

—. 1979. *Comments on sense and Meaning*, in *Posthumous writings*, (P. Long and R. White, translators), (Oxford, Basil Blackwell), 18–125.

—. 1980. *Philosophical and mathematical correspondence* (B. McGuinness, editor; H. Kaal, translator), Oxford, Basil Blackwell

GEACH, P.T. 1951. *On what there is*, Proceedings of the Aristotelian Society, Supplementary vol. XXV, 125–134.

—. 1975. *Names and identity,* in S. Guttenplan (editor), *Mind and language: Wolfson College Lectures 1974* (Oxford, Oxford University Press), 139–158.

—. 1976. *Saying and showing in Frege and Wittgenstein*, Acta Philosophica Fennica **28**, 54–70.

KLEMKE, E.D. 1968. (editor), *Essays on Frege*, Urbana, University of Illinois Press.

LONG, P. 1982. *Formal relations*, The Philosophical Quarterly **32**, 151–161.

—. 1984. *Universals: logic and metaphor*, in I. Dilman (editor), *Philosophy and life: essays on John Wisdom* (The Hague, Martinus Nijhof), 271–289.

QUINE, W.V.O. 1937. *New foundations for mathematical logic*, Amcrican Mathematical Monthly **44**, 70–80.

—. 1960. *Word and object*, Cambridge, Mass., The MIT Press.